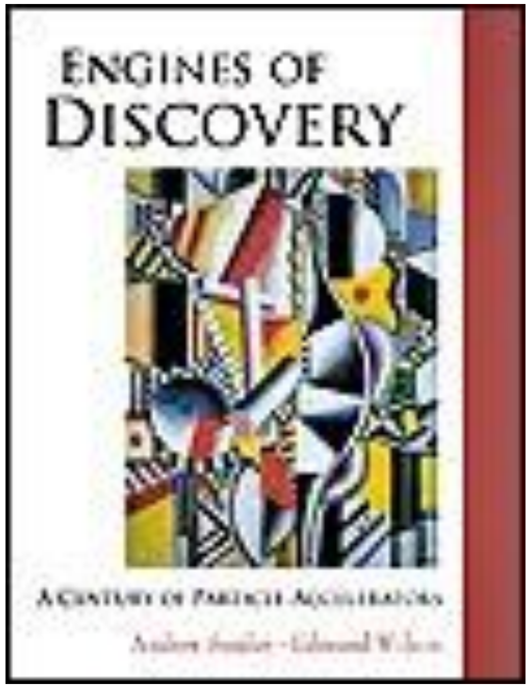
Accelerators for the future

Ted Wilson

Rutherford, Lawrence , Cockcroft, Walton, Alvarez, Oliphant, MacMillan, Richter, R.R Wilson, Wiik and Rubbia were physicists who, inspired by the goals of their own research, were stimulated to invent new acceleration techniques to achieve their ends. Modern physics has perhaps become too sophisticated now for individual imaginations to encompass both these creative activities. However, today's particle physicists will soon have to decide in which direction their research should lead them and make judgements on how they may be best served by future accelerators - whether future colliders should be linear or circular, whether energy or intensity is of greater importance - which particles: electrons protons, muons or neutrinos are of greatest interest. The choice must depend on the practical and affordable limits to each kind of machine. This talk will provide enough insight into the theory and design of accelerators to enable particle physicists to appreciate the limits to one of these kinds of machine: the circular collider.
Starting from the parameters which determine the luminosity and physics performance of the LHC: energy, intensity, bunch structure and transverse beam size, we will explore the basic theory of phase stability, betatron motion, instabilities and beam-beam forces behind each of these parameters.

Links



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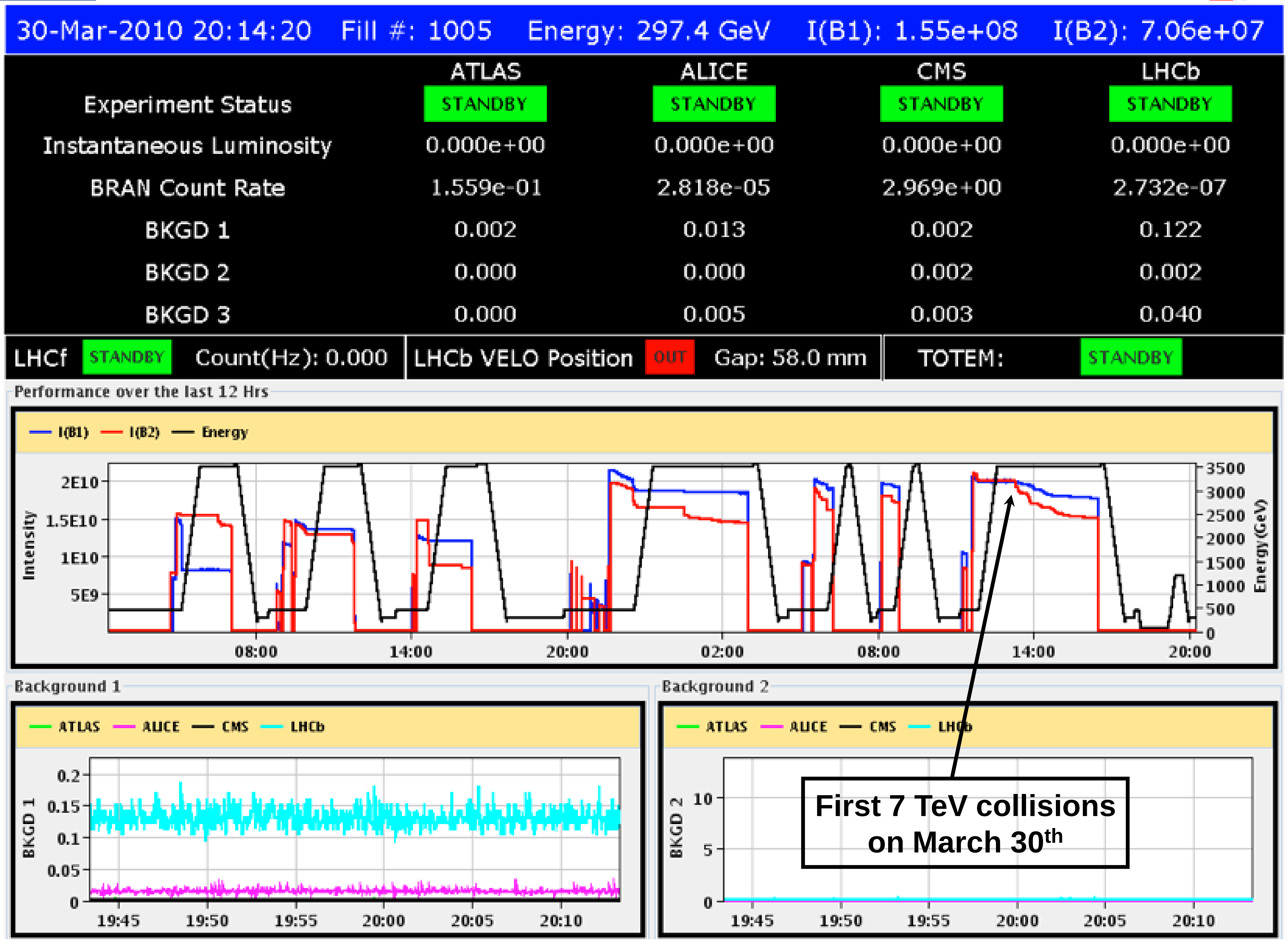
“Engines of Discovery”:

http://www.worldscibooks.com/physics/6272.html

http://www.enginesofdiscovery.com

“Particle Accelerators”

http://www.oup.com/uk/catalogue/?ci=9780198508298



First 7 TeV collisions on March 30th

LHC design parameters

Nominal LHC parameters	
Beam injection energy (TeV)	0.45
Beam energy (TeV)	7.0
Number of particles per bunch	1.15×10^{11}
Number of bunches per beam	2808
Max stored beam energy (MJ)	362
Norm transverse emittance (μm rad)	3.75
Colliding beam size (μm)	16
Bunch length at 7 TeV (cm)	7.55

- These are the key parameters of a collider – the LHC

- Why are they important for physics?

- What is the basic theory which limits each one of them?

What do experiments want?

High energy

B = bending field
ρ = bending radius
p = momentum
e = charge

$$\boxed{B\rho = \frac{p}{\mathrm{e}}}$$

Determined by the maximum field of bending dipoles, B

High luminosity

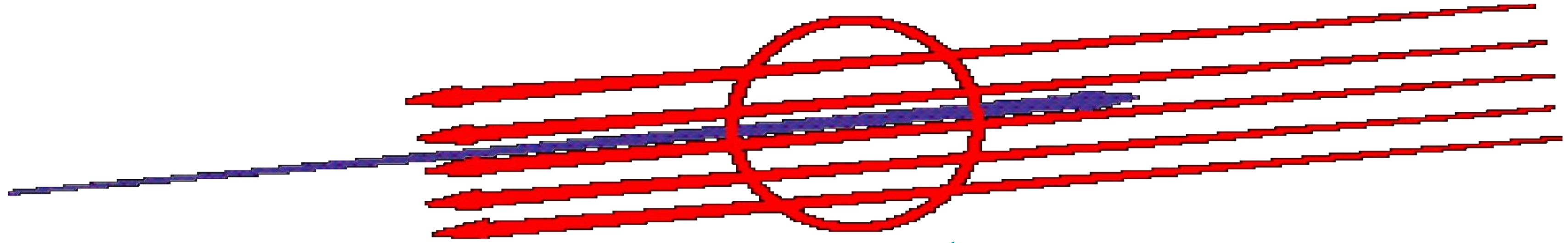
$$\boxed{\mathcal{L} = \frac{N^2 n_b f_{\mathrm{rev}}}{4\pi\sigma_x\sigma_y} F}$$

N = bunch population
n_b = number of bunches
f_{rev} = revolution frequency
$\sigma_{x,y}$ = colliding beam sizes
F = geometric factor

Depends on machine parameters: charge per bunch (N), num. of bunches (n_b) and transverse beam sizes (σ)

"Thus, to achieve high luminosity, all one has to do is make (lots of) high population bunches of low emittance to collide at high frequency at locations where the beam optics provides as low values of the amplitude functions as possible."

Luminosity (single bunches)



- Imagine a blue particle colliding with a beam of cross section area - A
- Probability of collision for an interaction is $\dfrac{\sigma}{A} \cdot N$
- For N particles in both beams $\dfrac{\sigma}{A} \cdot N^2$
- Suppose they meet *f* times per second at the revolution frequency

$$f_{rev} = \frac{\beta c}{2\pi R}$$

- Event rate

$$\boxed{\frac{f_{rev} N^2}{A}} \cdot \sigma$$

Make big

e.g. 10^{-25}

Make small

LUMINOSITY

$$\boxed{\approx 10^{30} \text{ to } 10^{34} \left[\mathrm{cm}^{-2}\ \mathrm{s}^{-1}\right]}$$

Luminosity is constrained by beam-beam forces.

With *k* bunches per beam and a round beam

$$L = \frac{N^2 fk}{4\pi\gamma\sigma^2}$$

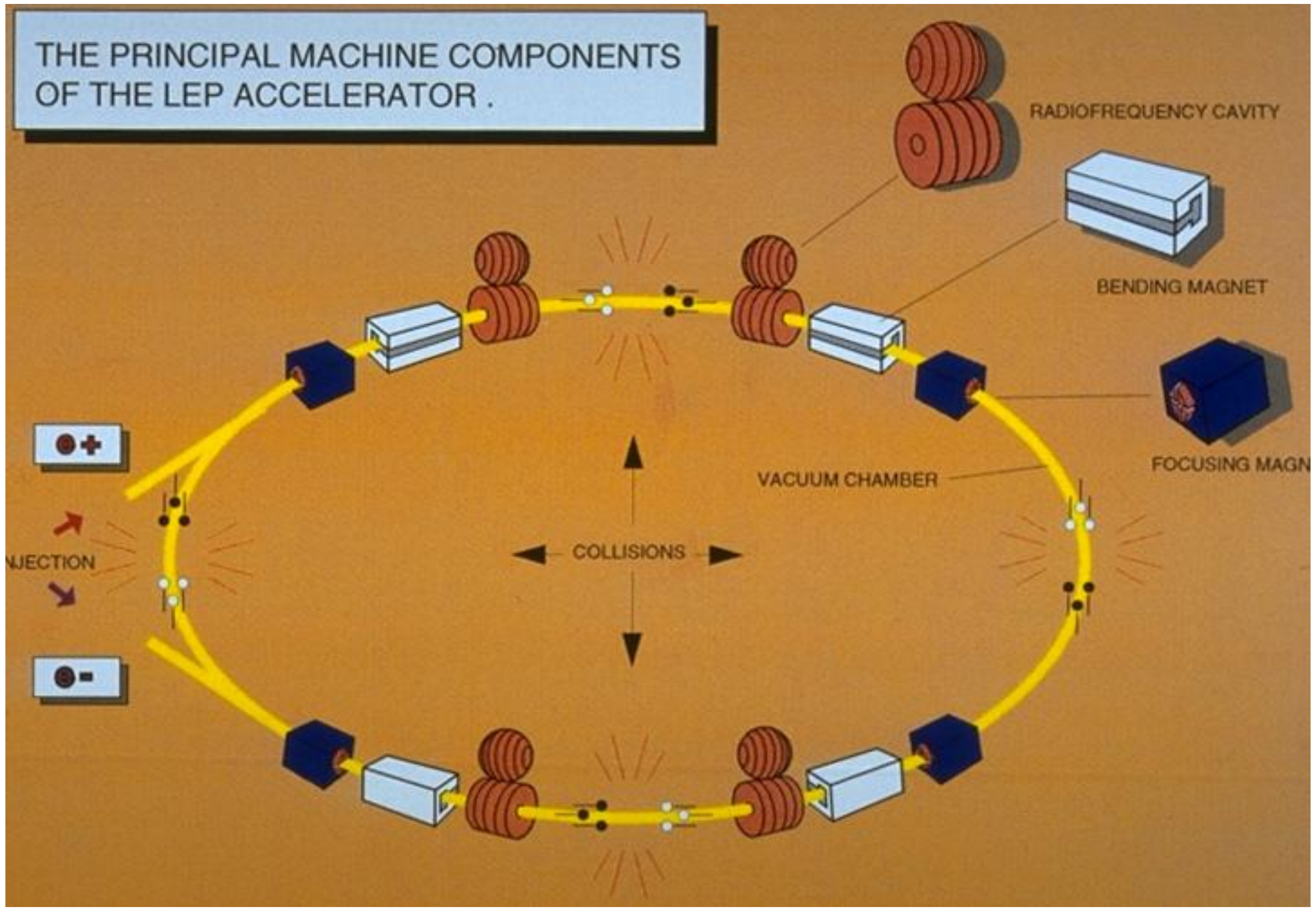
There is also a limit to performance called the linear beam-beam limit:

$$\Delta Q = \frac{\beta^* N r_0}{4\pi\gamma\sigma^2}$$

They have common parameters

$$\text{I must now digress to explain } \beta^* \text{ and } Q$$

Basic concepts



Charged particles are accelerated, guided and confined by **electromagnetic fields**.

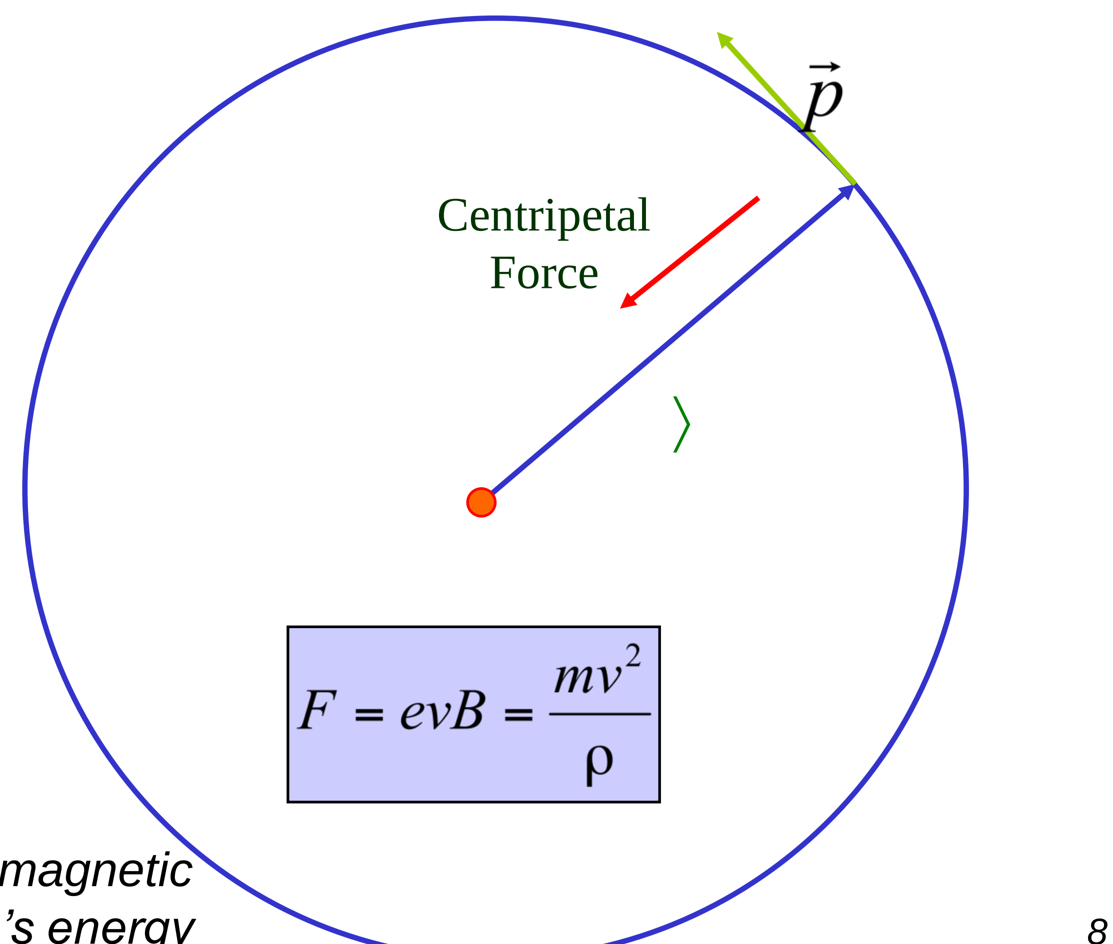
- Bending: Dipole magnets
- Focusing: Quadrupole magnets
- Acceleration: RF cavities

In synchrotrons, they are ramped together synchronously to match beam energy.

- Chromatic aberration: Sextupole magnets

Lorentz force $$\boxed{\vec{F} = e(\vec{v} \times \vec{B} + \vec{E})}$$

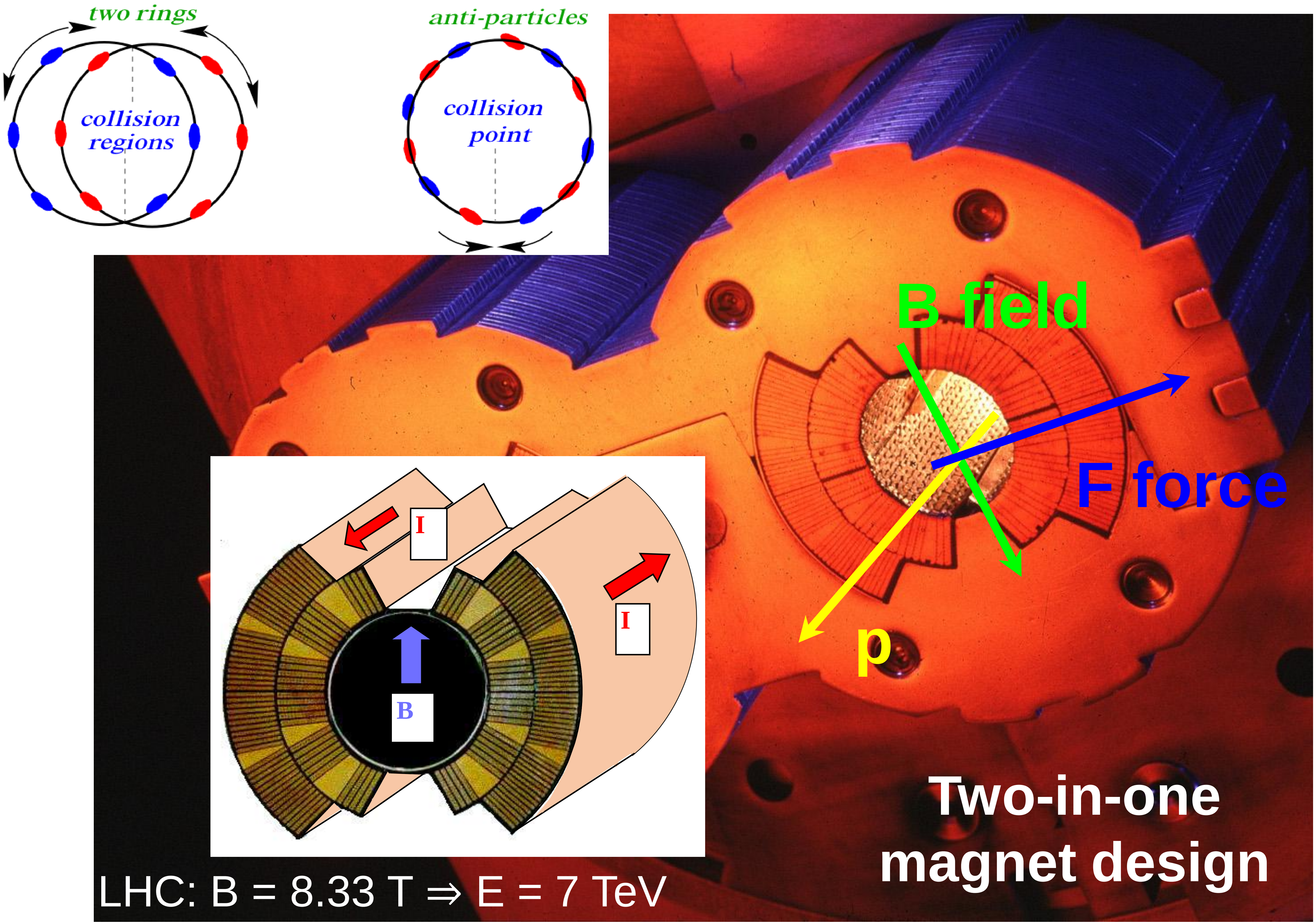
Magnetic rigidity $$\boxed{B\rho = \frac{mv}{e} = \frac{p}{e}}$$



LHC: ρ = 2.8 km given by LEP tunnel!

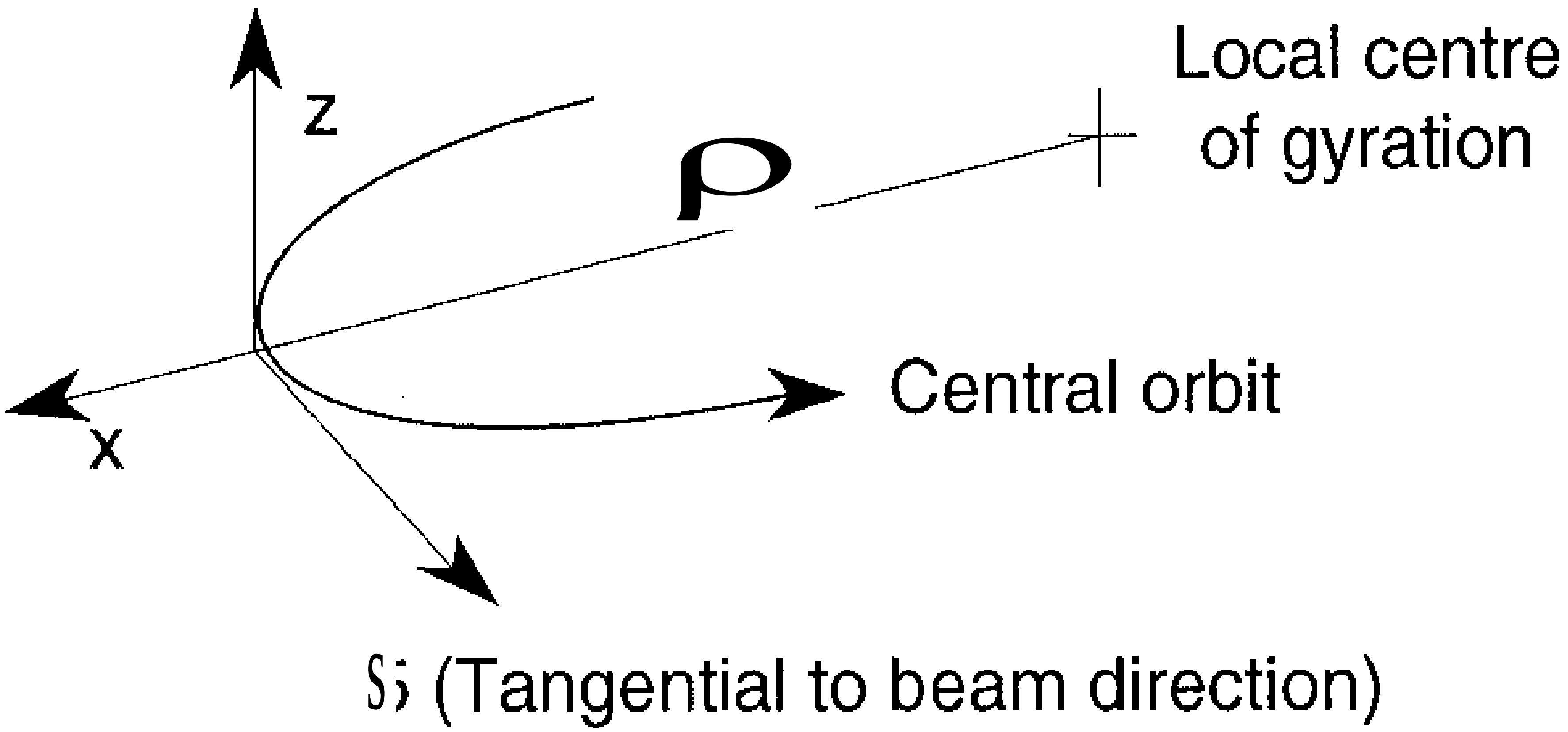
Fixes the relation between magnetic field and particle's energy

Bending

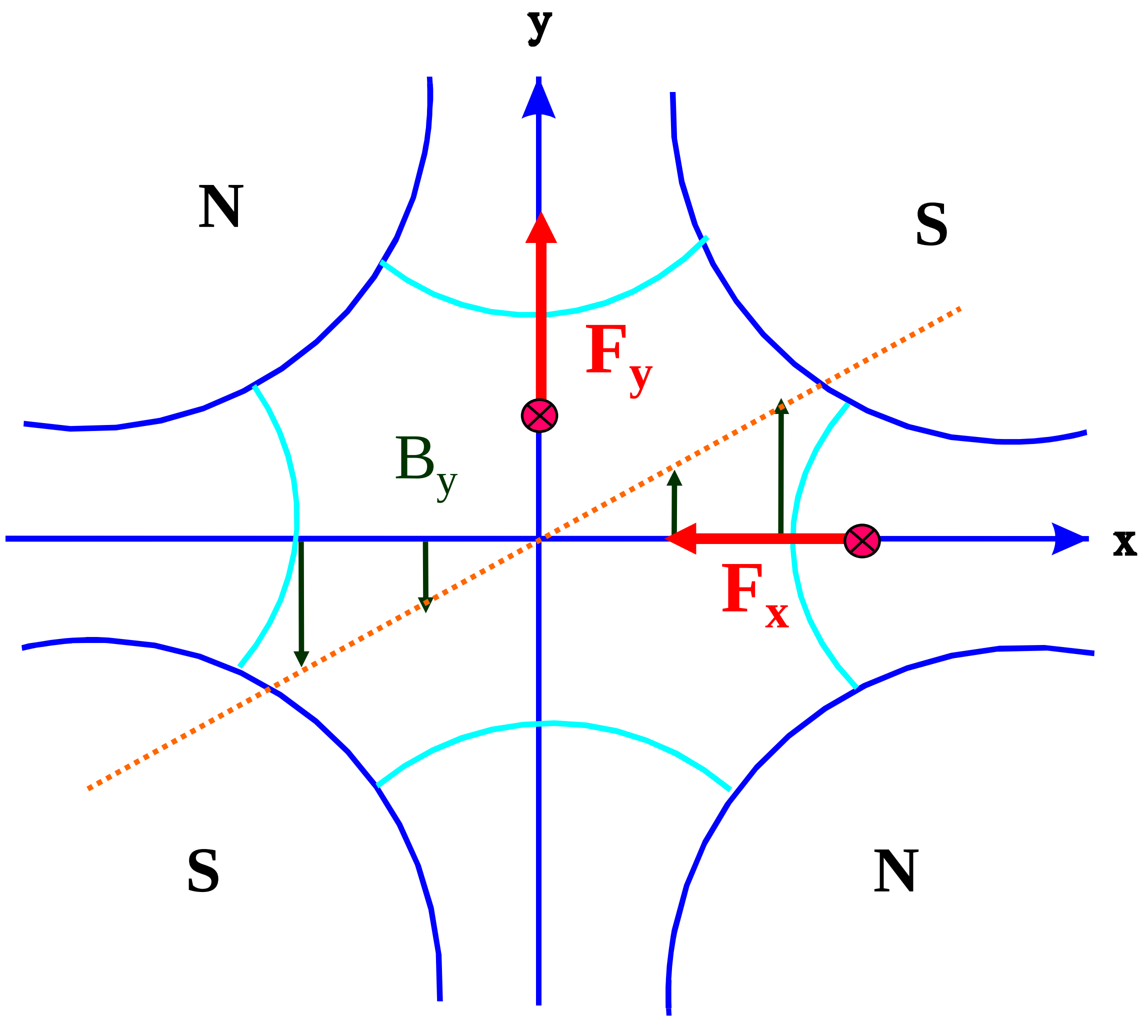


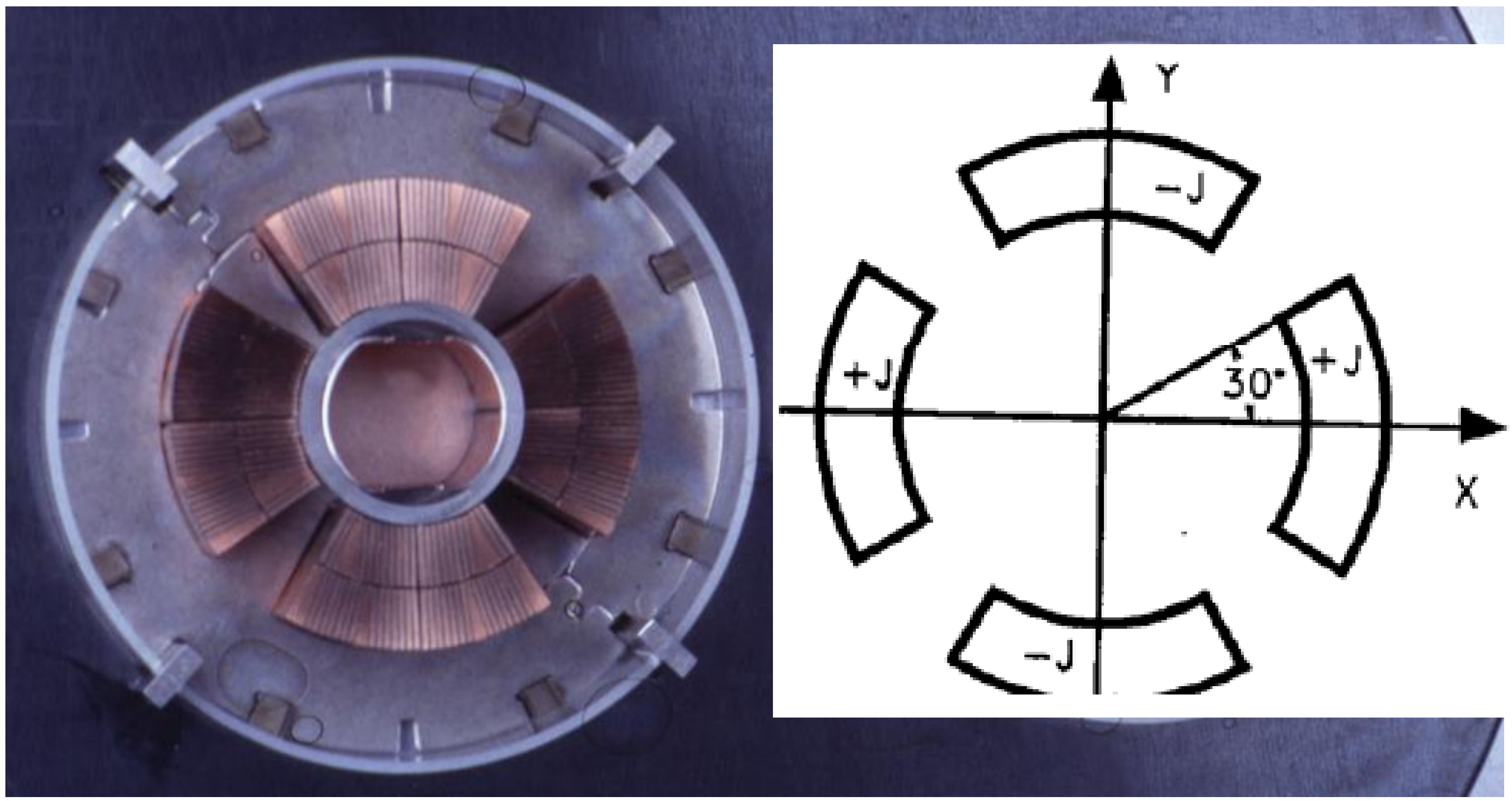
two rings
collision regions
anti-particles
collision point
B field
F force
p
I
I
B
Two-in-one magnet design
LHC: B = 8.33 T ⇒ E = 7 TeV

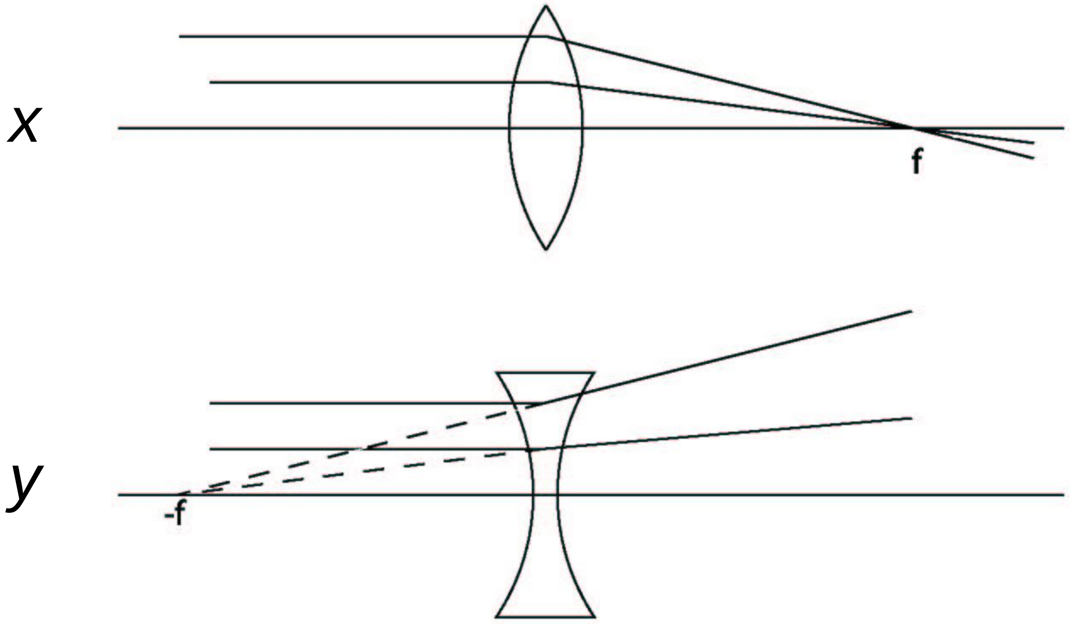
Transverse coordinates



Focusing





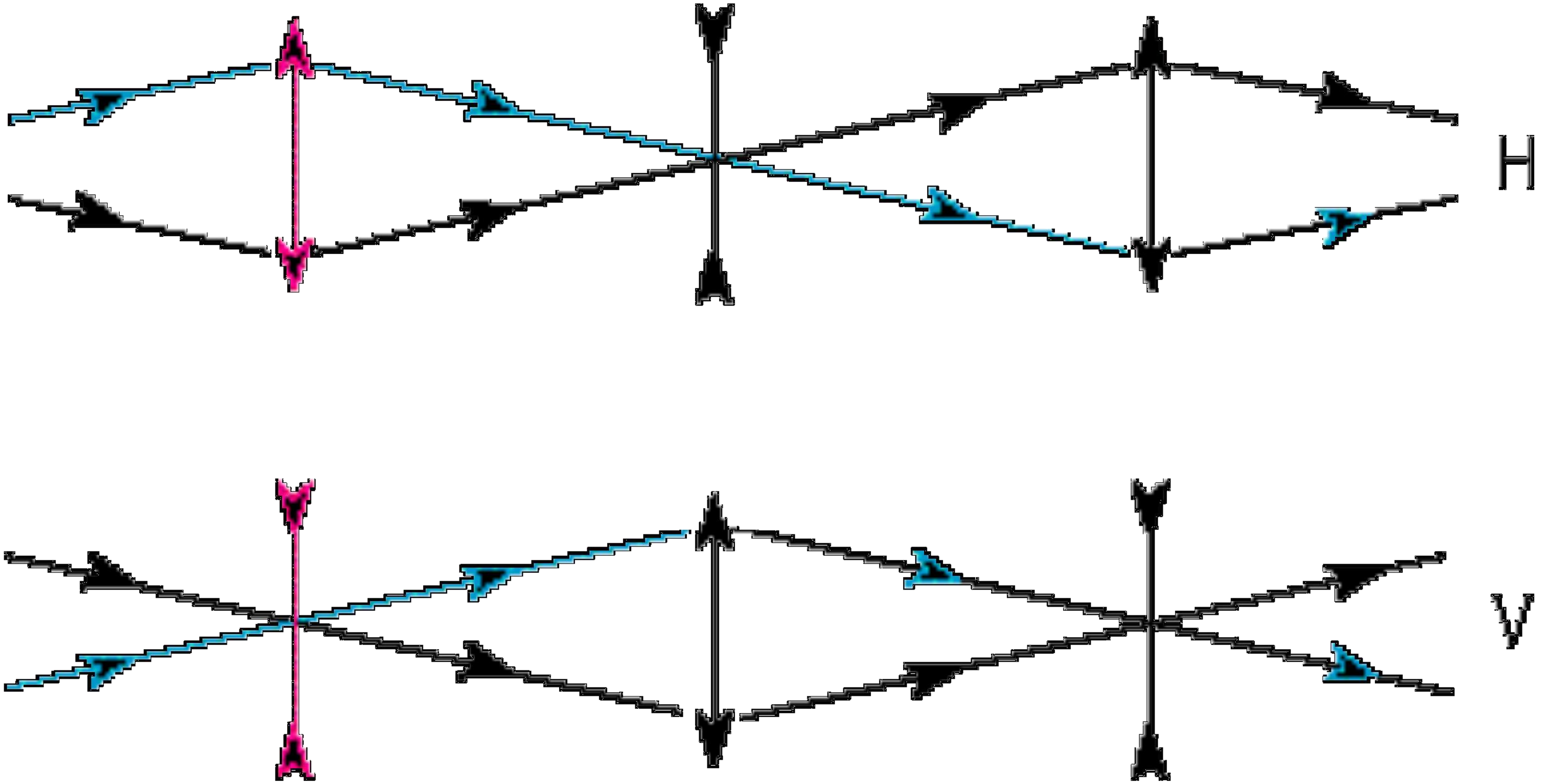


Transverse focusing is achieved with **quadrupole magnets**, which act on the beam like an optical lens.

Linear increase of the magnetic field along the axes (no effect on particles on axis).

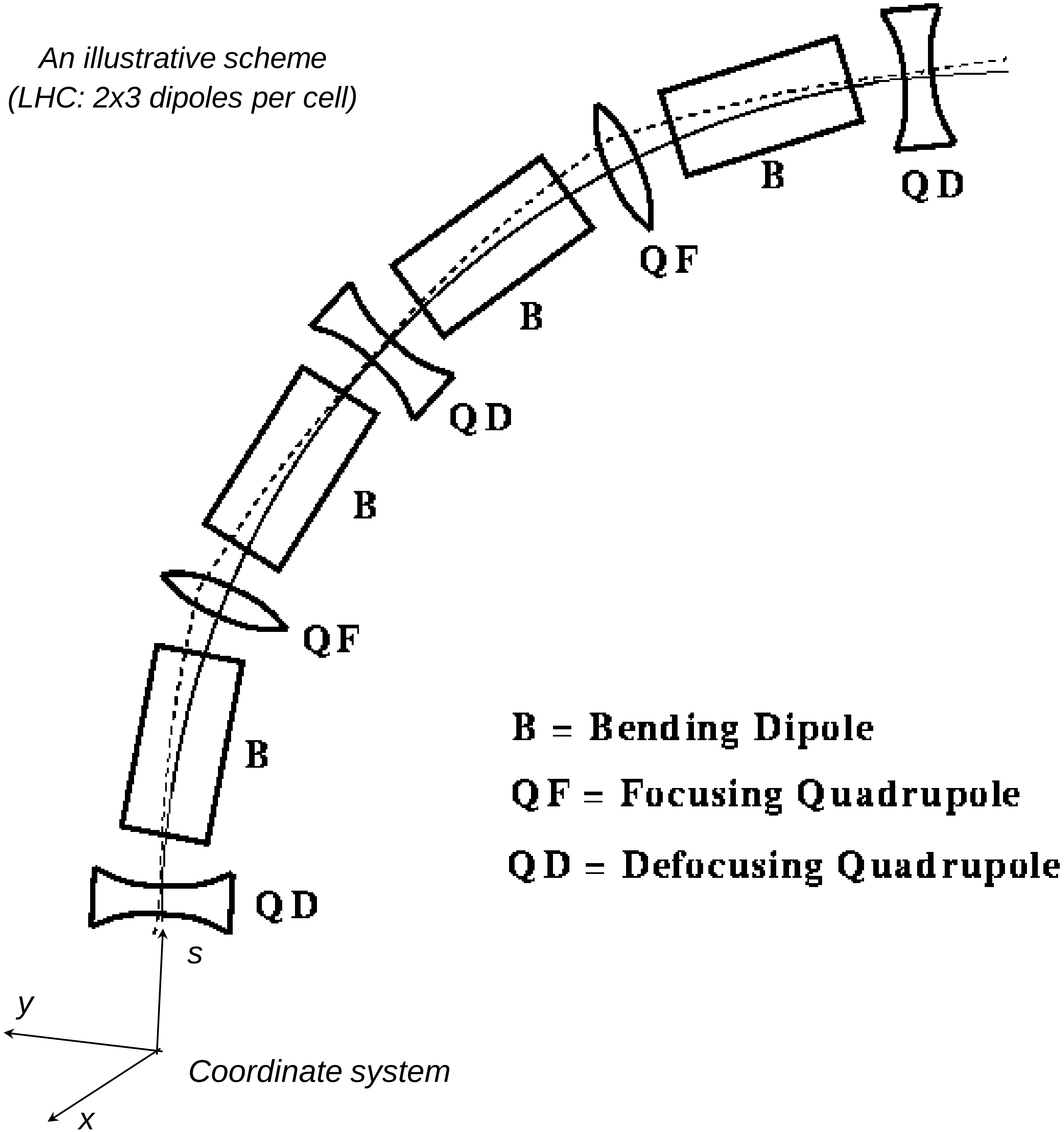
Focusing in one plane, **de-focusing** in the other!

Alternating gradients



Alternating gradient lattice

An illustrative scheme
(LHC: 2x3 dipoles per cell)



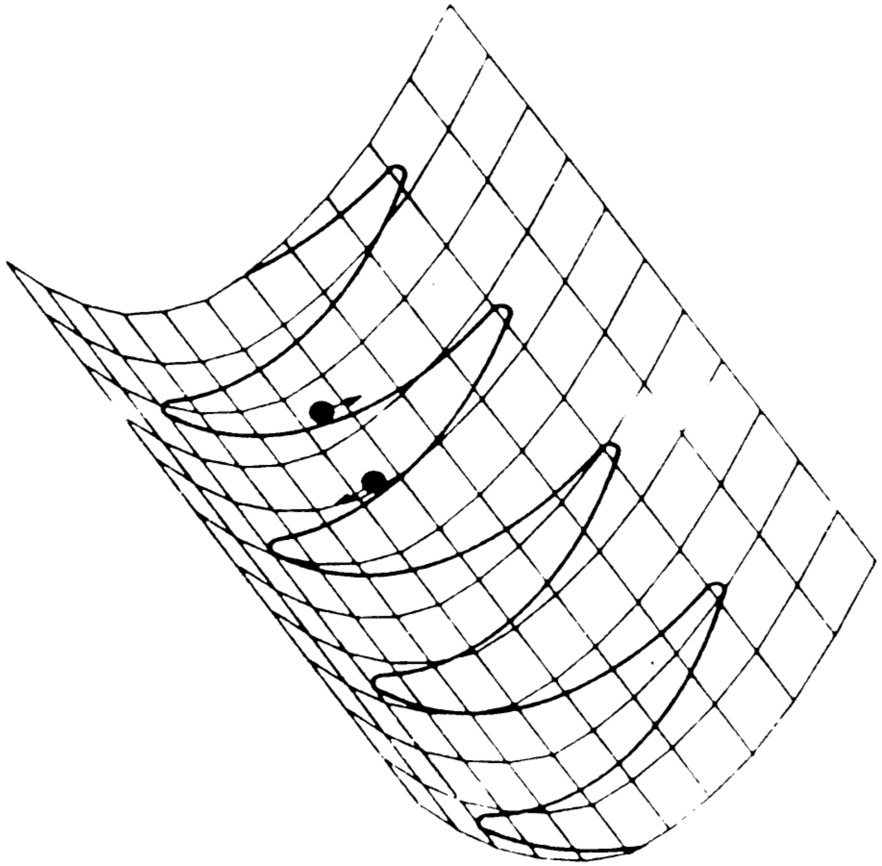
Coordinate system

One can find an arrangement of quadrupole magnets that provides net focusing in both planes (“strong focusing”).

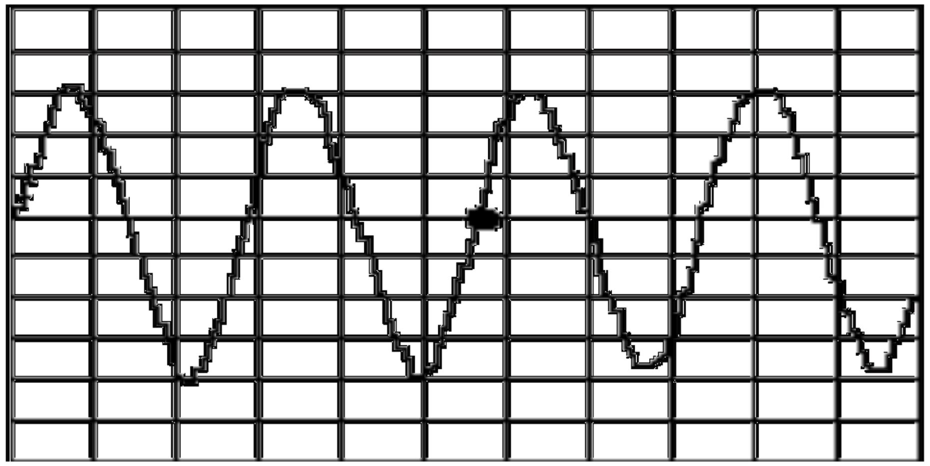
Dipole magnets keep the particles on the circular orbit.

Quadrupole magnets focus alternatively in both planes.

Focusing, Gutter ad Hamiltonian



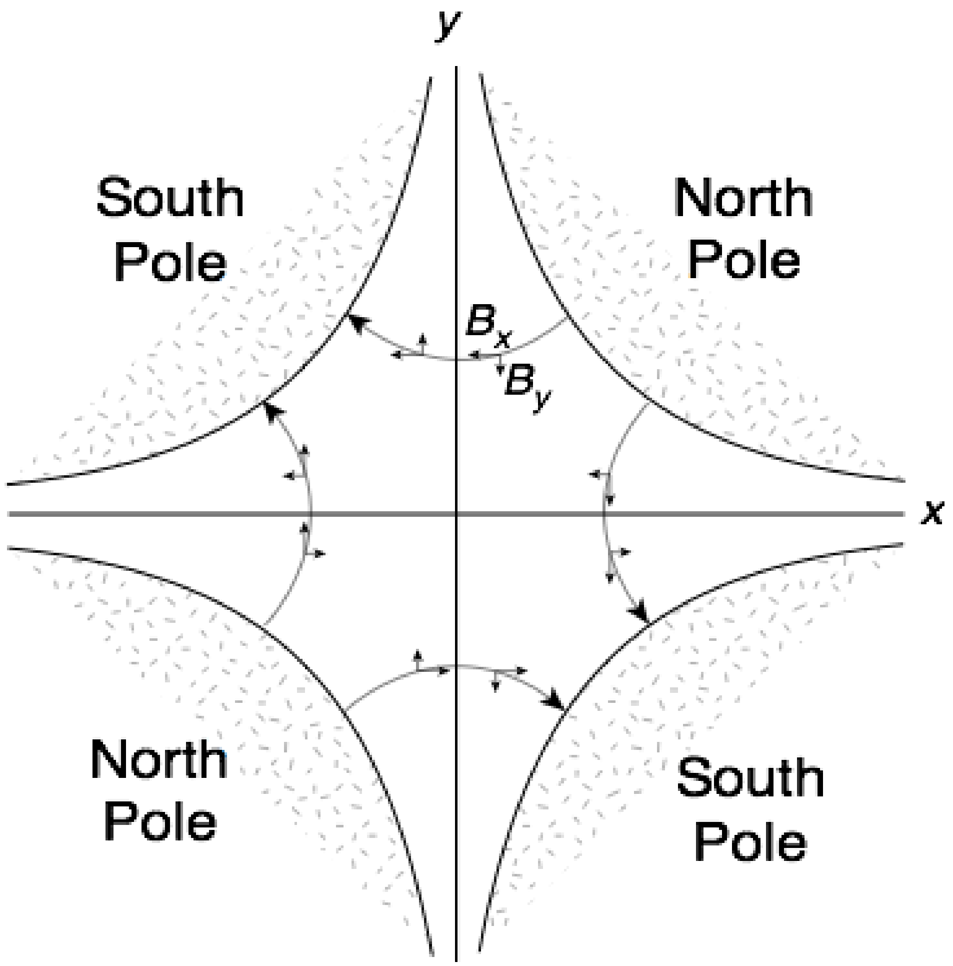
$$\mathcal{H} = \frac{(x')^2}{2} - \frac{A_s}{(B\rho)}$$



$$\mathcal{H} = \frac{(x')^2}{2} + \frac{k(s)x^2}{2}$$

But what you may ask is B-rho

Transverse equation of motion



Magnetic field [T] : $$B_y = \frac{\partial B_y}{\partial x} \times x$$

Field gradient [T m^{-1}] : $$g = \frac{\partial B_y}{\partial x}$$

Normalized grad. [m^{-2}] : $$K = \frac{g}{p_0/e} = \frac{1}{f}$$

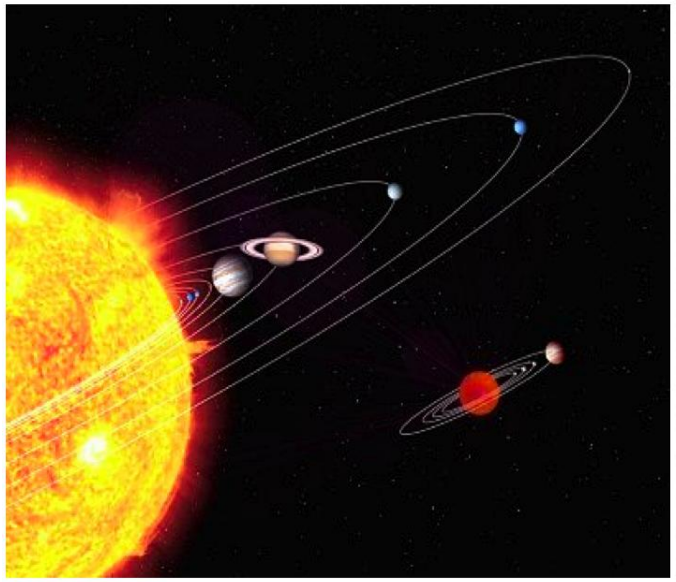
$$\boxed{x'' + K(s)x = 0}$$

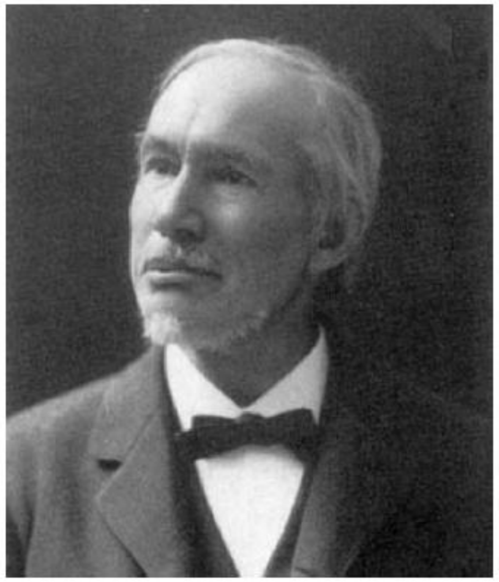
Hill's equation

K(s) describes the distribution of focusing strength along the lattice.

and its solution

$$x(s) = \sqrt{\varepsilon_x \beta_x} \cos[\phi(s) + \phi_0]$$

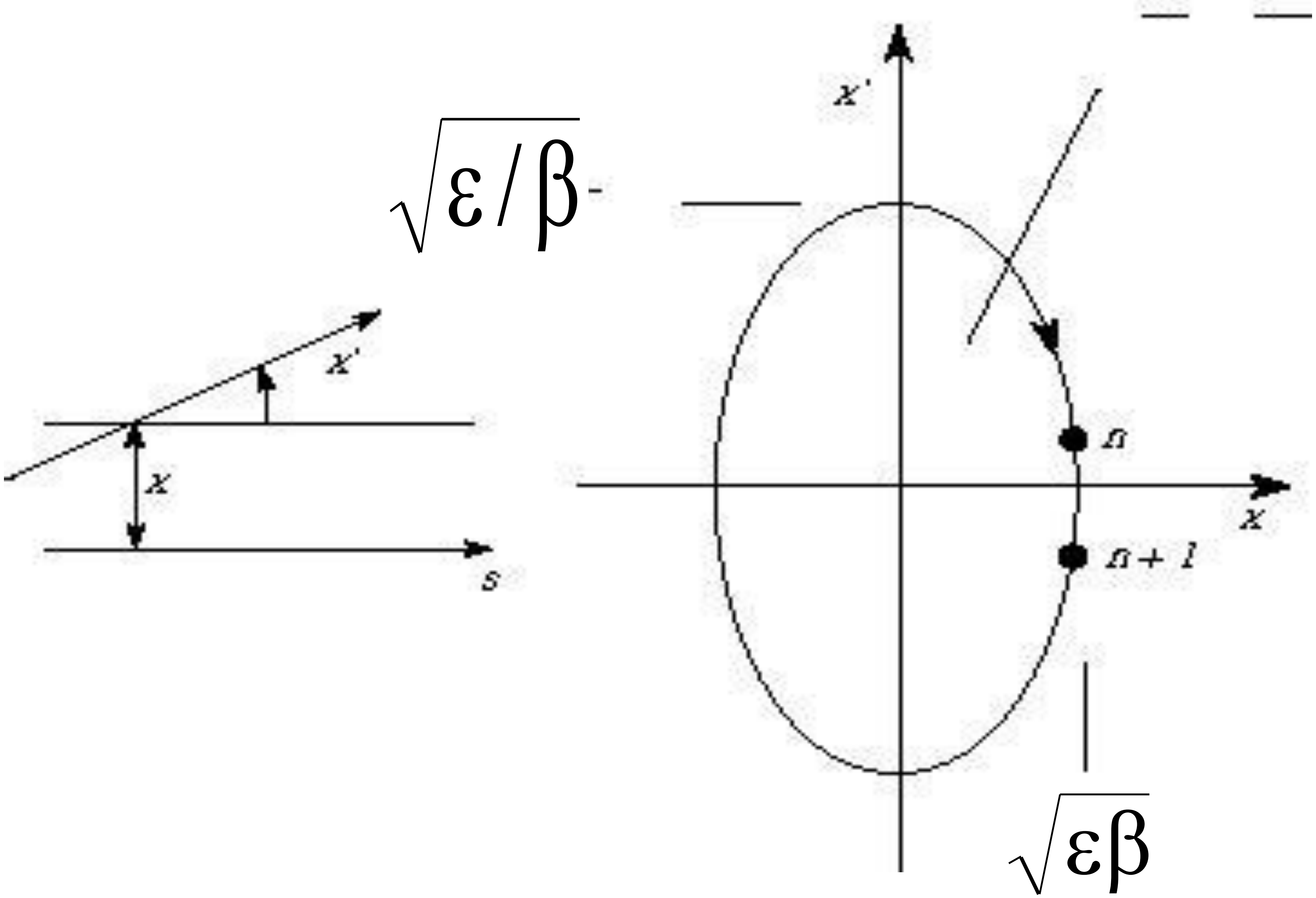




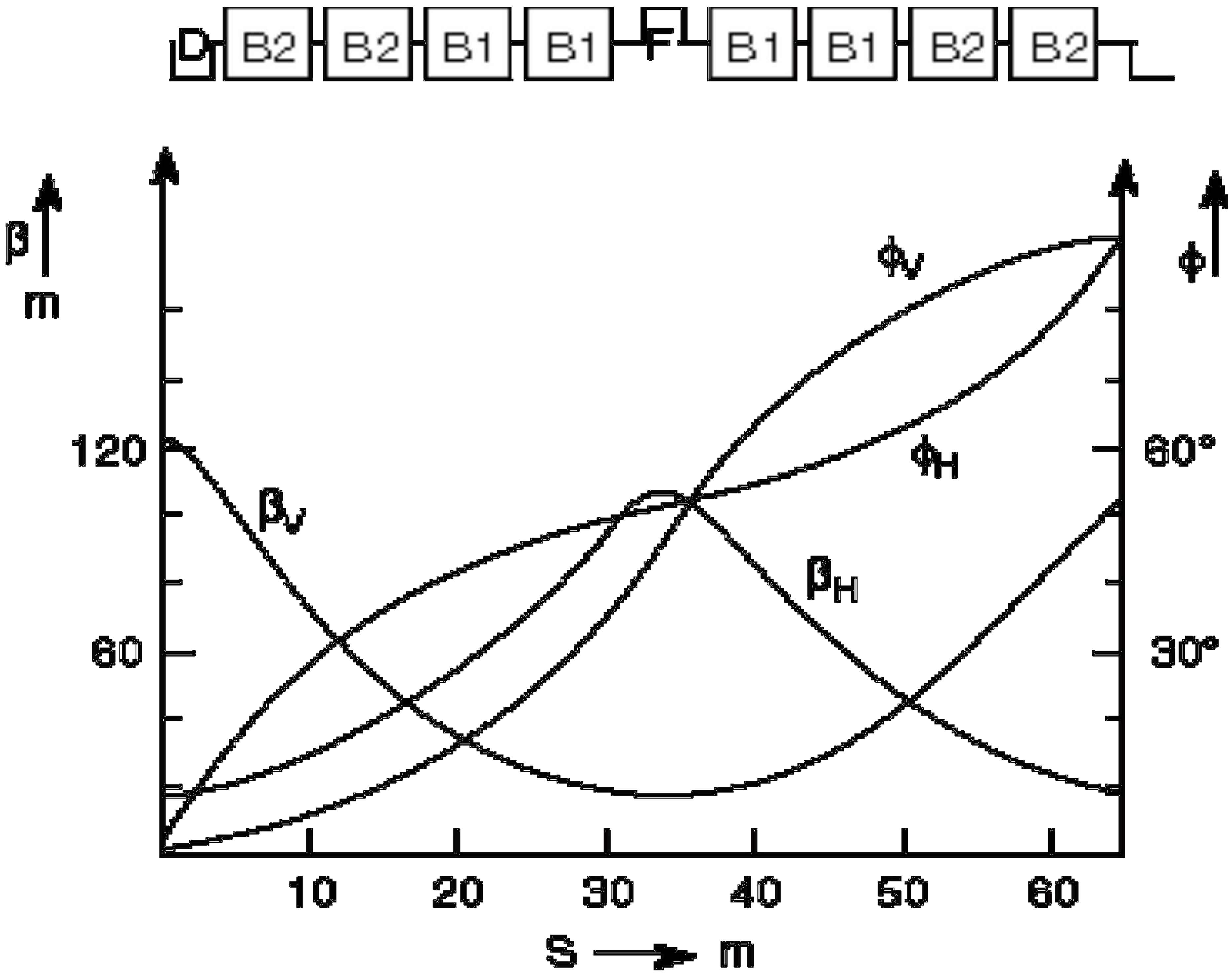
G. Hill, 1838-1914

Transverse ellipse

$$\text{Area} = \pi\sqrt{\varepsilon\beta}.\sqrt{\varepsilon/\beta} = \pi\varepsilon$$

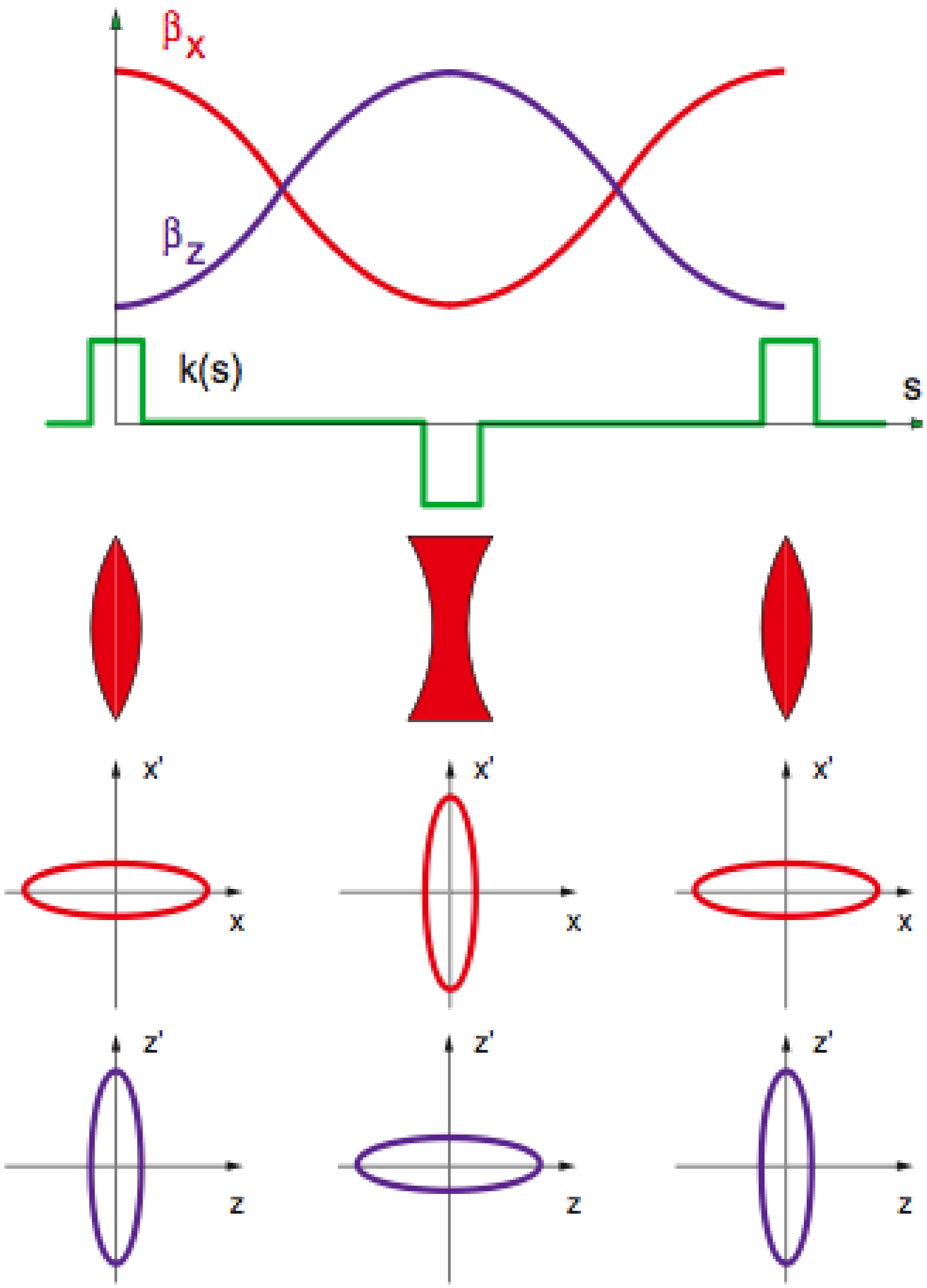


The lattice

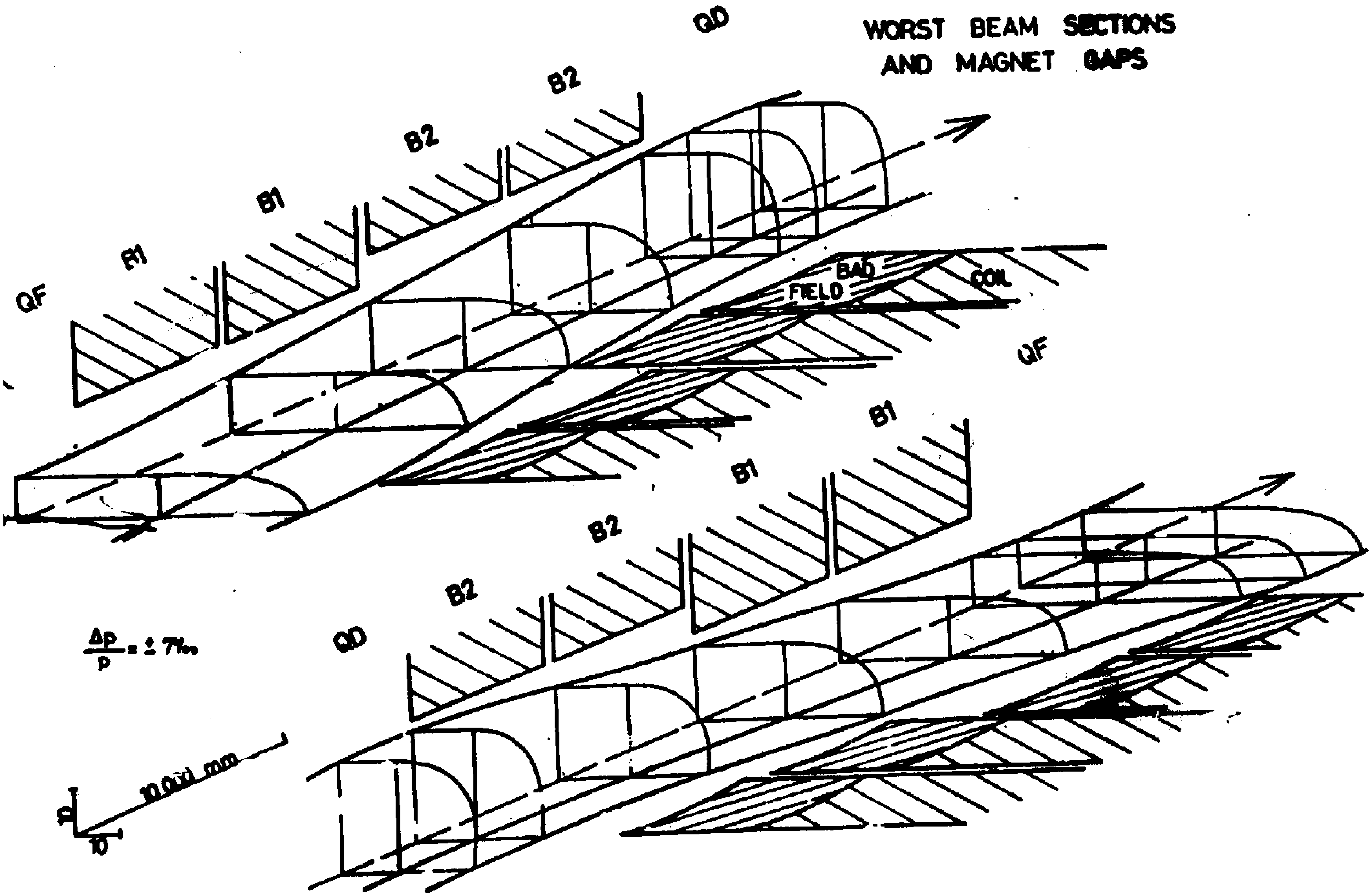


D
B2
B2
B1
B1
F
B1
B1
B2
B2
β
m
120
60
β_V
β_H
ϕ_V
ϕ_H
ϕ
60°
30°
10
20
30
40
50
60
S
m

Emittance and beam size (ii)



Beam sections



after,pct

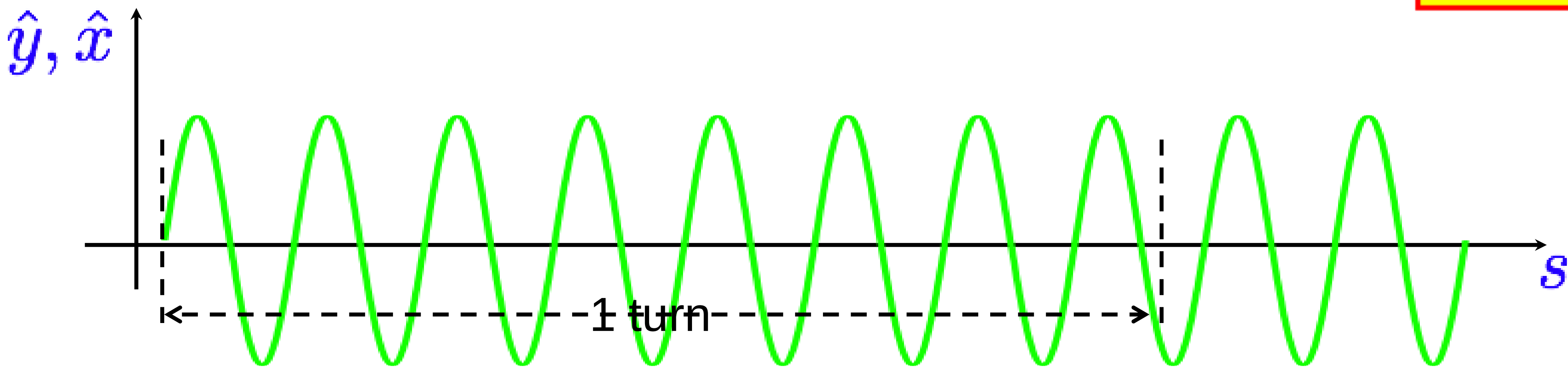
Betatron tune

Betatron phase advance over 1 turn:

$$\mu = \oint \frac{ds}{\beta(s)}$$

Betatron tune:

$$\boxed{Q \equiv \frac{1}{2\pi} \oint \frac{ds}{\beta(s)}}$$

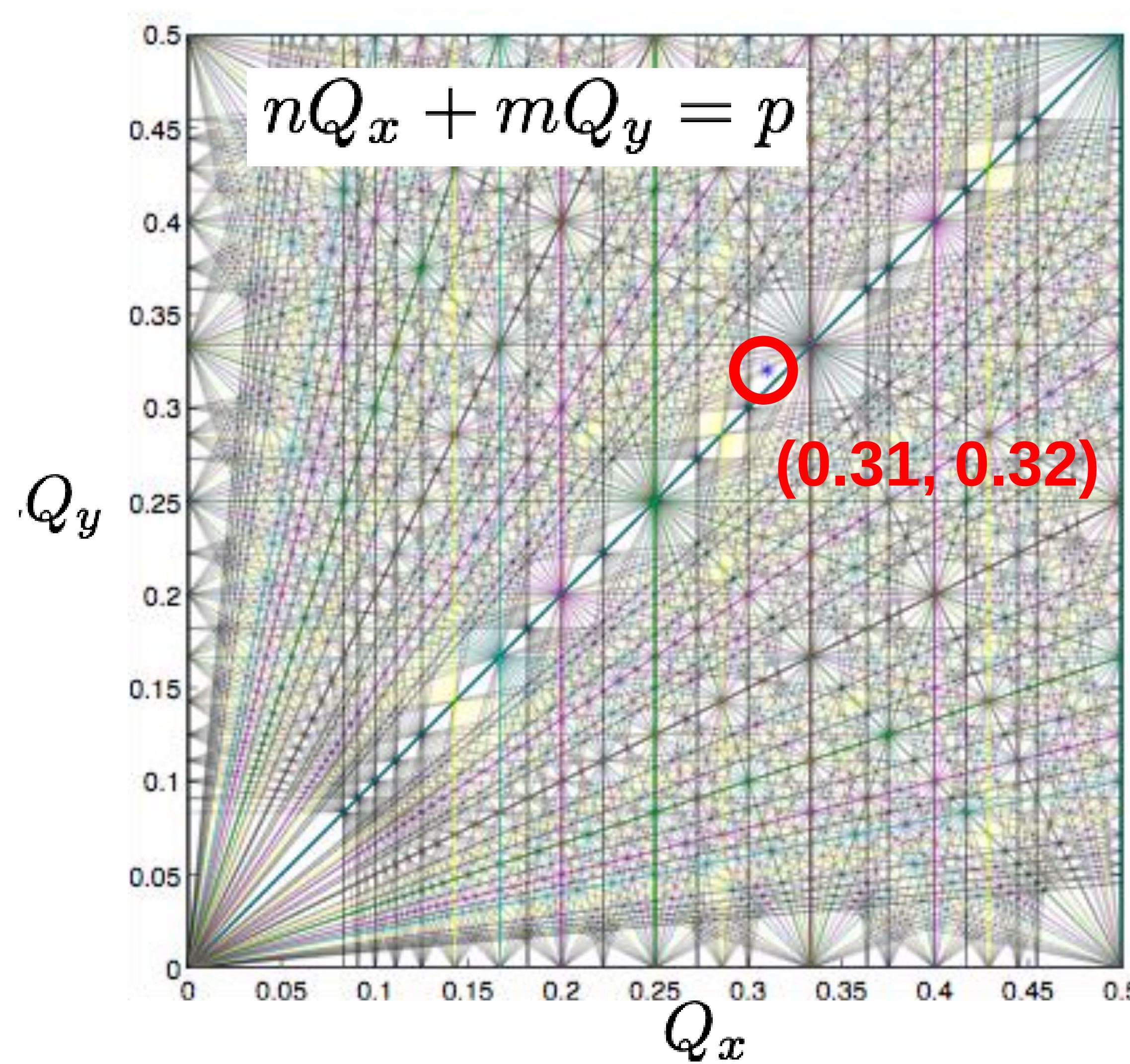


The tune is the **number of betatron oscillations per turn**.

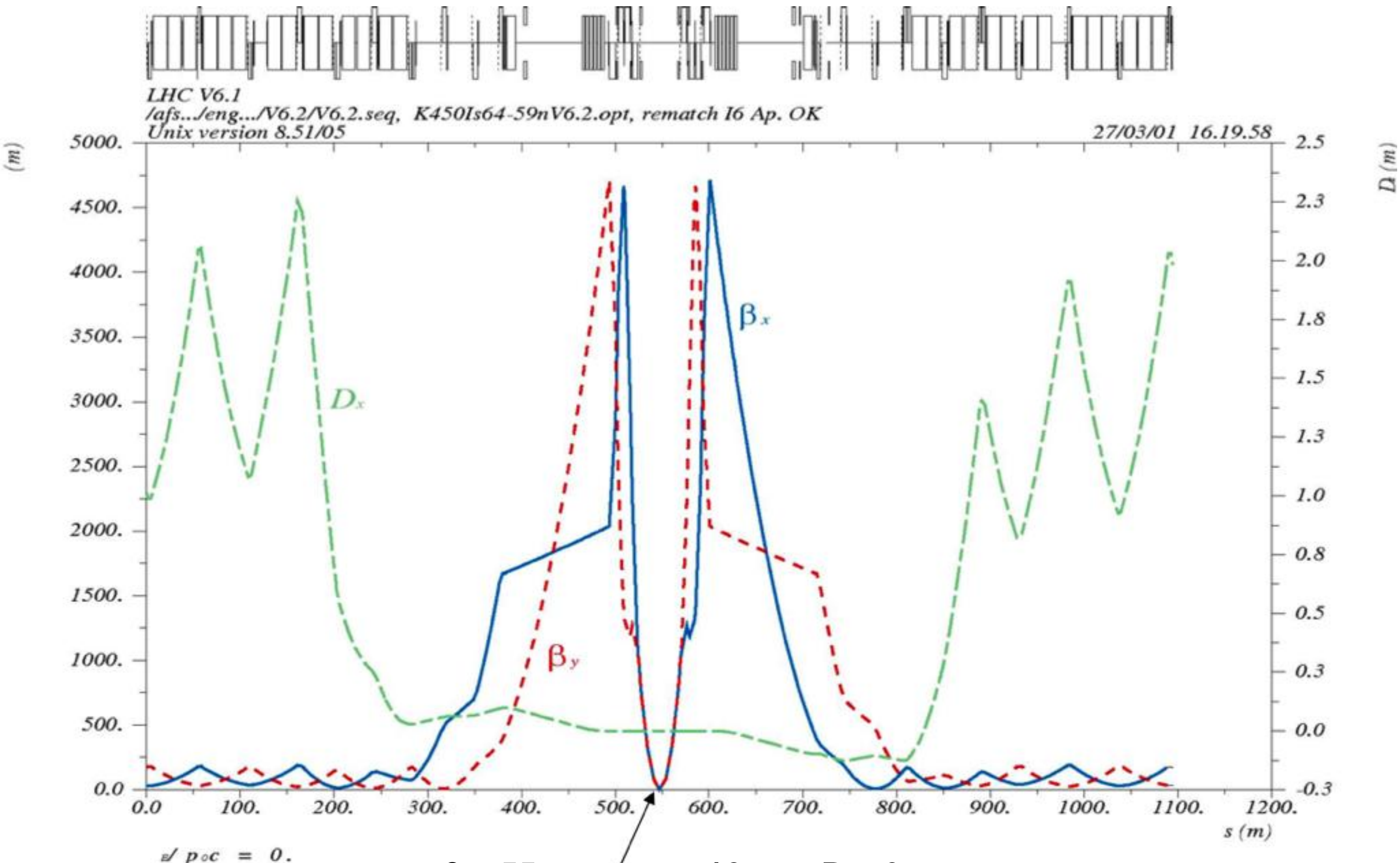
We *normally* only care about the **fractional part** of the tune! 64.31 is 0.31!

The operating tune values (**working point**) must be chosen to avoid resonance.

The tune values must be controlled to within better than 10^{-3}, during all machine phases (ramp, squeeze, ...)

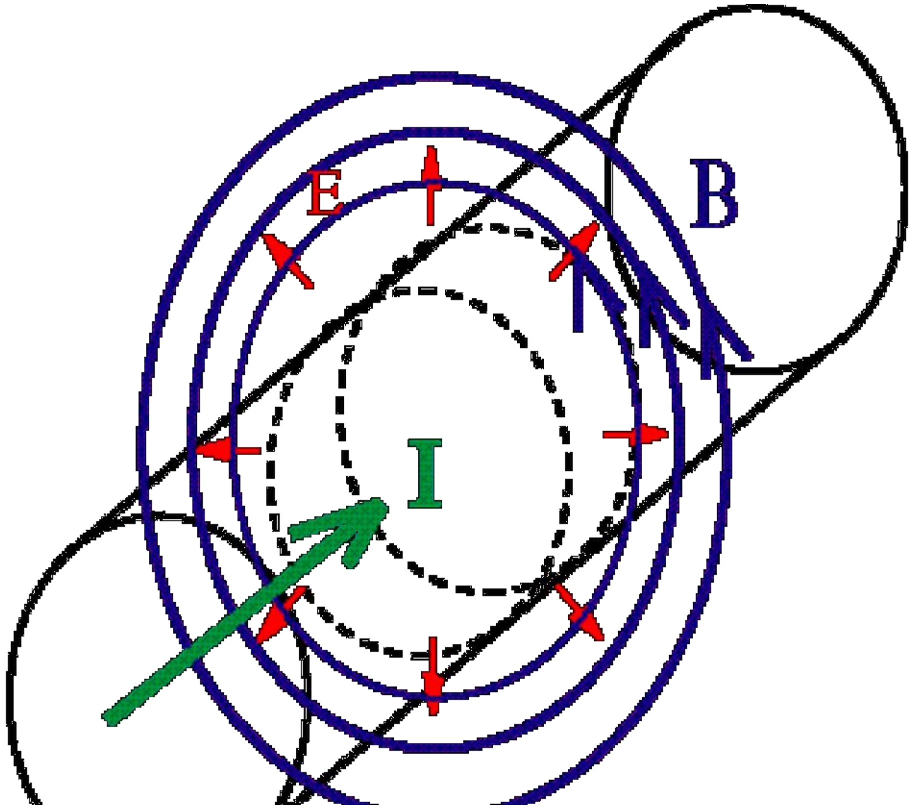


Beta functions for IP1 and IP5



β = 55 cm → σ = 16 μm; D = 0m

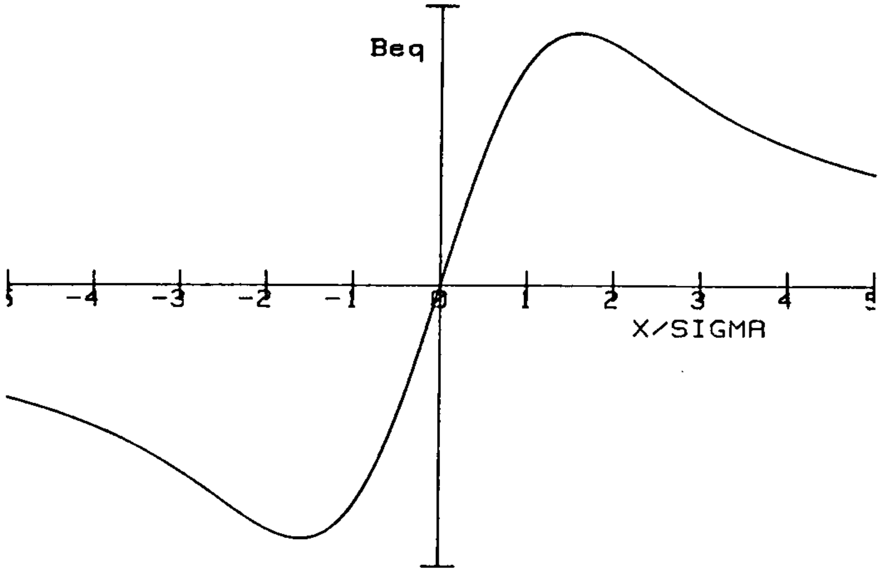
Beam Beam Forces



$$\Delta Q = \frac{\beta^* N r_0}{4\pi\gamma\sigma^2} \le 0.01$$

$$L = \frac{N^2 fk}{4\pi\gamma\sigma^2}$$

ΔQ



Juggling produces

$$L = \frac{N^2 fk\gamma}{r_0\beta^*}\Delta Q$$

End of digression to explain transverse motion

LHC design parameters

Nominal LHC parameters	
Beam injection energy (TeV)	0.45
Beam energy (TeV)	7.0
Number of particles per bunch	1.15×10^{11}
Number of bunches per beam	2808
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Colliding beam size (μm)	16
Bunch length at 7 TeV (cm)	7.55

- These are the key parameters of a collider – the LHC

- Why are they important for physics?

- What is the basic theory which limits each one of them?

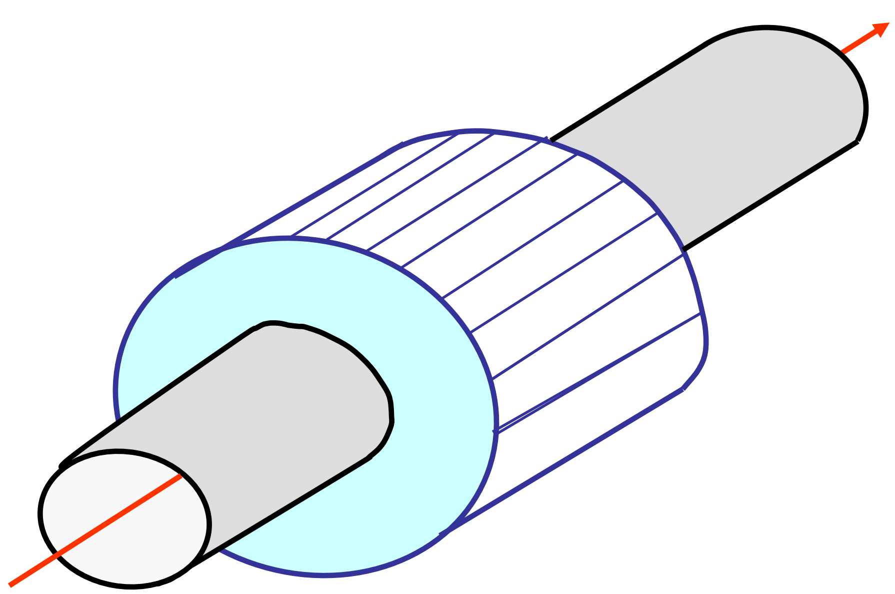
Acceleration

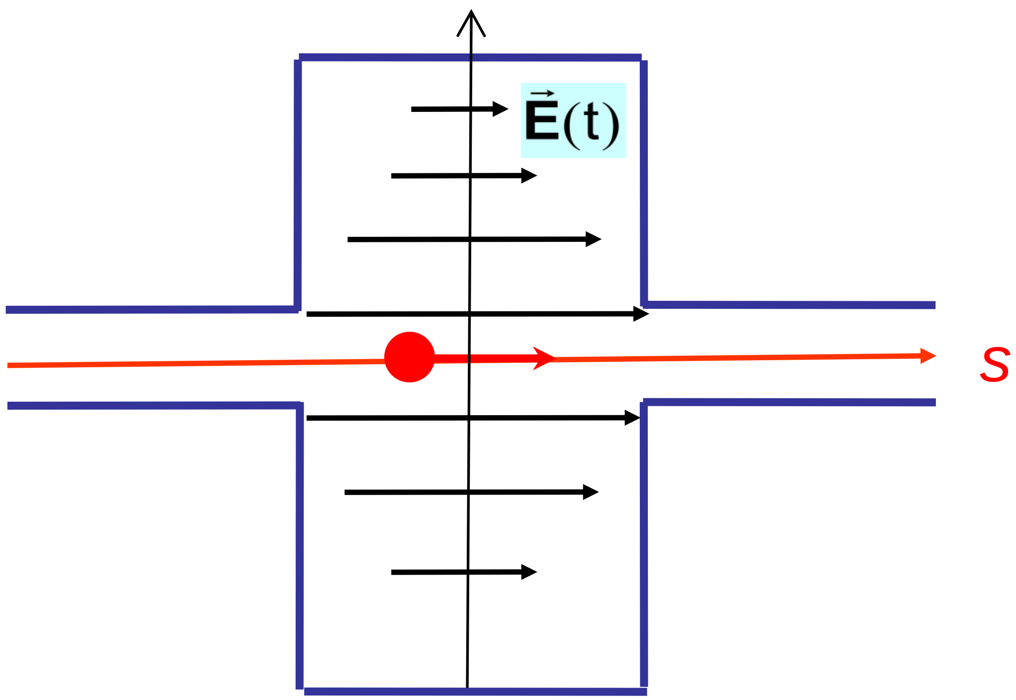
Acceleration is performed with electric fields fed into **Radio-Frequency (RF) cavities**.
RF cavities are basically resonators tuned to a selected frequency.

In circular accelerators, the acceleration is done with small steps at each turn.

LHC: 8 RF cavities per beam (400 MHz), located in point 4

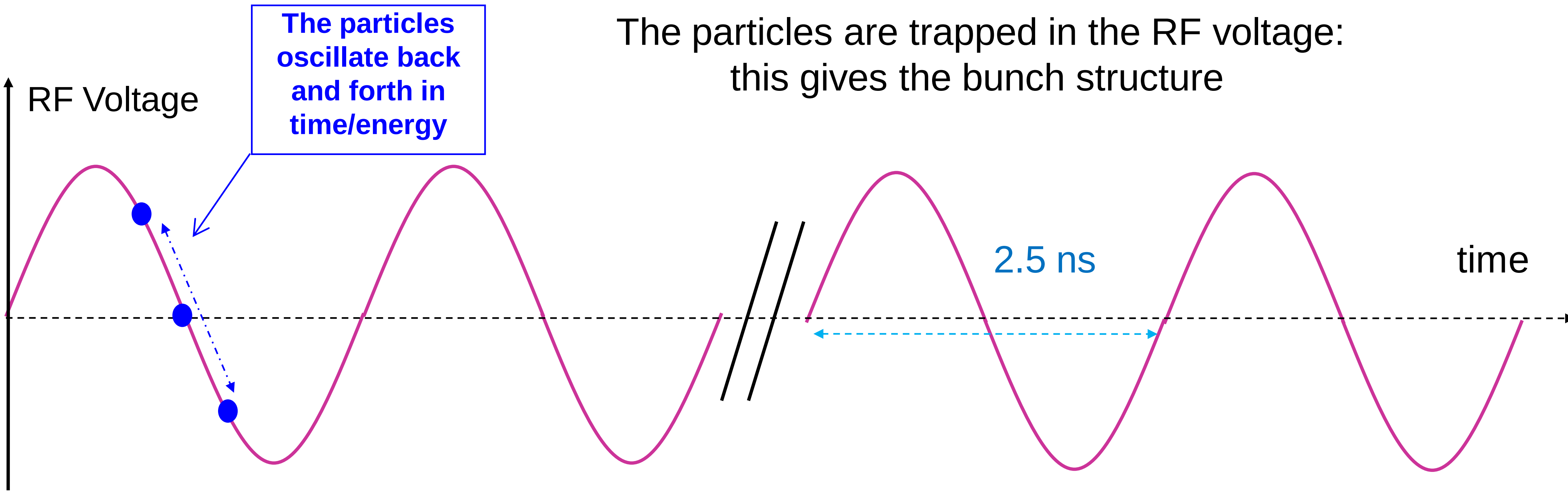
At the LHC, the acceleration from **450 GeV** to **7 TeV** lasts ~ 20 minutes (nominal!), with an average energy gain of ~ 0.5 MeV on each turn.
[Today, we ramp at a factor 4 less energy gain per turn than nominal!]

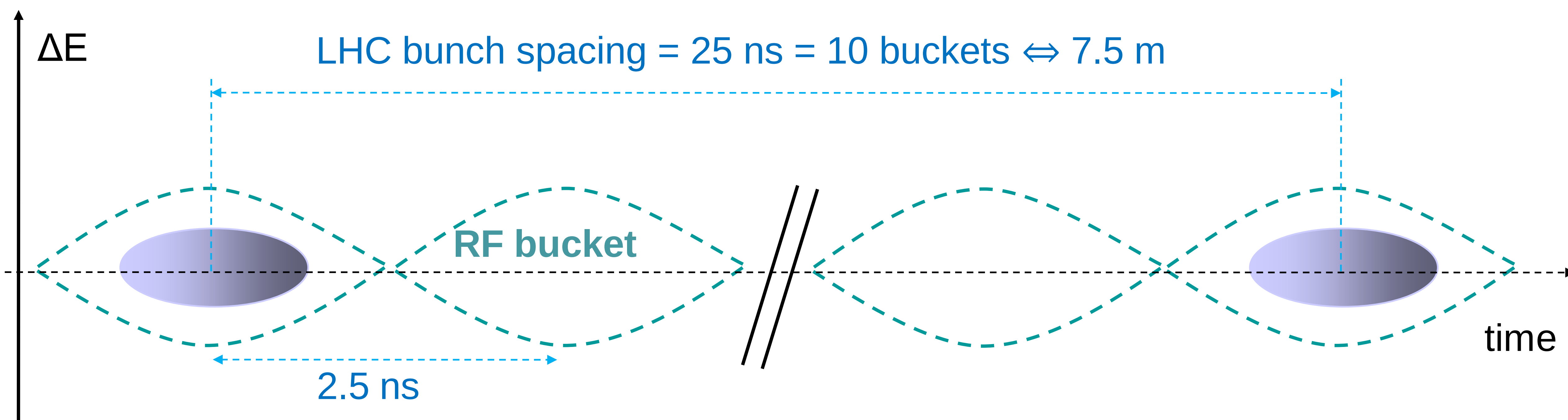




Buckets and bunches

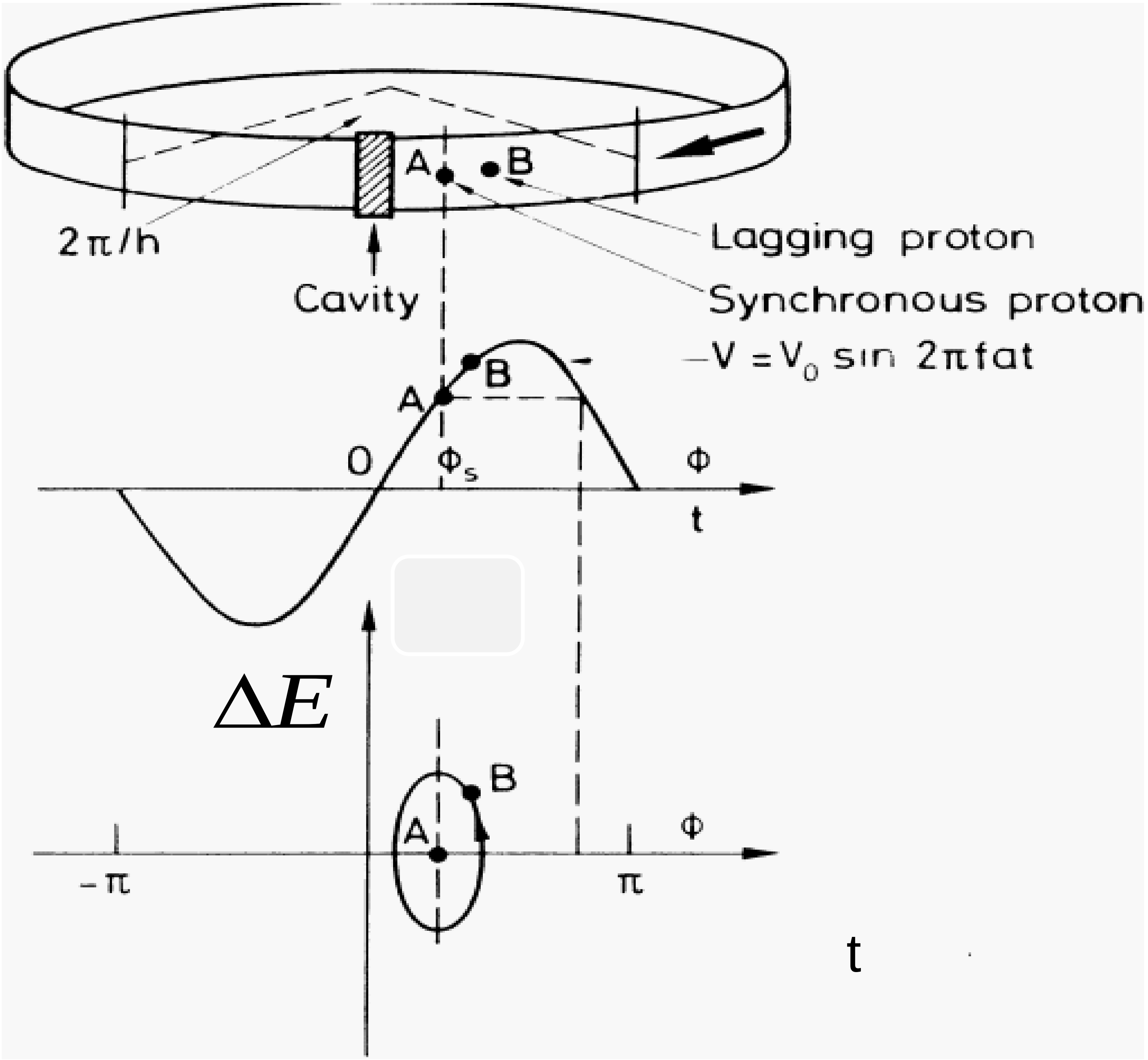
The particles are trapped in the RF voltage:
this gives the bunch structure





	450 GeV	7 TeV
RMS bunch length	11.2 cm	7.6 cm
RMS energy spread	0.031%	0.011%

Phase stability



- But is speed more important than path length
- Digress again for DIspersion?

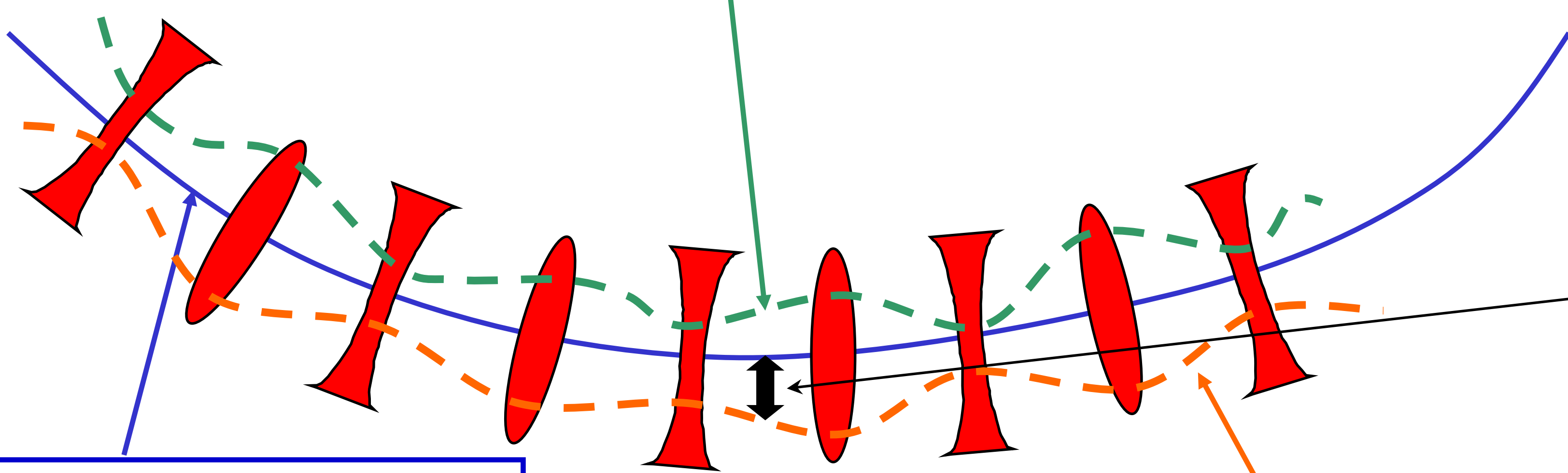
Dispersion

Dipole
=
spectrometer

Closed orbit for $p < p_0$

Lattice property

Particle's momentum error



$$\Delta x(s) = D(s) \times \frac{\Delta p}{p}$$

Central design orbit = closed orbit for $p=p_0$

Closed orbit for $p > p_0$

$$x'' + K(s)x = \frac{1}{\rho}\frac{\Delta p}{p_0}$$

Non-homogeneous Hill's equation

$$x(s) = A\sqrt{\beta_x(s)}\cos[\phi(s) + \phi_0] + D(s) \times \frac{\Delta p}{p}$$

D(s) = dispersion function. Periodic in *s*.

Transition - does an accelerated particle catch up - it has further to go

$$f = \frac{\beta c}{2\pi R}\ , \qquad (\beta = v / c)$$

$$\boxed{\eta_{rf} = \frac{\Delta f / f}{\Delta p / p} = \frac{p}{\beta}\frac{d\beta}{dp} - \frac{p}{R}\frac{dR}{dp} = \frac{1}{\gamma^2} - \frac{\bar{D}}{R_0}}$$

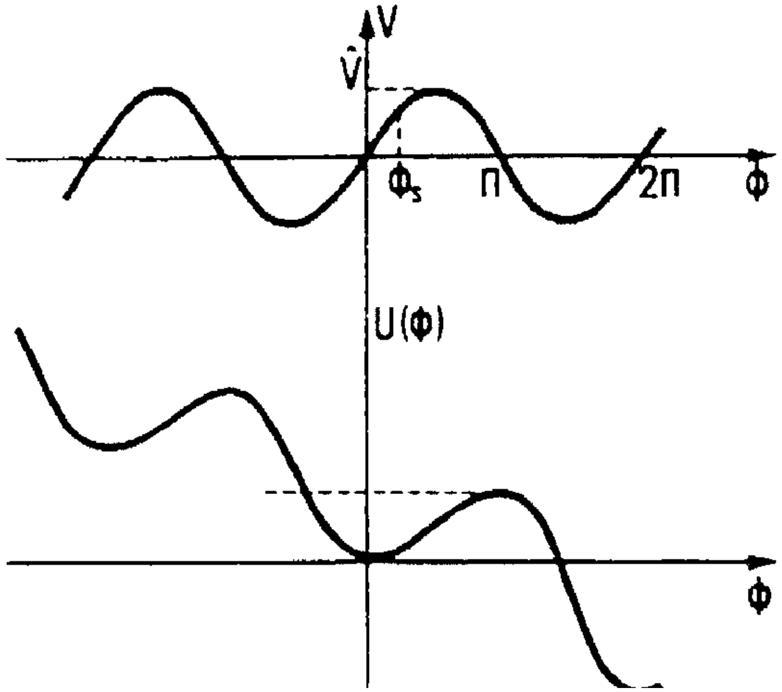
$$\Delta E = V_0 \left(\sin\phi - \sin\phi_s \right)$$

$$\ddot{\phi} = -\frac{2\pi V_0 h \eta f^2}{E_0 \beta^2 \gamma} \left(\sin\phi - \sin\phi_s \right)$$

Transition - does an accelerated particle catch up - it has further to go

Above is the voltage applied

Below it is integrated by phase which is a potential



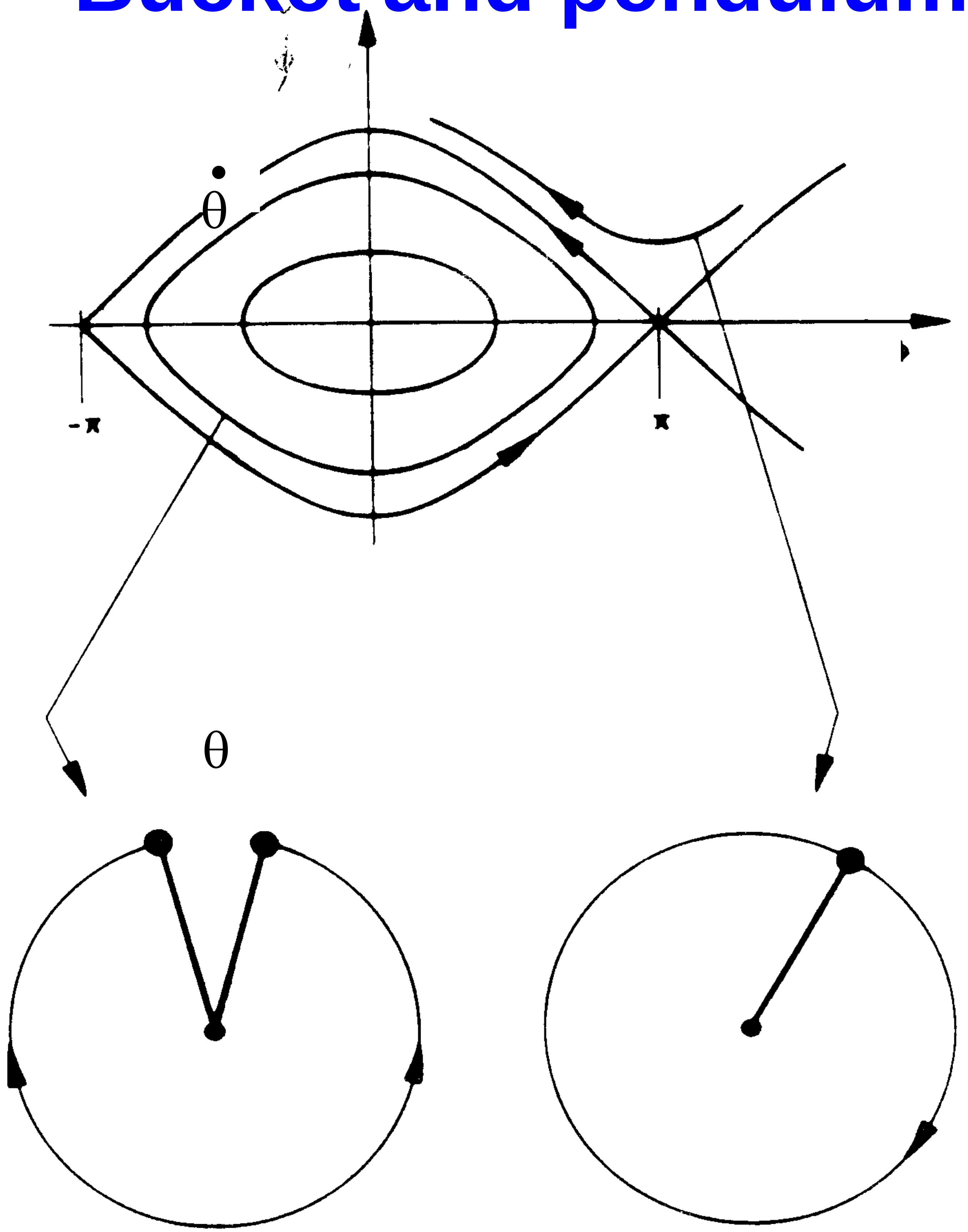
Here is an expression for energy (quasi Hamiltonian)

$$\frac{\dot{\phi}^2}{2} - \frac{\Omega_s^2}{\cos\phi_s}\left(\cos\phi + \phi\sin\phi_s\right) = const.$$

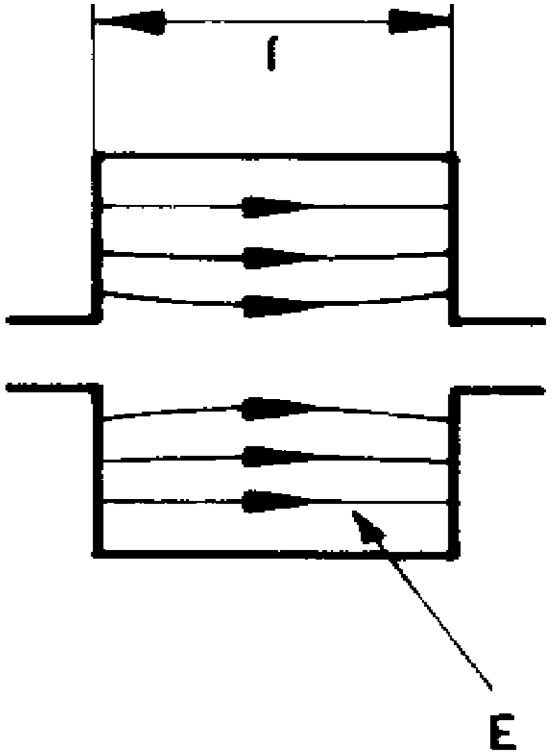
From which we can derive the equation of motion

$$\ddot{\phi} = -\boxed{\frac{2\pi V_0 h\,\eta f^{\,2}}{E_0\beta^2\gamma}}\left(\sin\phi - \sin\phi_s\right)$$

Bucket and pendulum

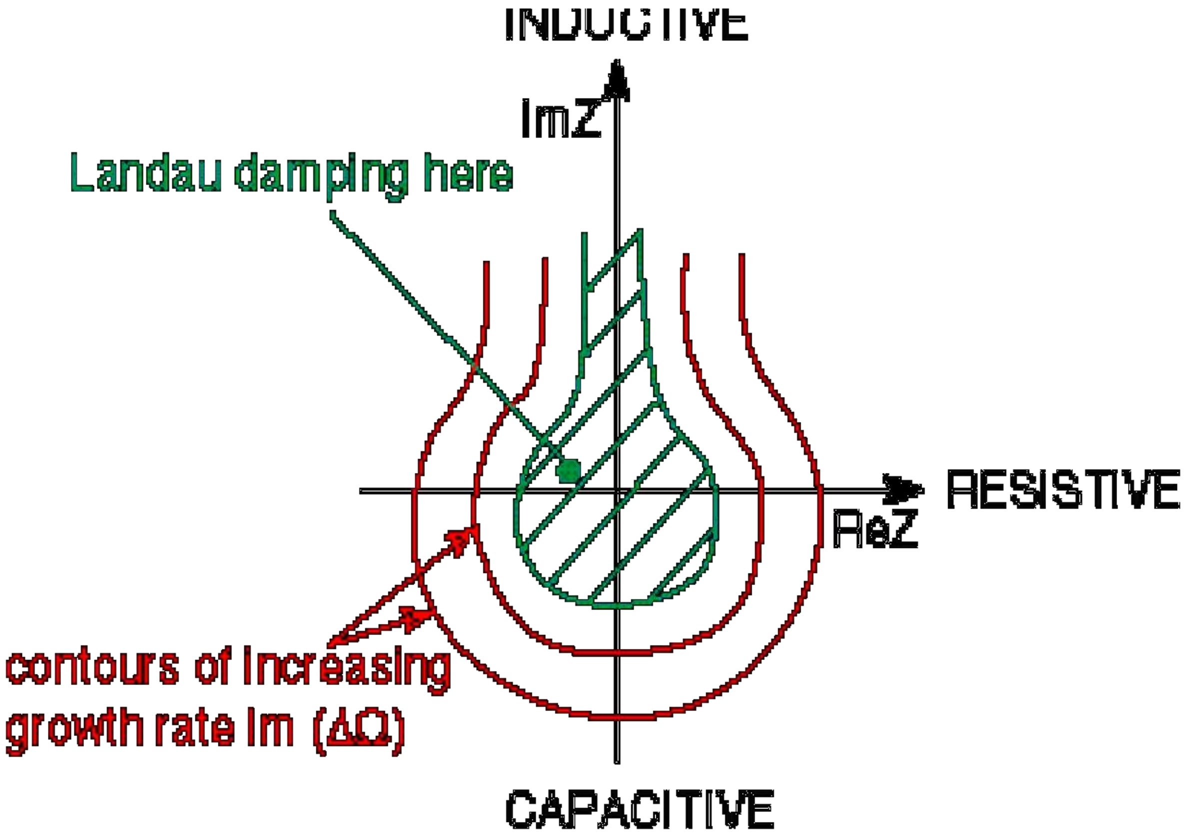


Instability



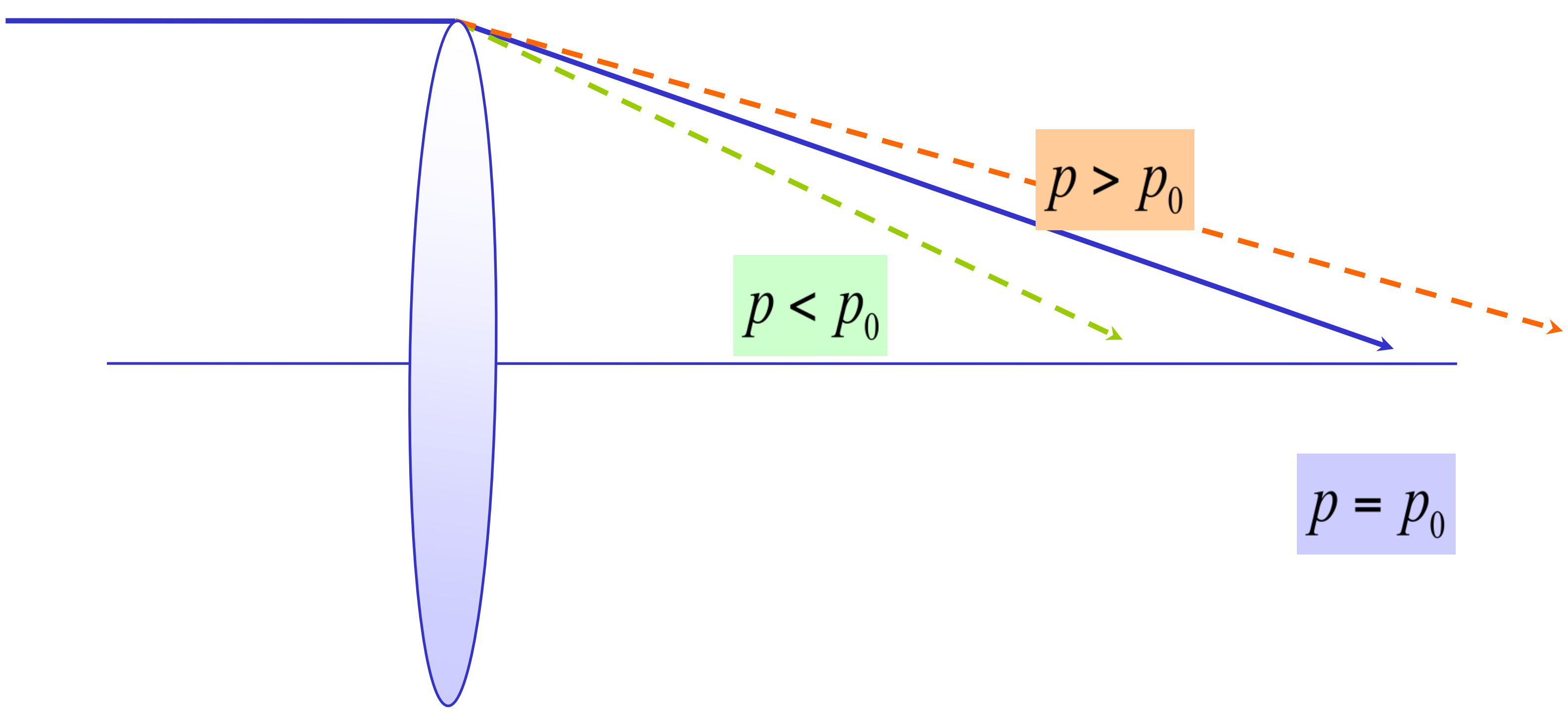
$$I = \hat{I}\ e^{-i\omega t}$$

$$U = \hat{U}\ e^{-i\omega t}$$



$$Z = X + iY = \frac{U}{I}$$

Chromaticity



$$Q' = \frac{\Delta Q}{\Delta p / p}$$

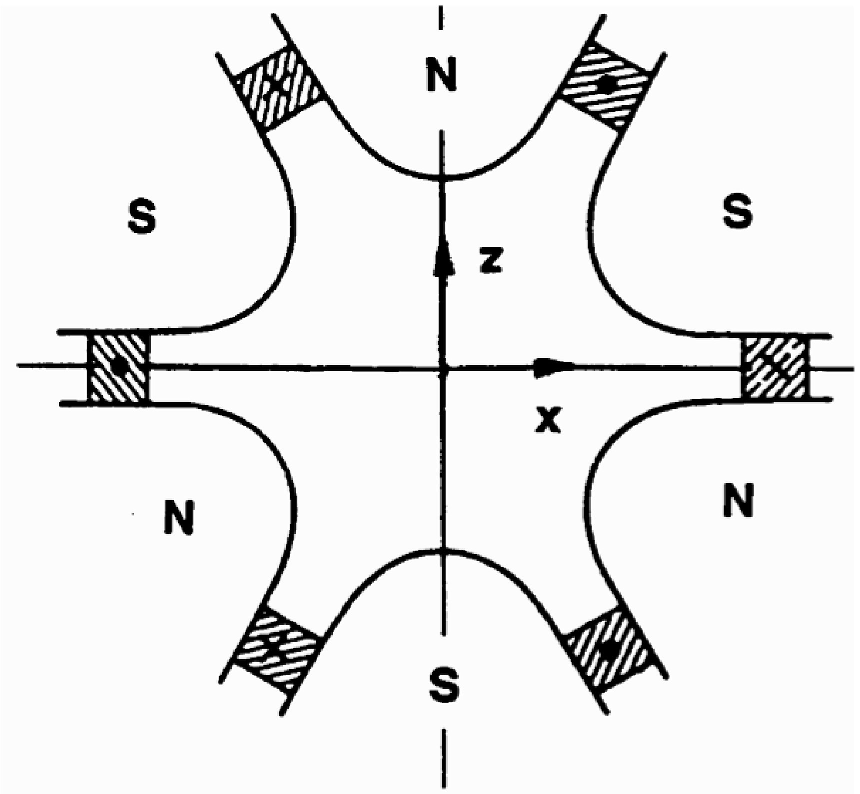
Particles with different energies have different betatron tunes.

<u>Bad</u> for the beam:
- Adds a tune spread
- Instabilities ("head-tail")

Focusing error from momentum errors ~ ***-K Δp/p***

Chromaticity corrections is done with **sextupole magnets**. The field changes as x^2.

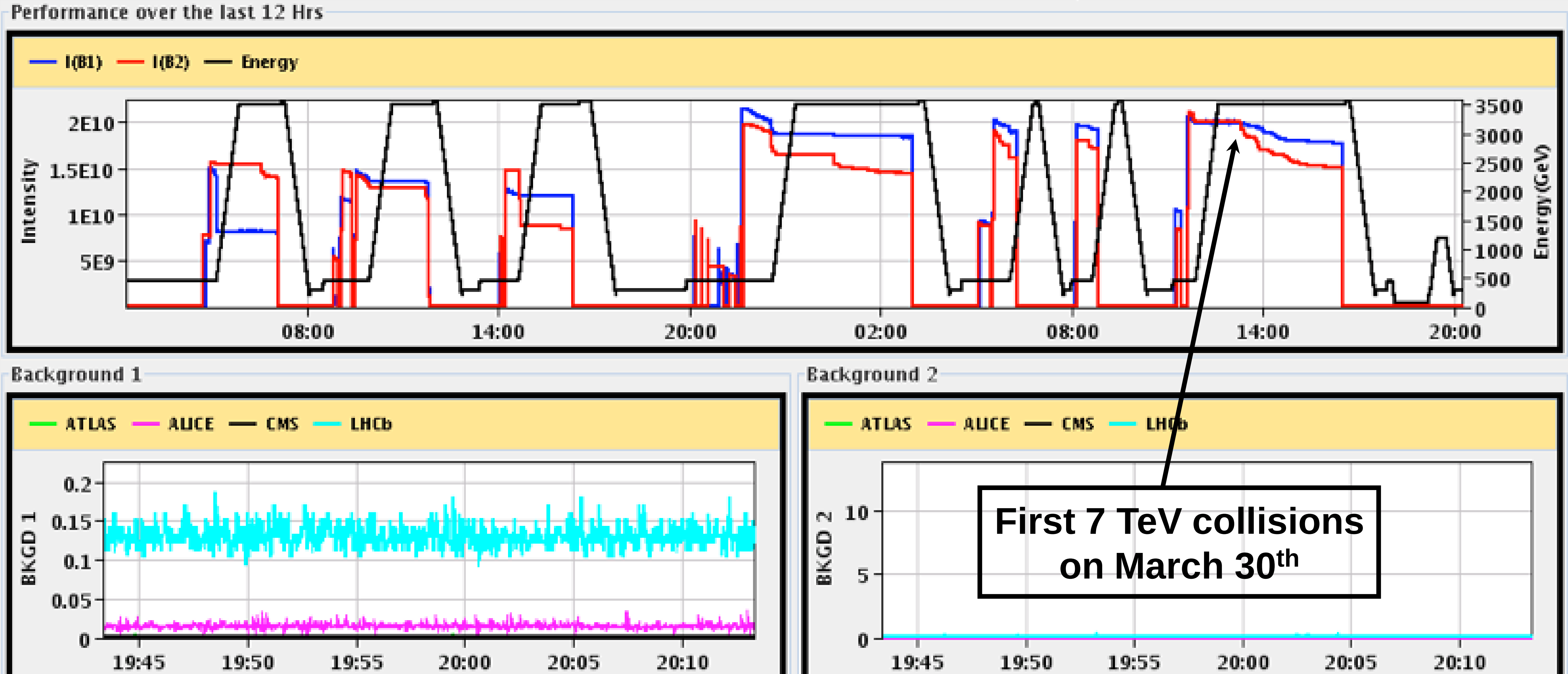
LHC:
2 sextupole families per plane per beam for chromaticity correction.



30-Mar-2010 20:14:20 Fill #: 1005 Energy: 297.4 GeV I(B1): 1.55e+08 I(B2): 7.06e+07

	ATLAS	ALICE	CMS	LHCb
Experiment Status	STANDBY	STANDBY	STANDBY	STANDBY
Instantaneous Luminosity	0.000e+00	0.000e+00	0.000e+00	0.000e+00
BRAN Count Rate	1.559e-01	2.818e-05	2.969e+00	2.732e-07
BKGD 1	0.002	0.013	0.002	0.122
BKGD 2	0.000	0.000	0.002	0.002
BKGD 3	0.000	0.005	0.003	0.040

LHCf STANDBY Count(Hz): 0.000 | LHCb VELO Position OUT Gap: 58.0 mm | TOTEM: STANDBY



Conclusions

- The limits to circular collider luminosity performance include:
  - Beta value at the IP
  - Beam-beam tune shift
  - Beam impedance and instabilities
  - Number of rf bunches
  - Synchrotron radiation (future machines)
  - Engineering viability
  - Funding and site availability

If anything goes wrong...

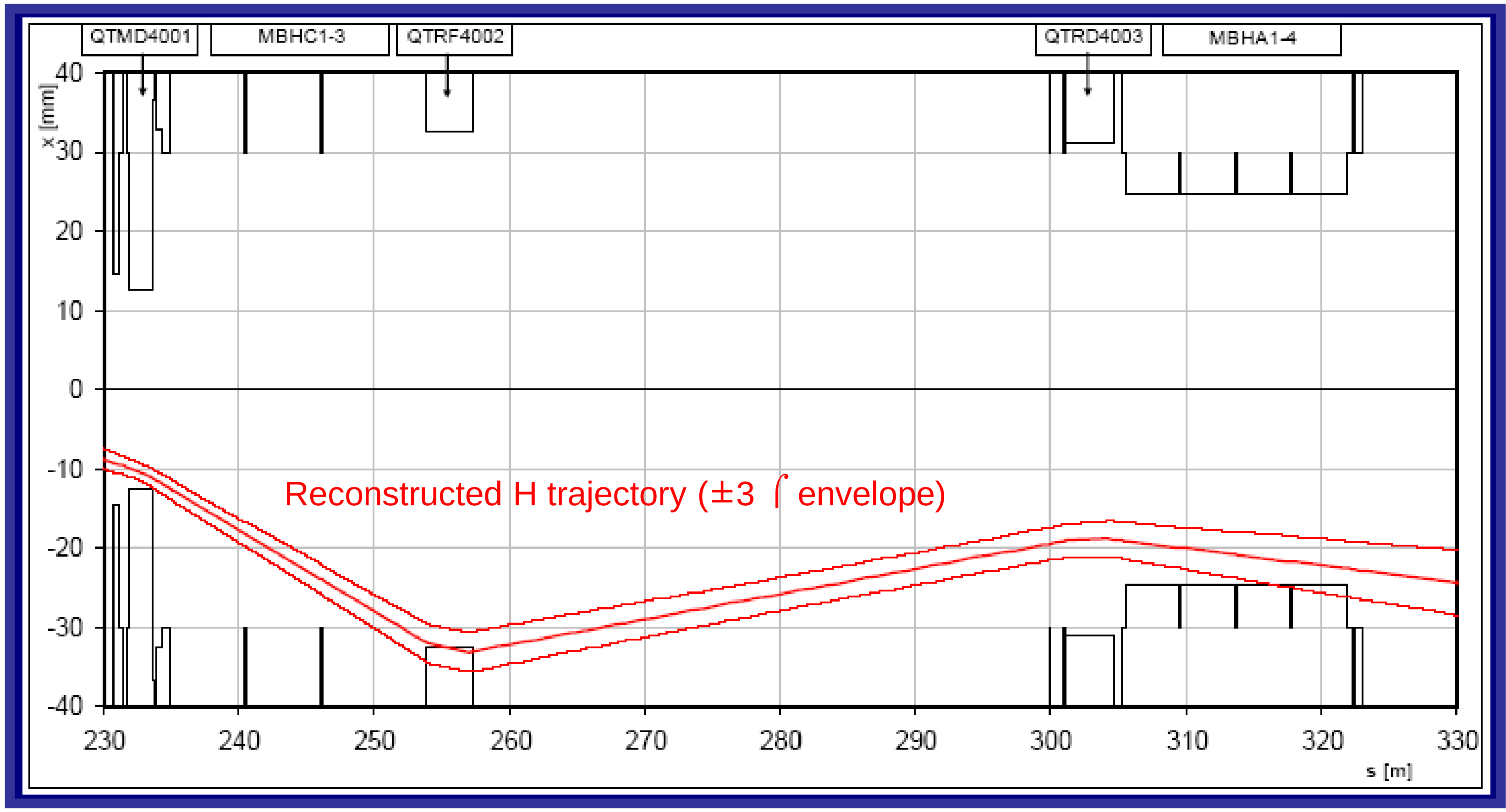
Failure in SPS during setting-up of LHC beam (25/10/04)

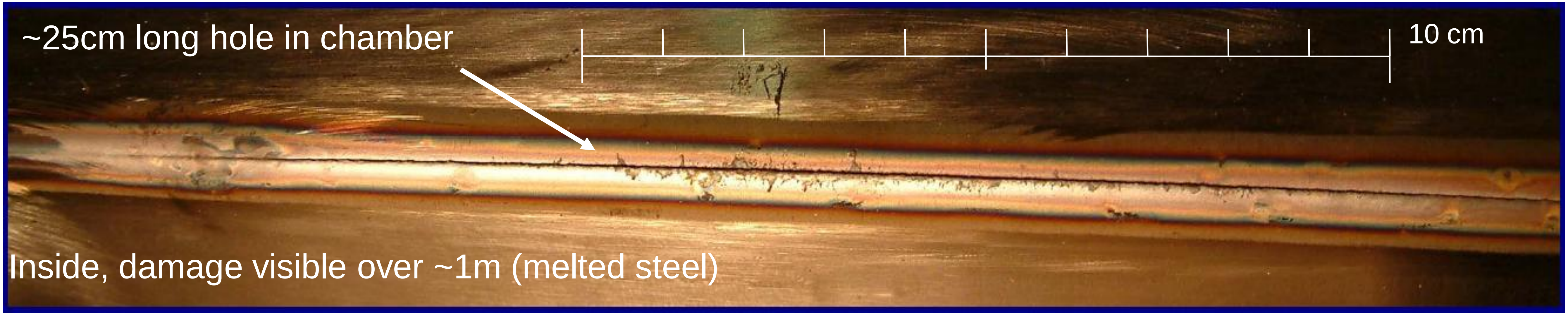
Extraction septum supply tripped due to EMC from the beam

In 11ms the field dropped 5%

3.4×10^{13} p+ @ 450GeV were wrongly extracted onto aperture

Chamber and magnet were damaged and had to be replaced





B. Goddard