From T2K to Hyper-K

David Hadley, University of Warwick

RAL Seminar, February 2018

Outline

Long baseline neutrino oscillation at T2K

Hyper-K Detector

Systematic uncertainty challenges and solutions

Neutrino Oscillations

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Weak flavour eigenstates ≠ Mass eigenstates
Neutrinos produced and detected in their weak flavour states

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \mathrm{U}_{\mathrm{MNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Unitary PMNS mixing matrix parameterised with 3 angles and **CP violating phase**
θ_{ij}, $\boldsymbol{\delta_{CP}}$

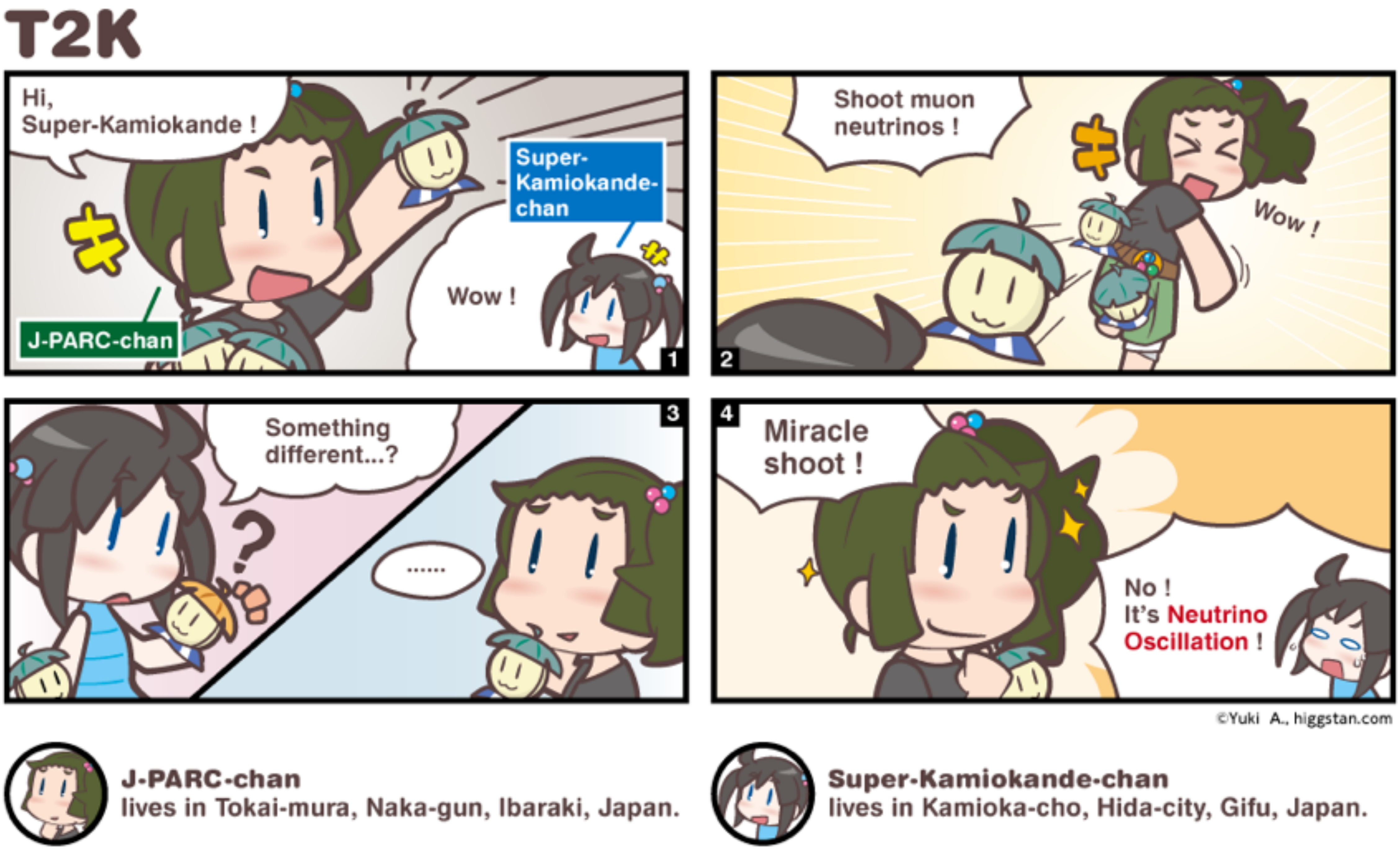
Relative phase difference between due to mass difference, $\boldsymbol{\Delta m^2}$

Appearance probability:

$$P_{\mu \to e} \approx \sin^2 \theta_{23} \cdot \sin^2 2\theta_{13} \cdot \sin^2 \left(\frac{\Delta m^2 L}{4E_\nu} \right)$$

+ higher order terms involving δ_{CP}

Neutrino Oscillations



Higgstan [http://higgstan.com/4koma-t2k/]

Neutrino Oscillations

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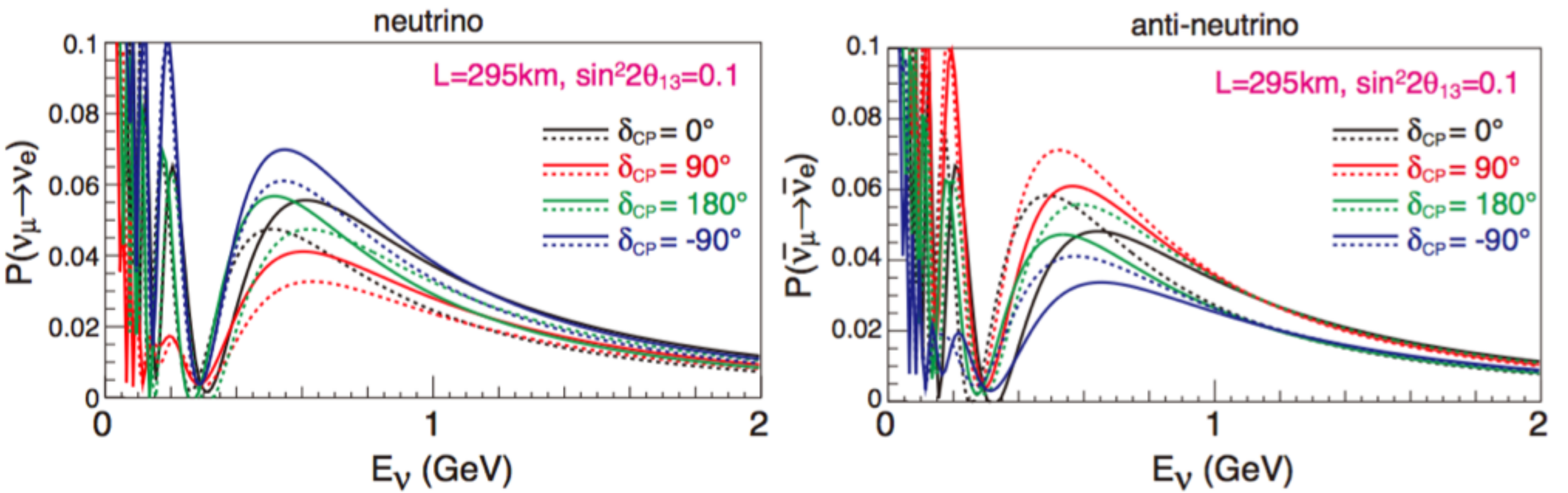
Typically perform experiment at fixed L with wide range of E



CP violation ~ 20% effect at 1st oscillation maximum
Much larger effect at 2nd oscillation maximum

Neutrino Oscillations

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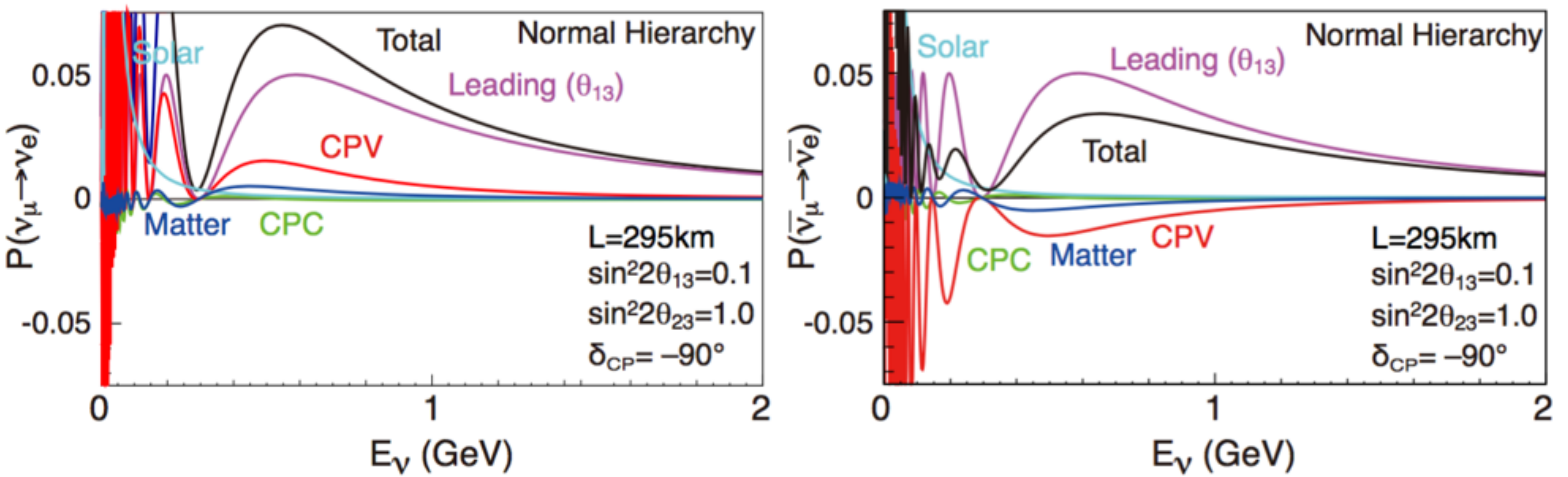
Typically perform experiment at fixed L with wide range of E



CP violation ~ 20% effect at 1st oscillation maximum
Much larger effect at 2nd oscillation maximum

What we actually measure:

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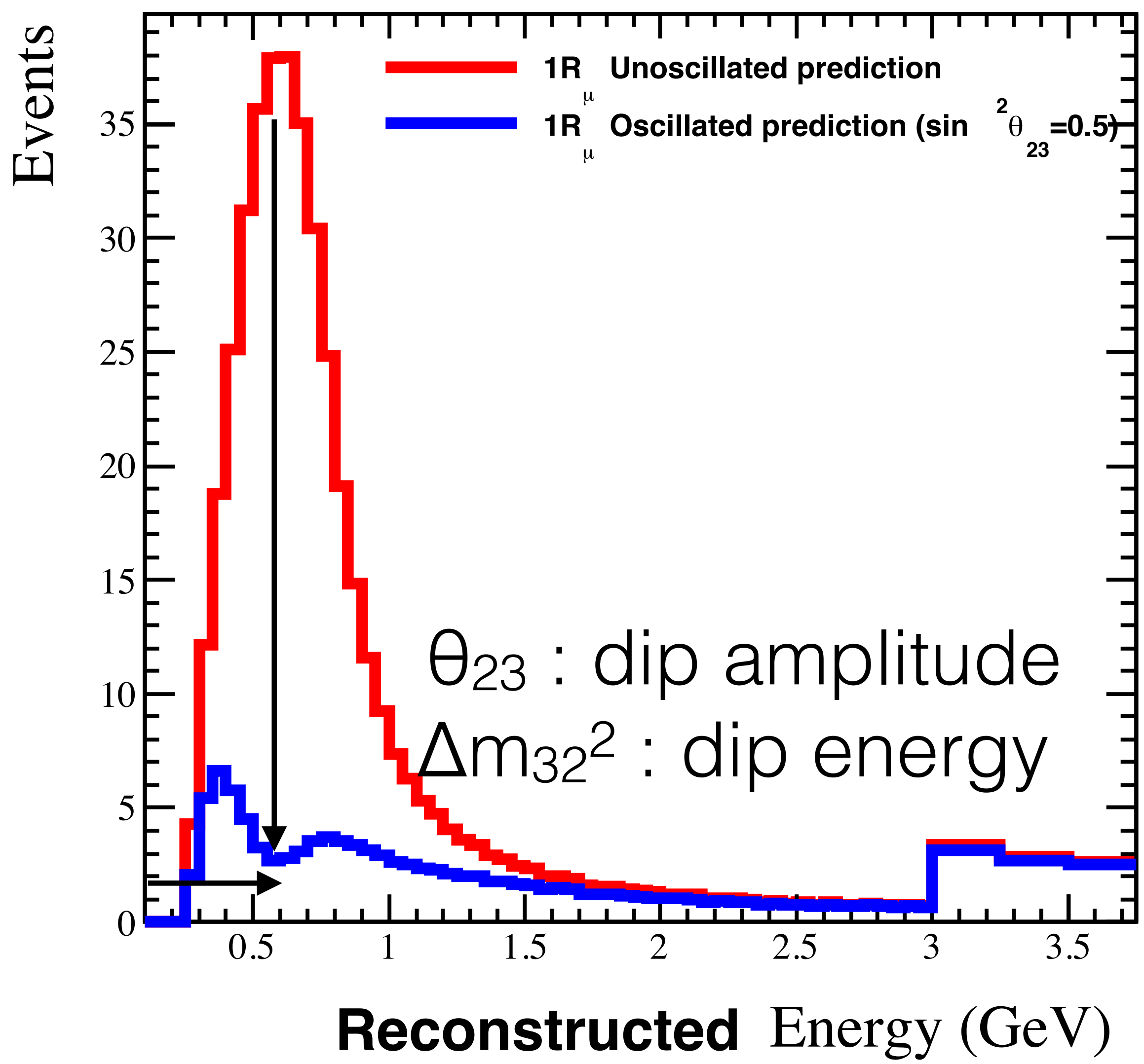
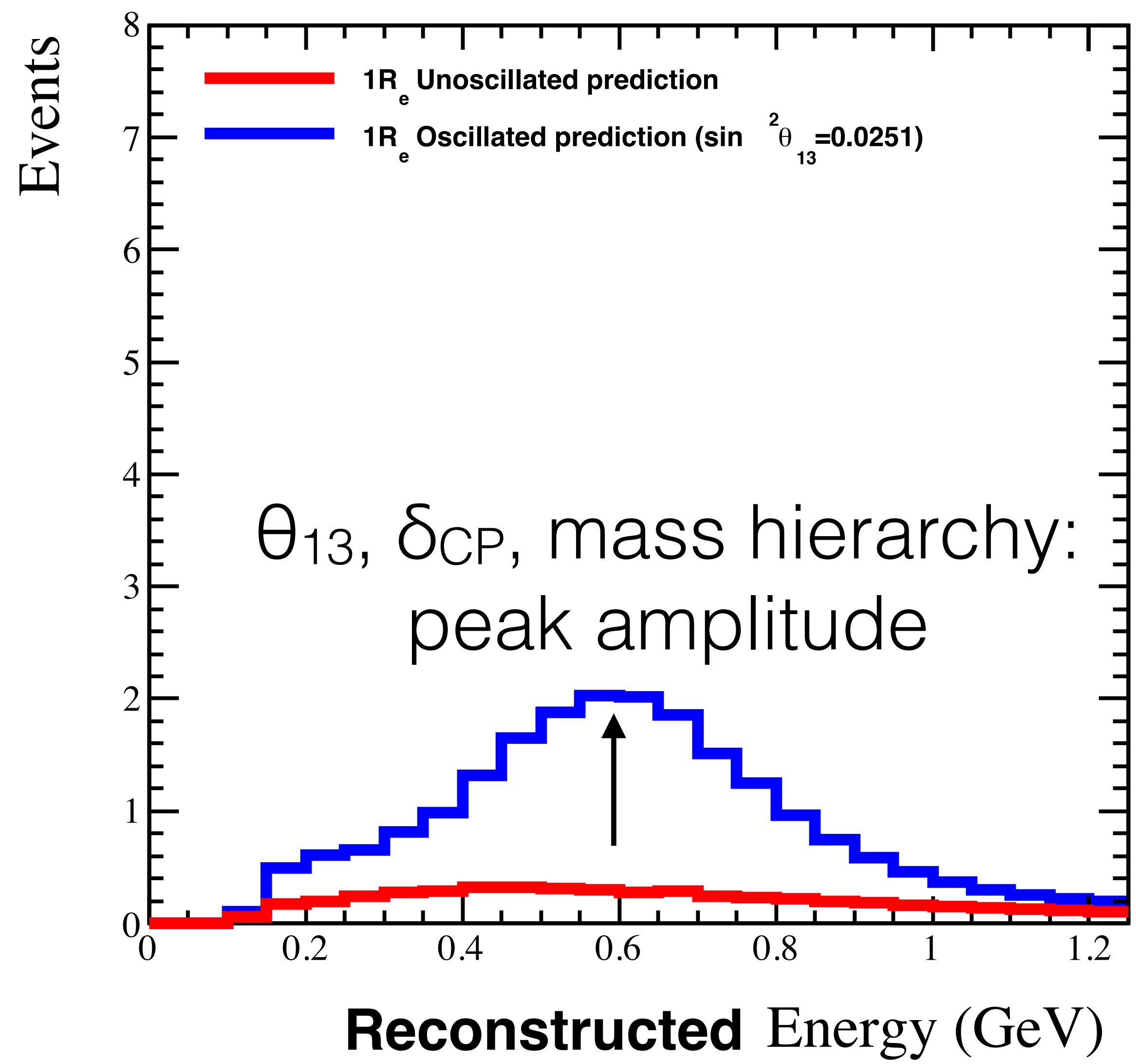
ν_μ disappearance



ν_e appearance

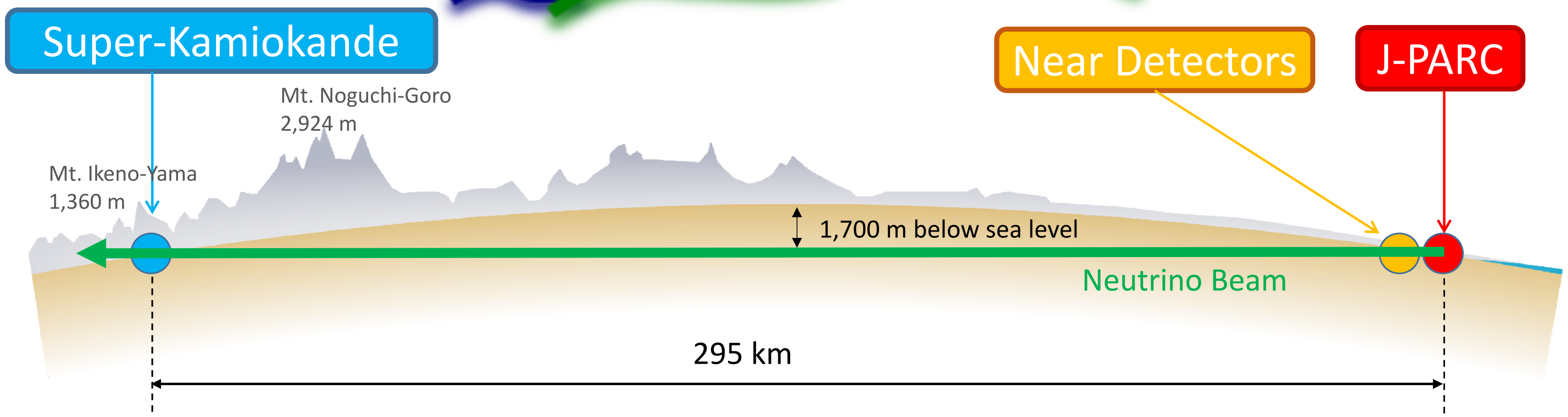


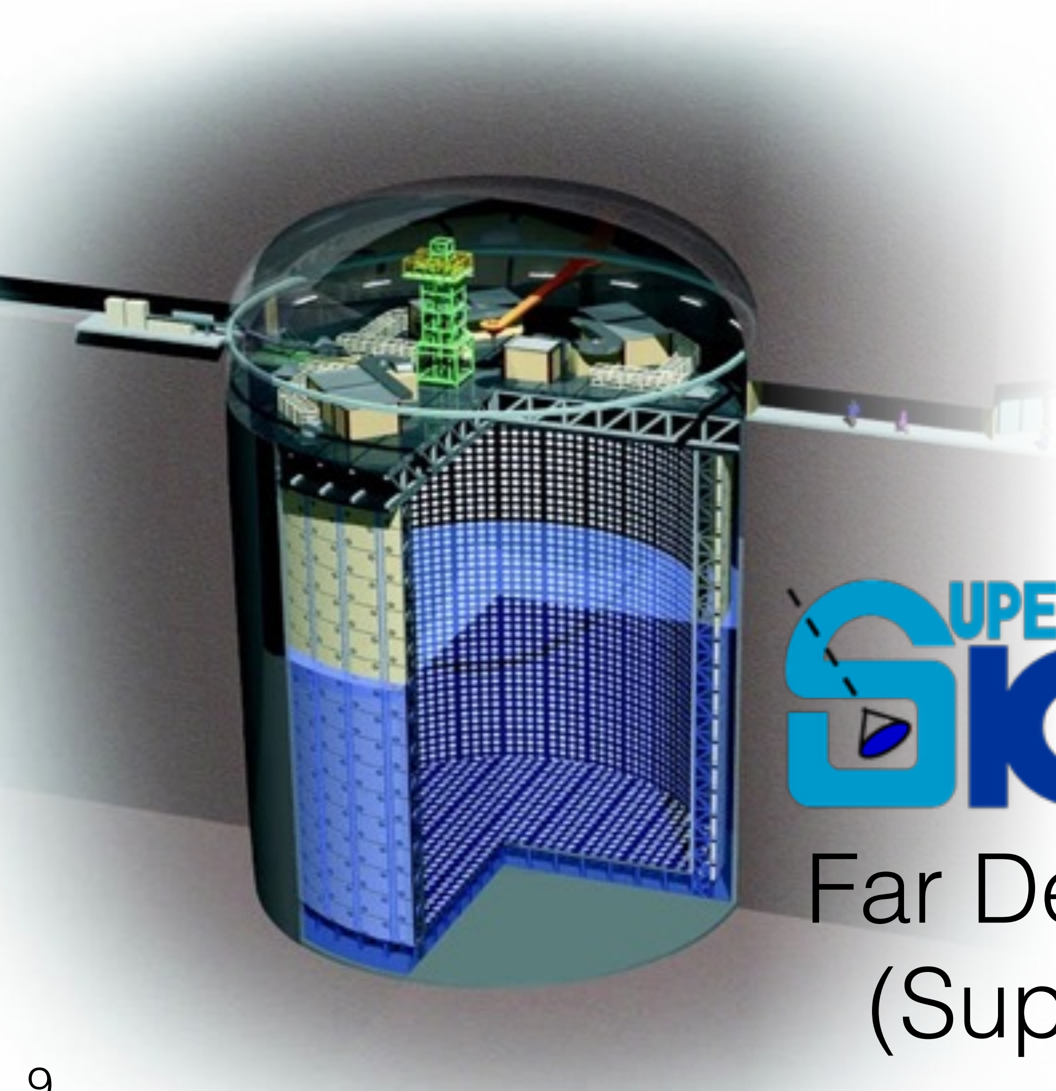
Measurement precision limited by:

- Statistics
- Neutrino energy reconstruction
- Knowledge of unoscillated spectrum and background contamination

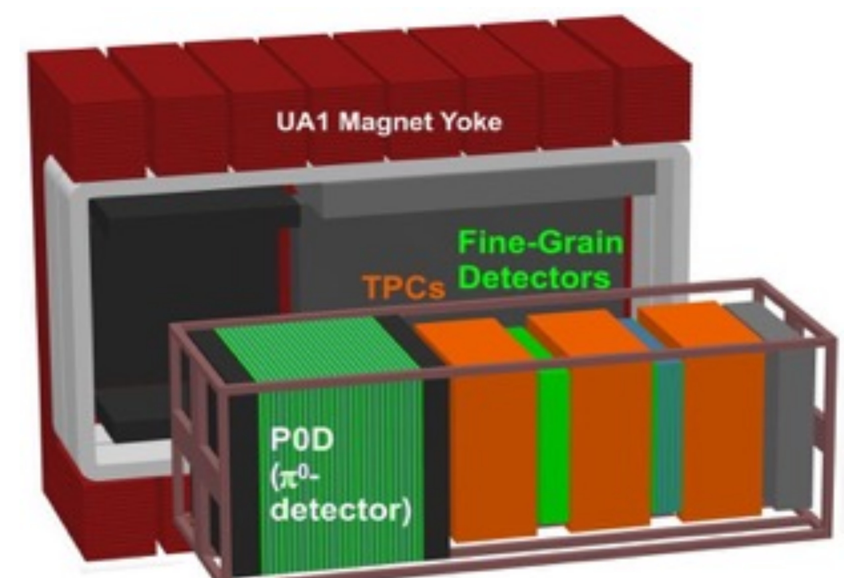
Accelerator based Neutrino Oscillation Experiments

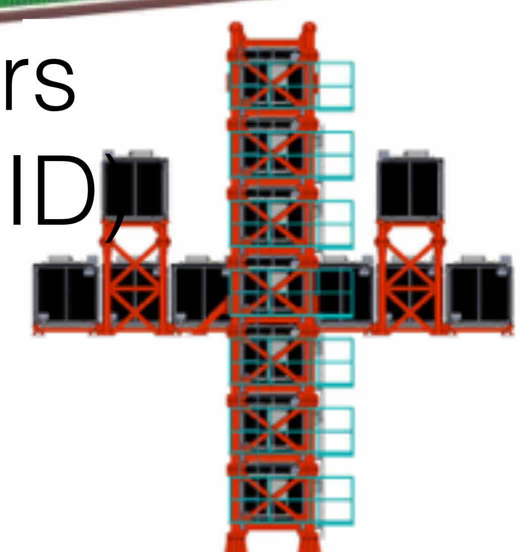
T2K





Far Detector
(Super-K)





Near Detectors
(ND280+INGRID)

T2K ν_e appearance

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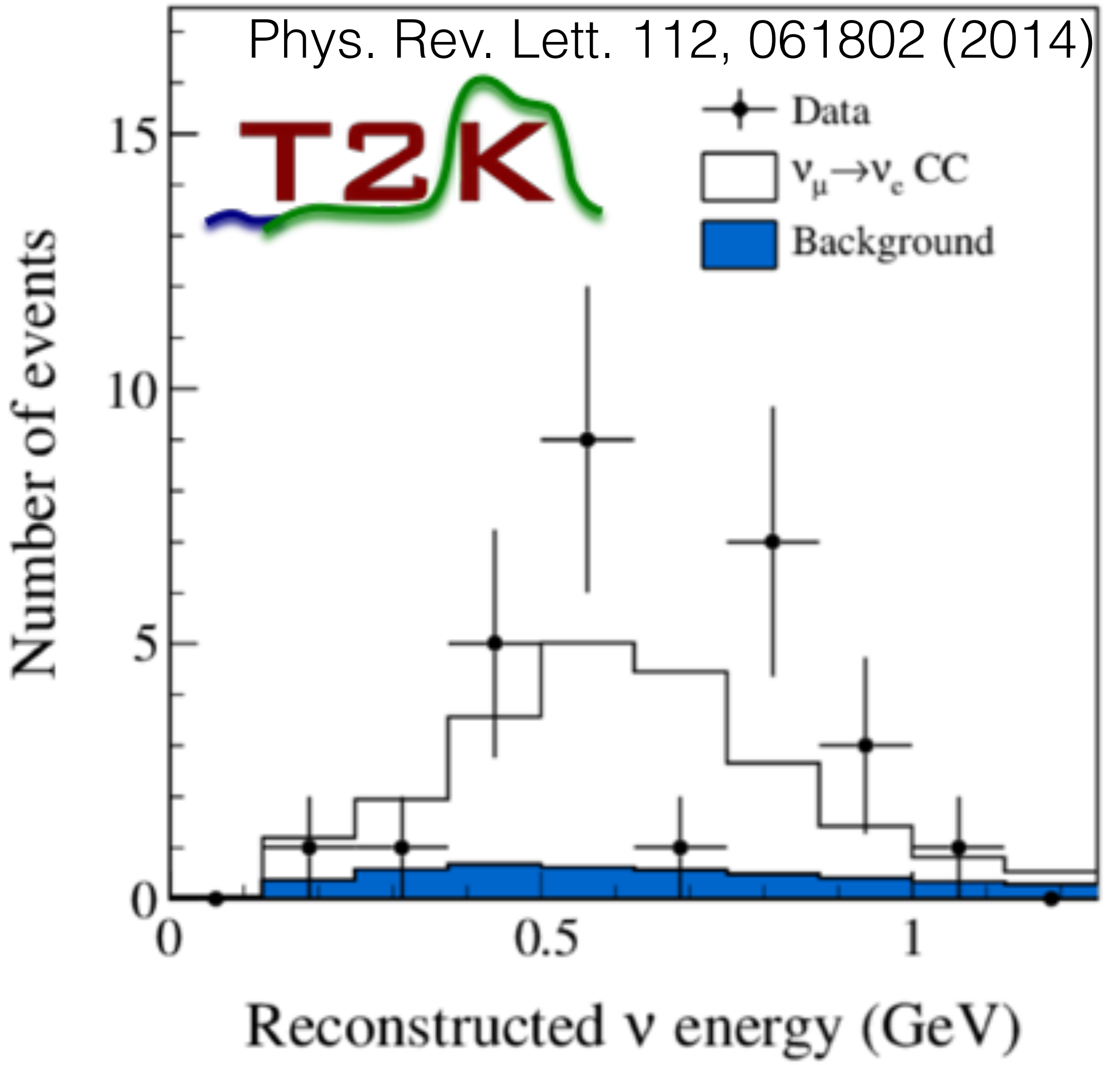
2013: ν_e appearance established

28 events observed (4.3 expected background)

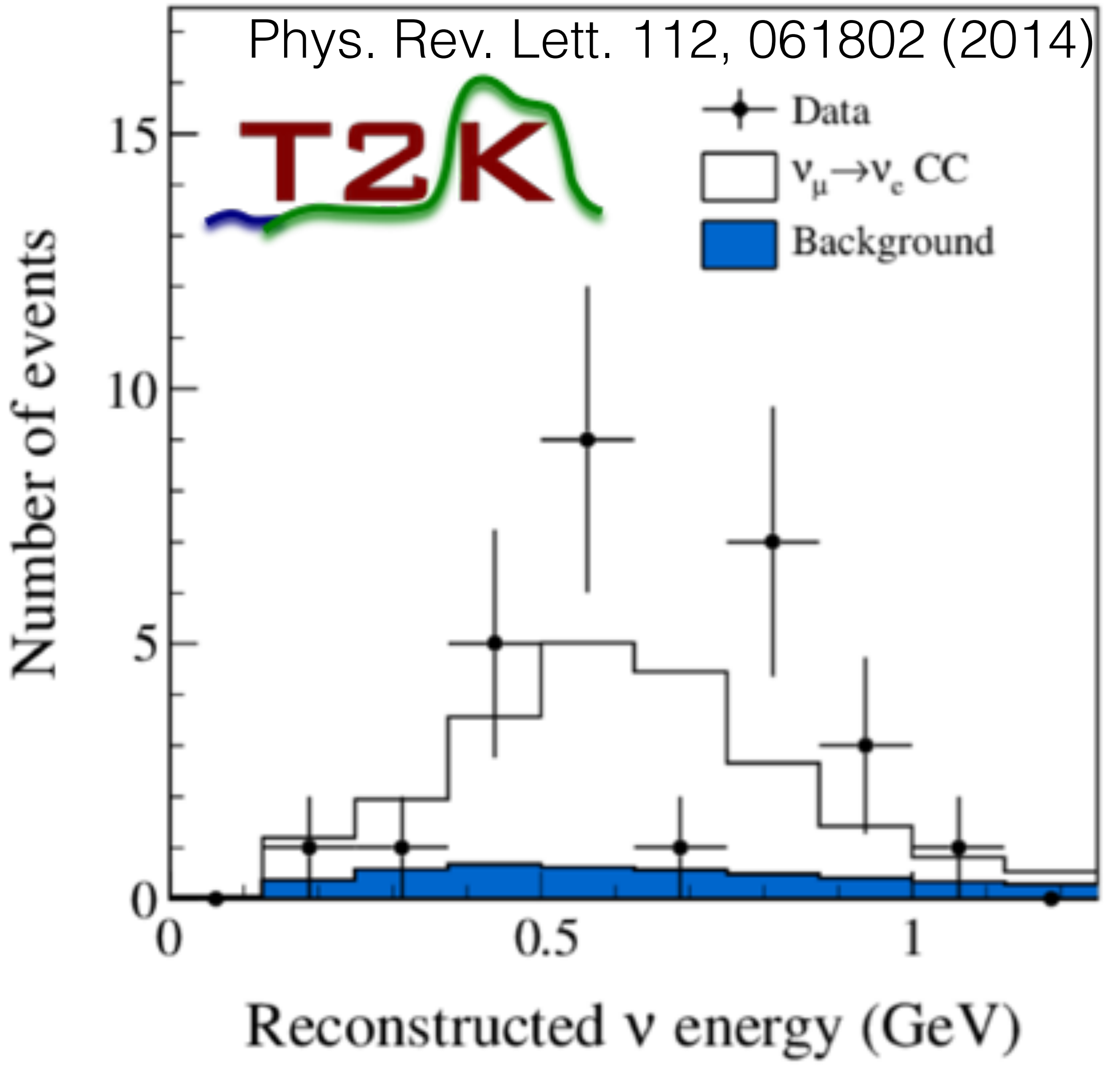


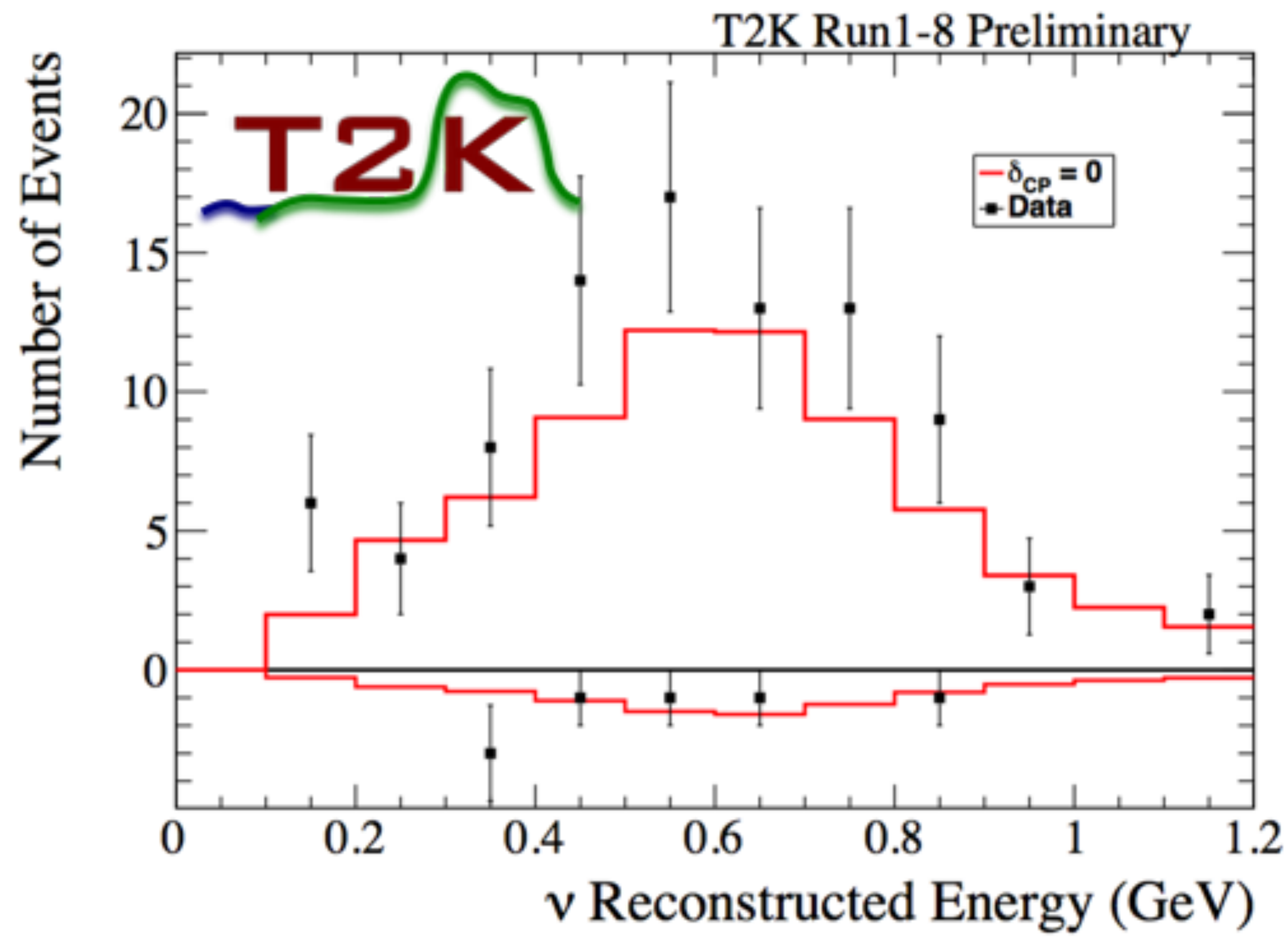
effect is large, opens the way to leptonic CP violation δ_{CP}.

T2K ν_e appearance

2013: ν_e appearance established→2017: "indications" of CP violation

28 events observed (4.3 expected background)

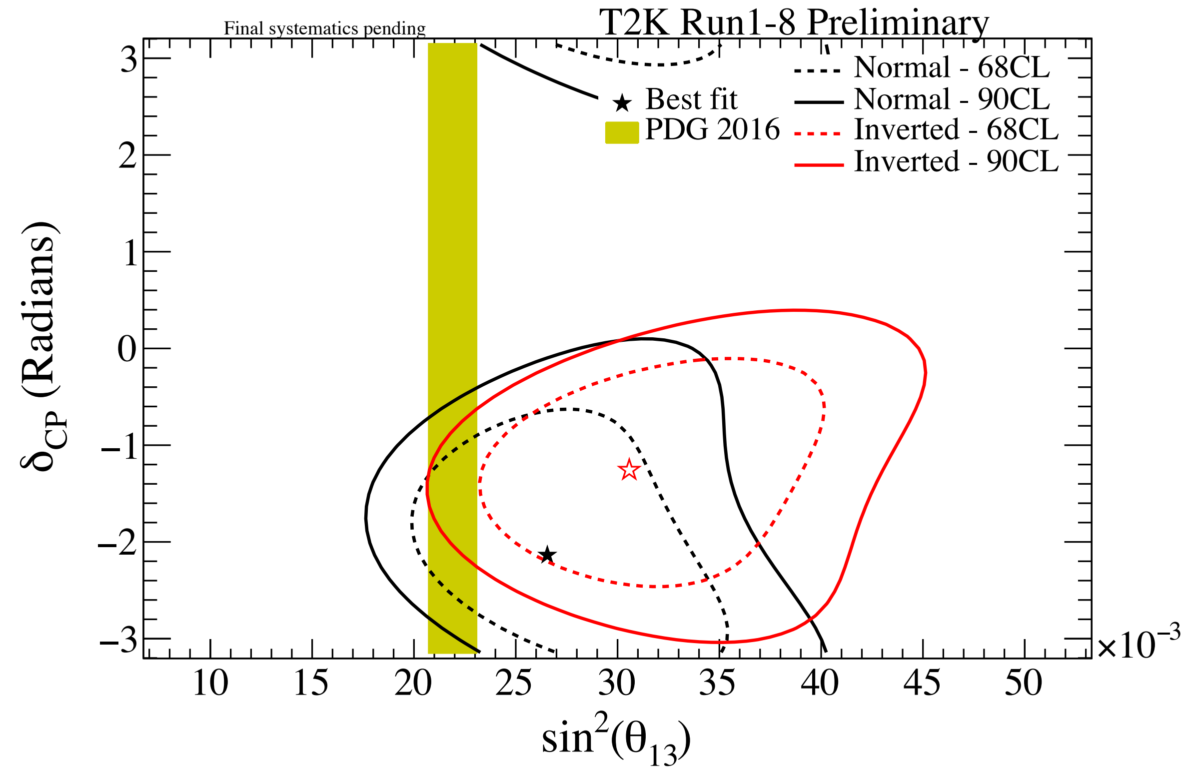




effect is large, opens the way to leptonic CP violation δ_{CP}.

Small ν_e excess and $\bar{\nu}_e$ deficit
Current measurement based on 74+7 events in single ring sample

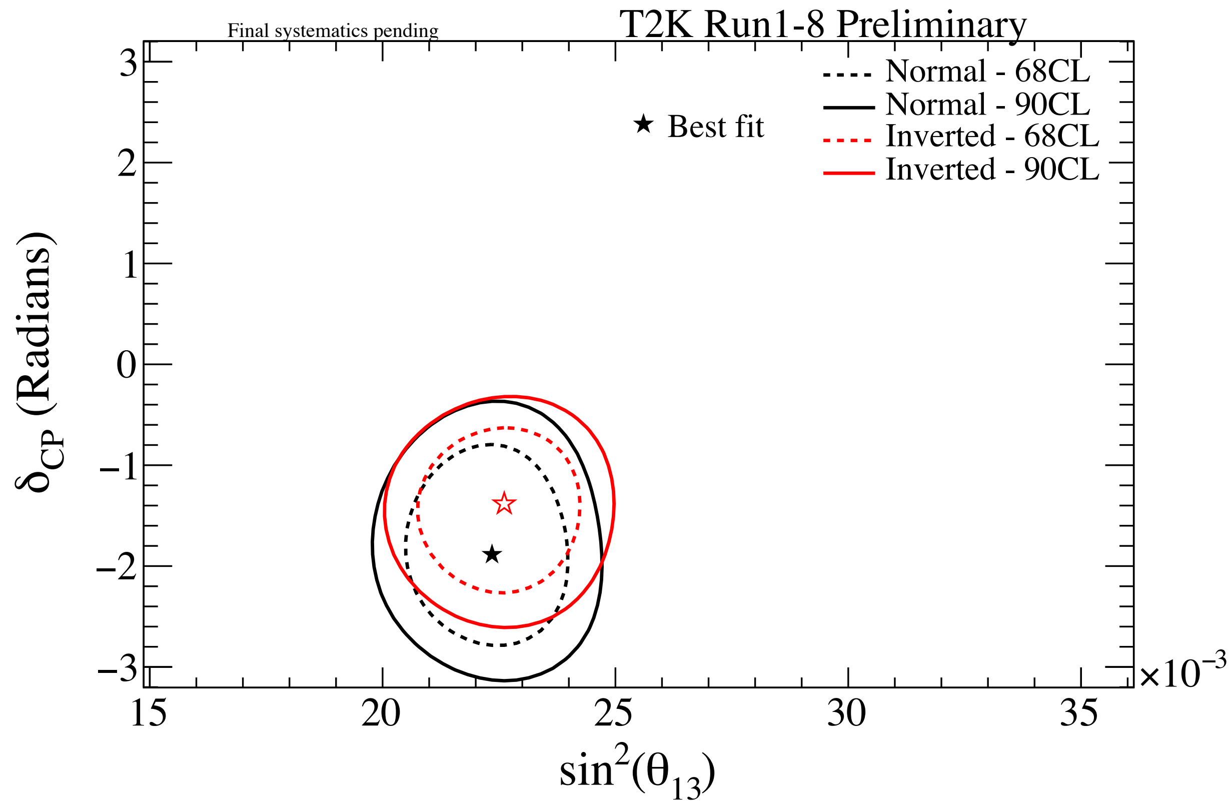
First Indications of CP violation

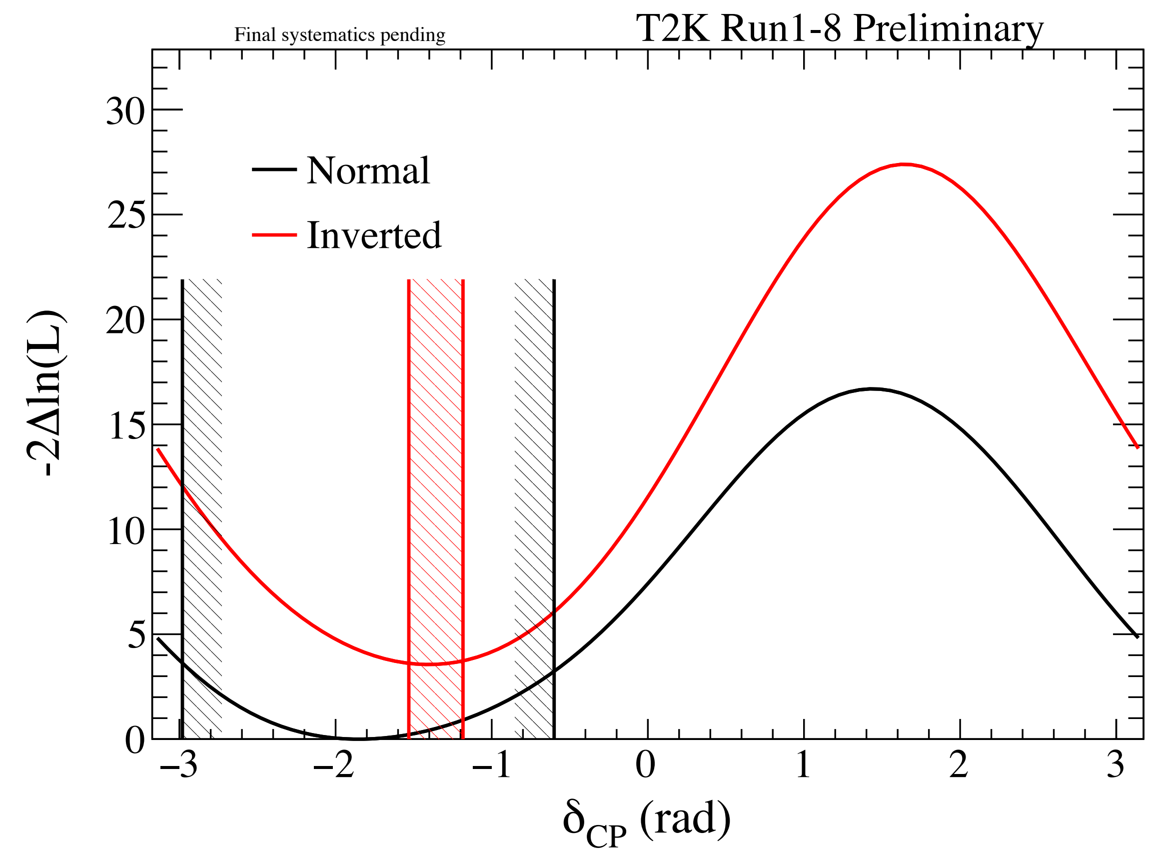


CP conserving values excluded at 2σ

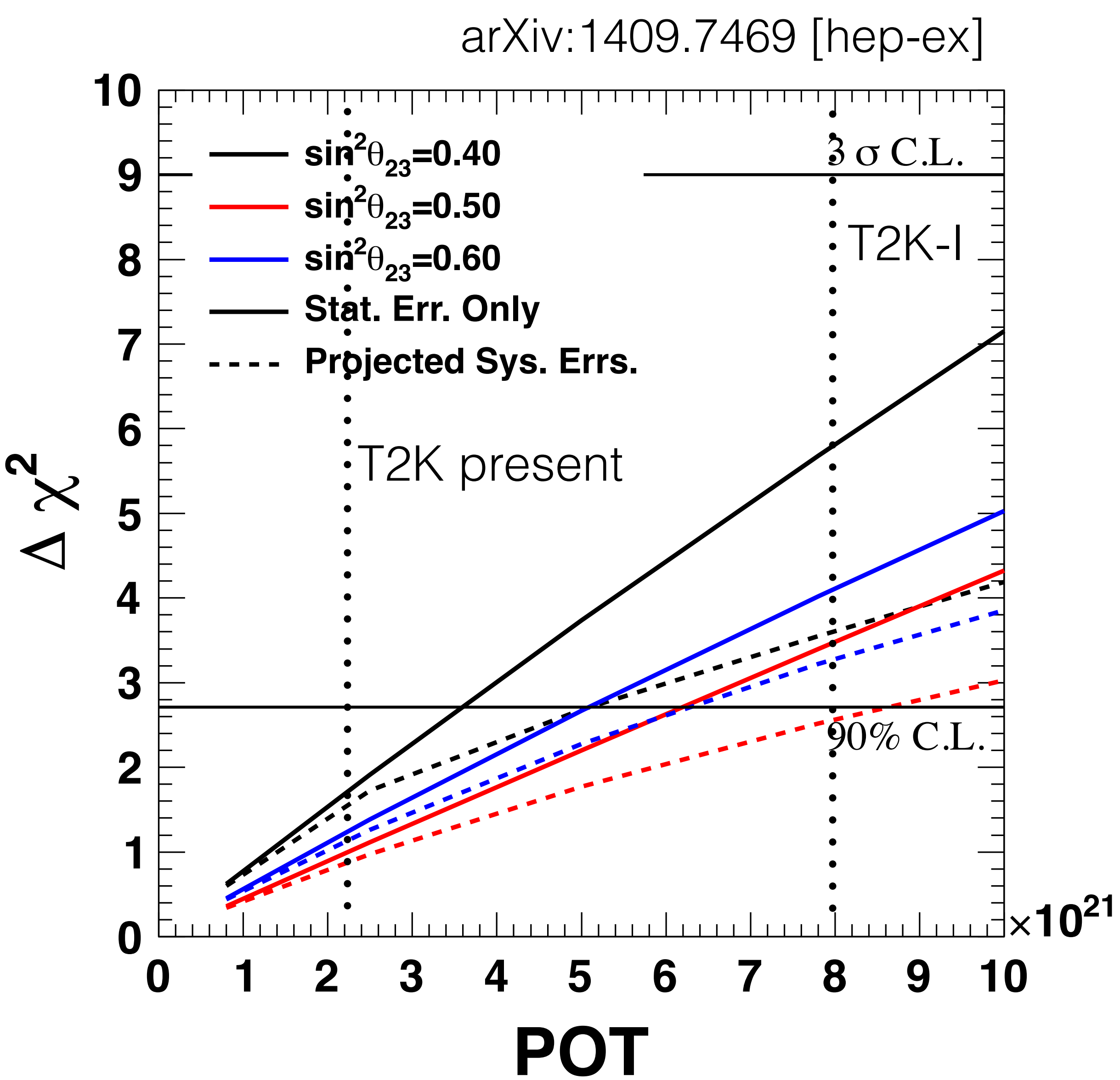
Statistically limited

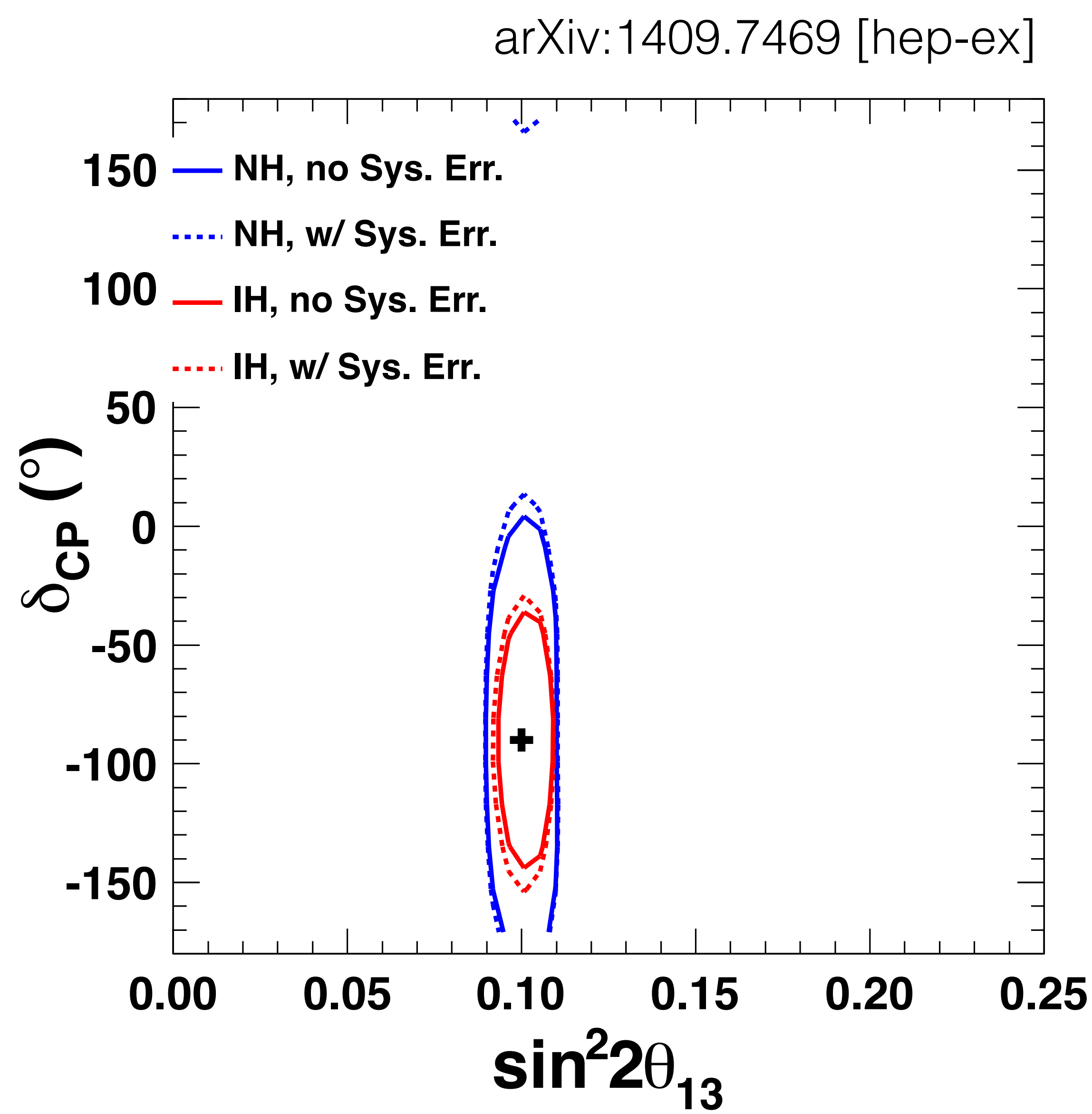
Dependent on reactor $\overline{\nu}_e$ disappearance measurement





T2K Projected Sensitivity





~2.5σ projected significance if *maximal CP violation*.

to firmly establish CP violation we will need **Hyper-K**!

Kamiokande Detectors

Kamiokande
680 tonne
fiducial mass
(1983)

Kamiokande Detectors

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680 tonne
fiducial mass
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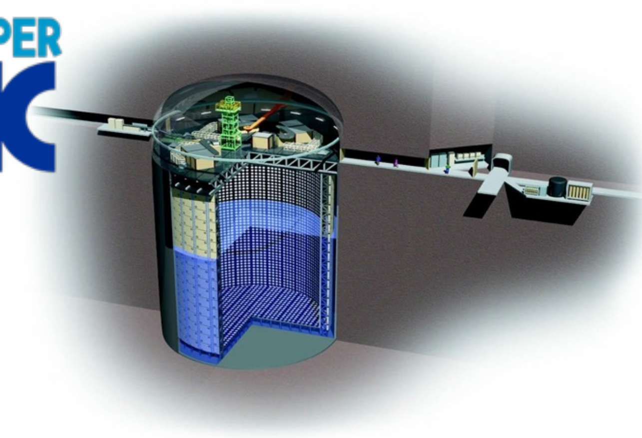
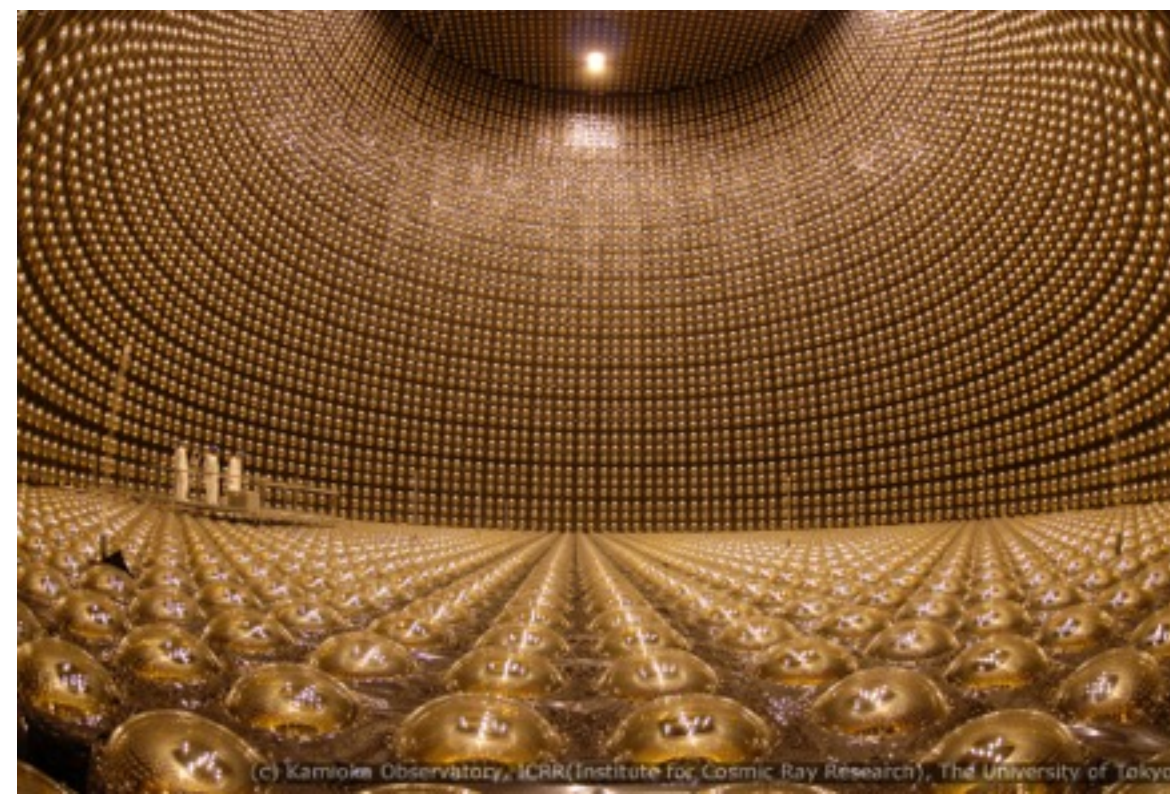


Super-Kamiokande
22.5kt fiducial mass
(33x Kamiokande)
(1996)



Kamiokande Detectors

Kamiokande
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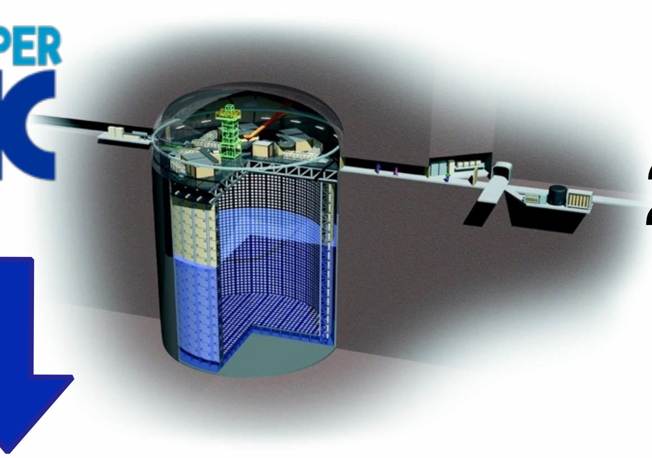

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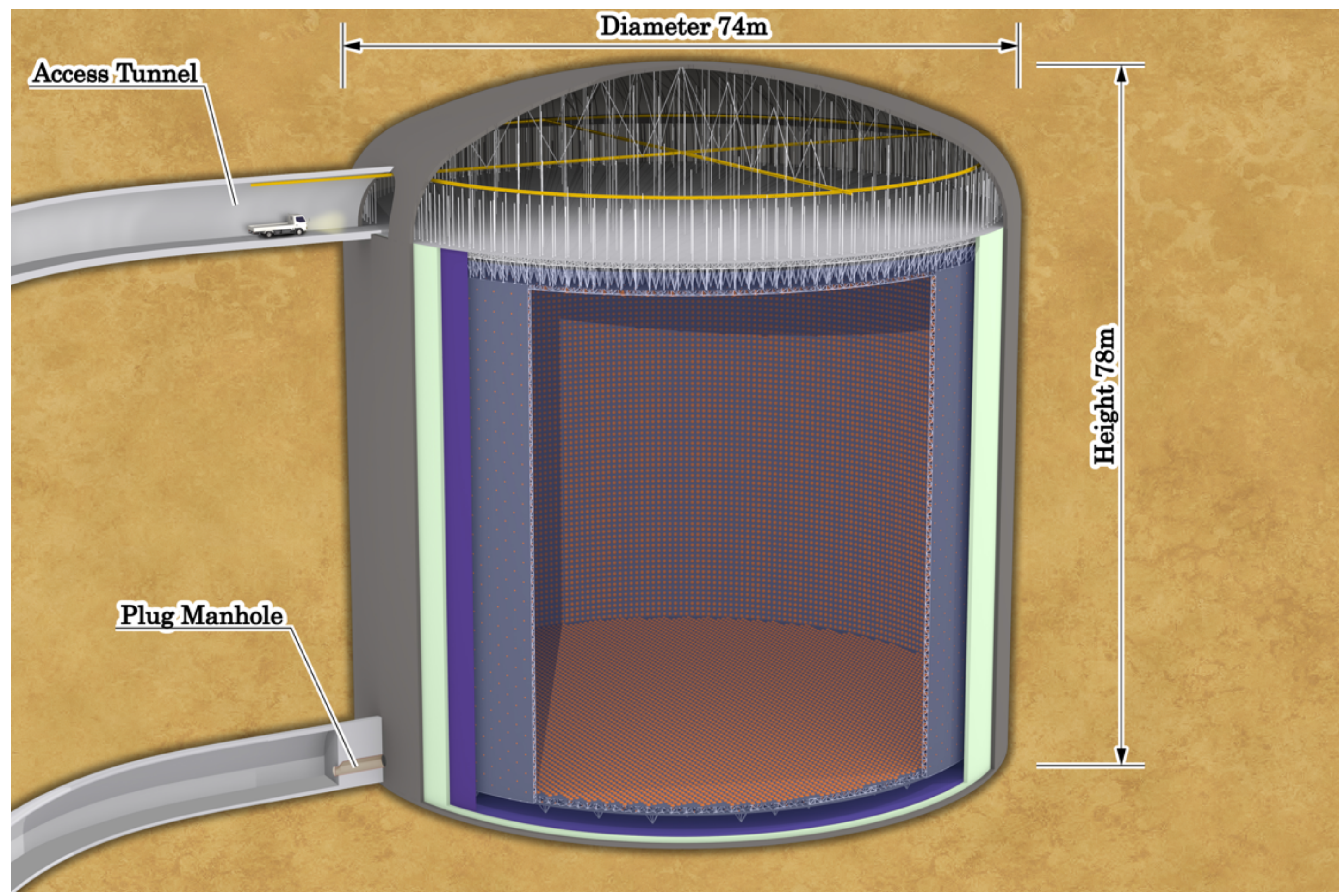

Hyper-Kamiokande
187 kt fiducial mass per tank
(2026?)

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Hyper-K Collaboration

Growing international collaboration: 14 countries, ~300 people

Why Water Cherenkov?

Scalability

Water is cheap, non-toxic, liquid at room temperature

we already know how to build big water WC detectors

Proven technology

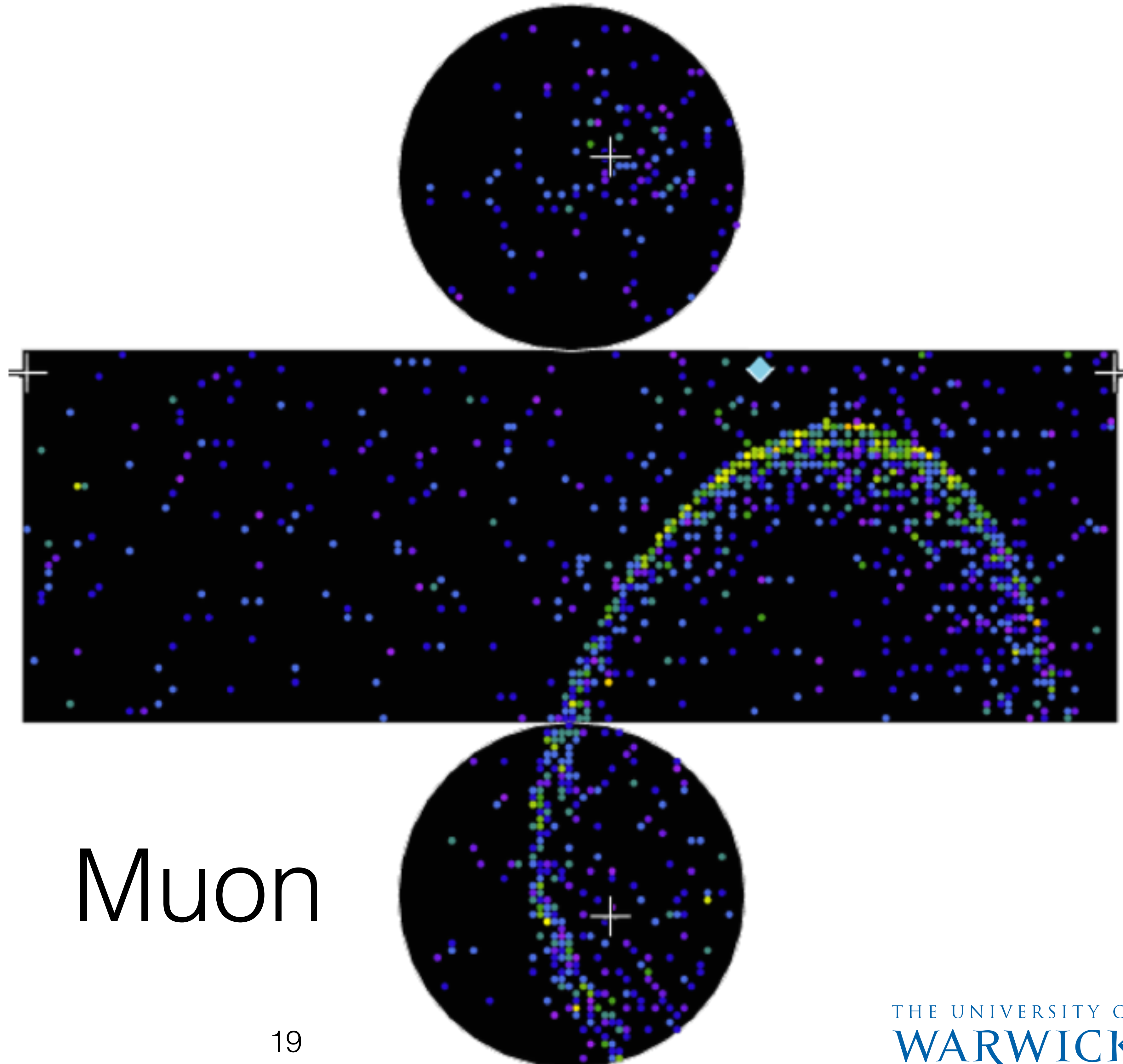
many years of experience from Super-K

low risk

Excellent performance

based on real Super-K and T2K performance

Water Cherenkov Technique



Muon

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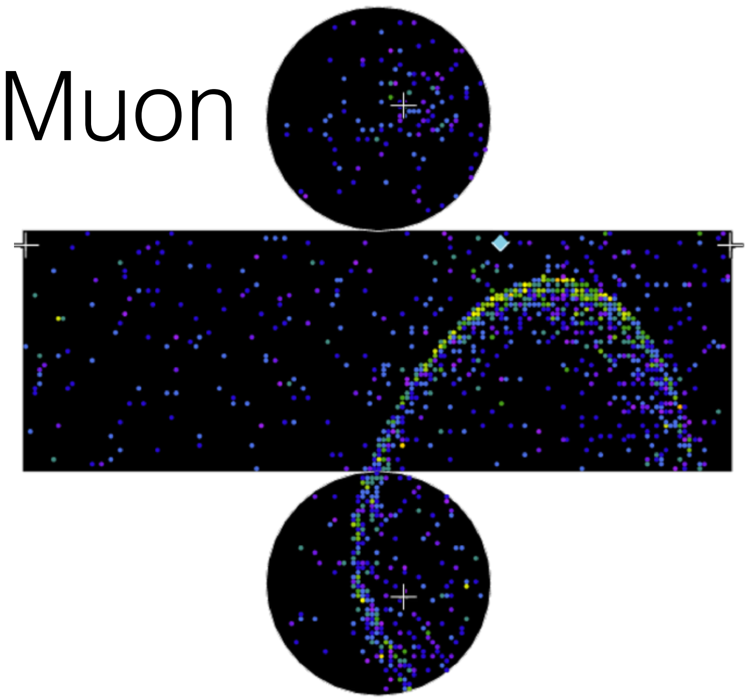
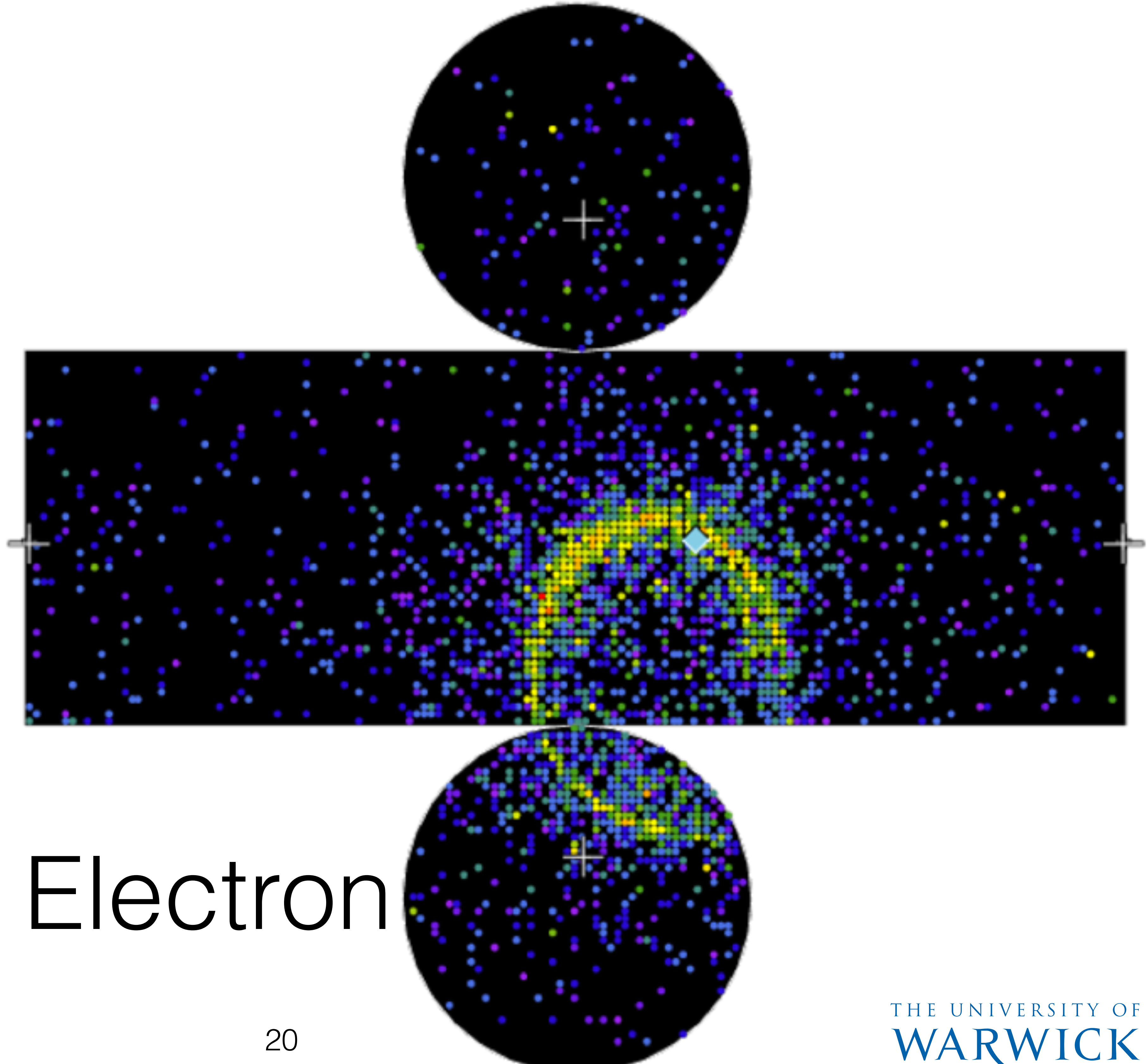
Water Cherenkov Technique

Muon

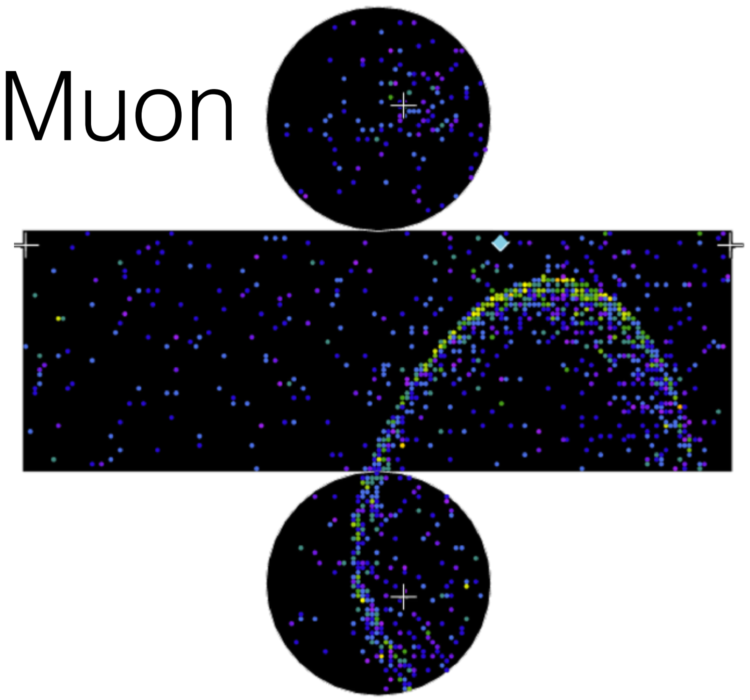




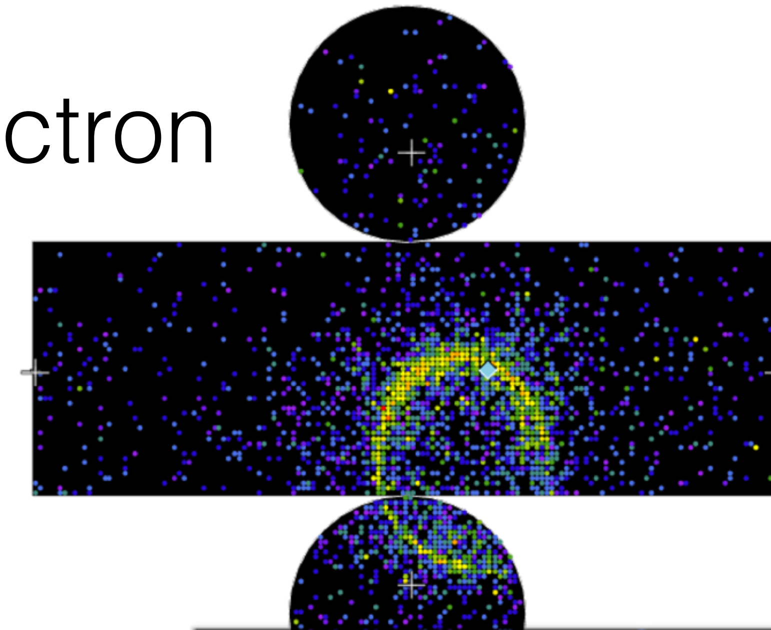
Electron

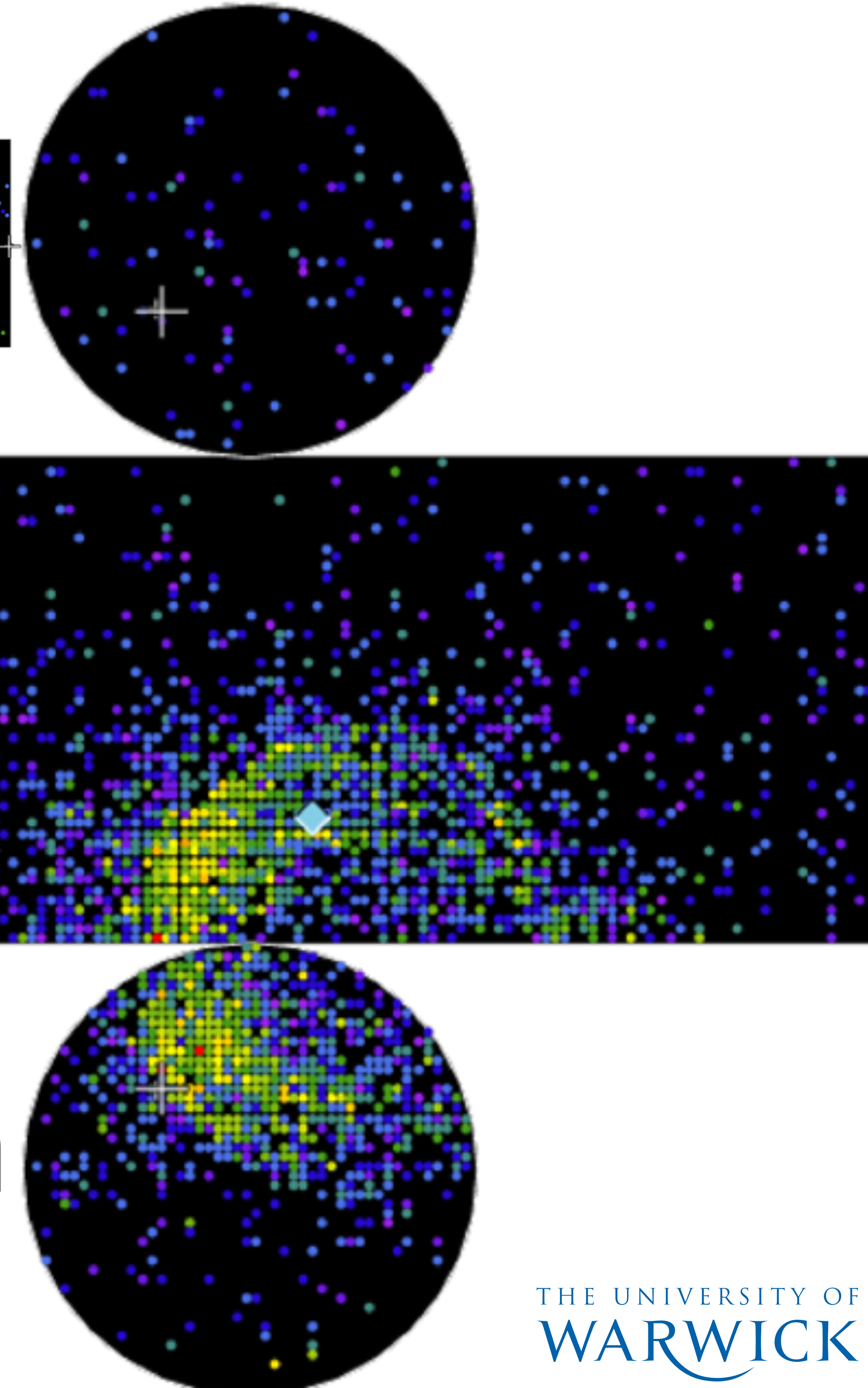
Water Cherenkov Technique

Muon



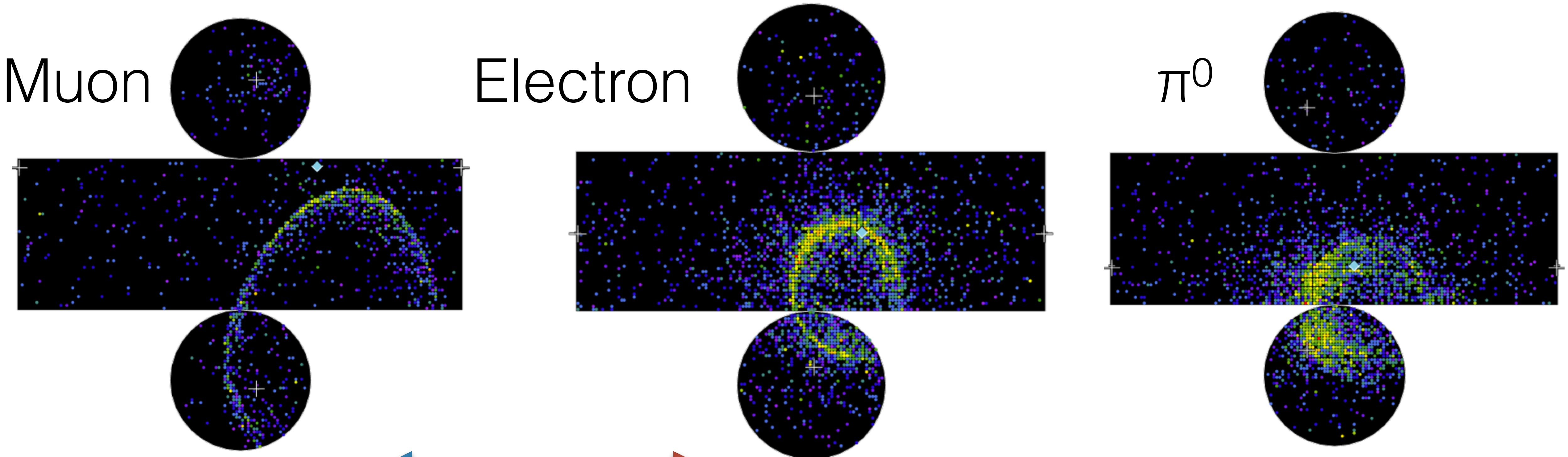
Electron

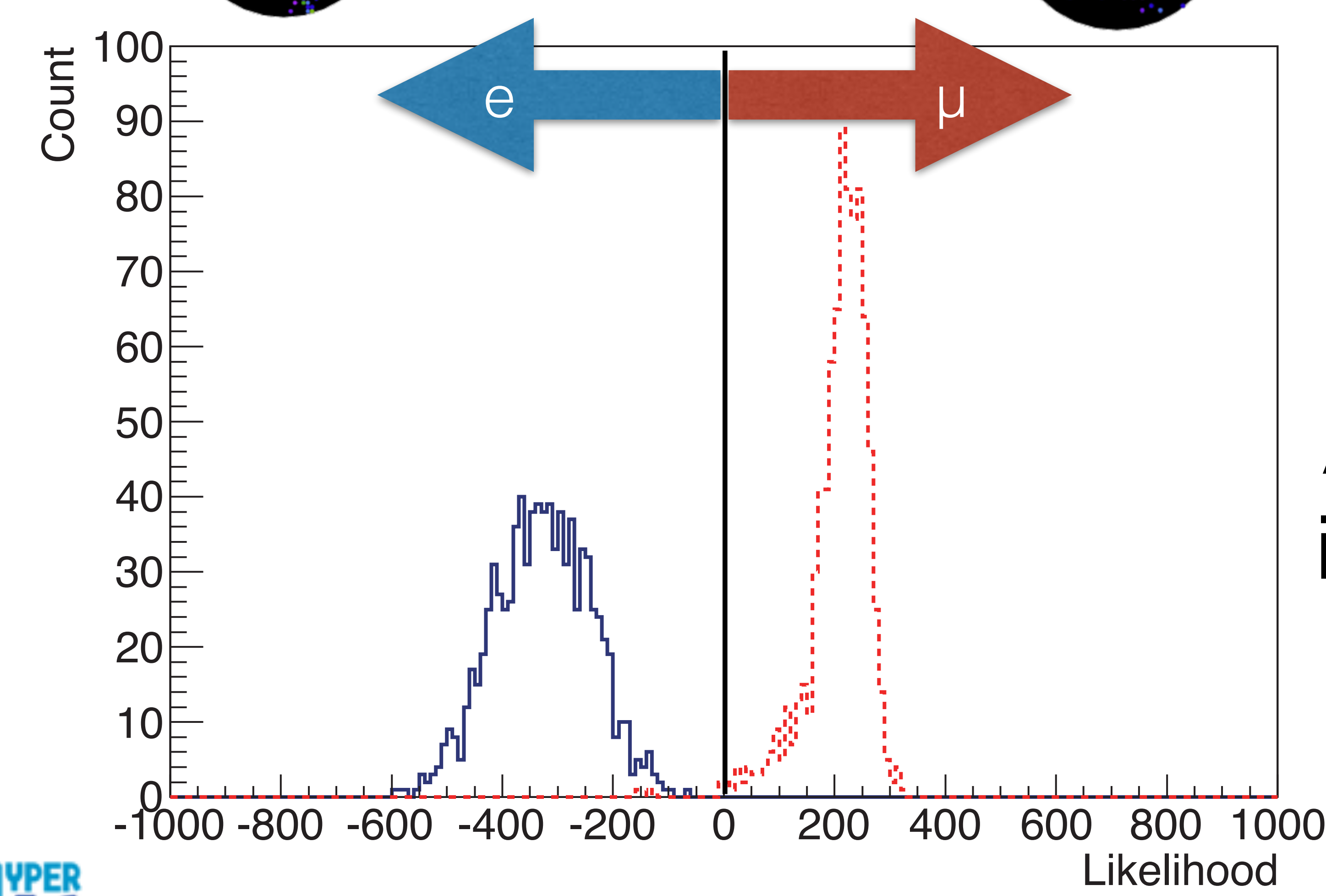




Neutral Pion

Water Cherenkov Technique





Excellent PID performance

Accelerator ν_e background is dominated by irreducible intrinsic ν_e.

Neutrino Energy Measurement

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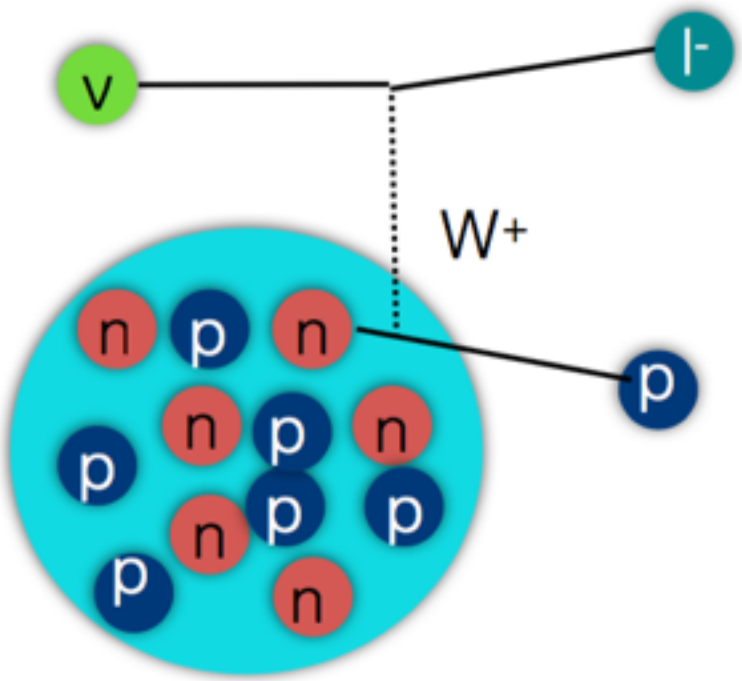


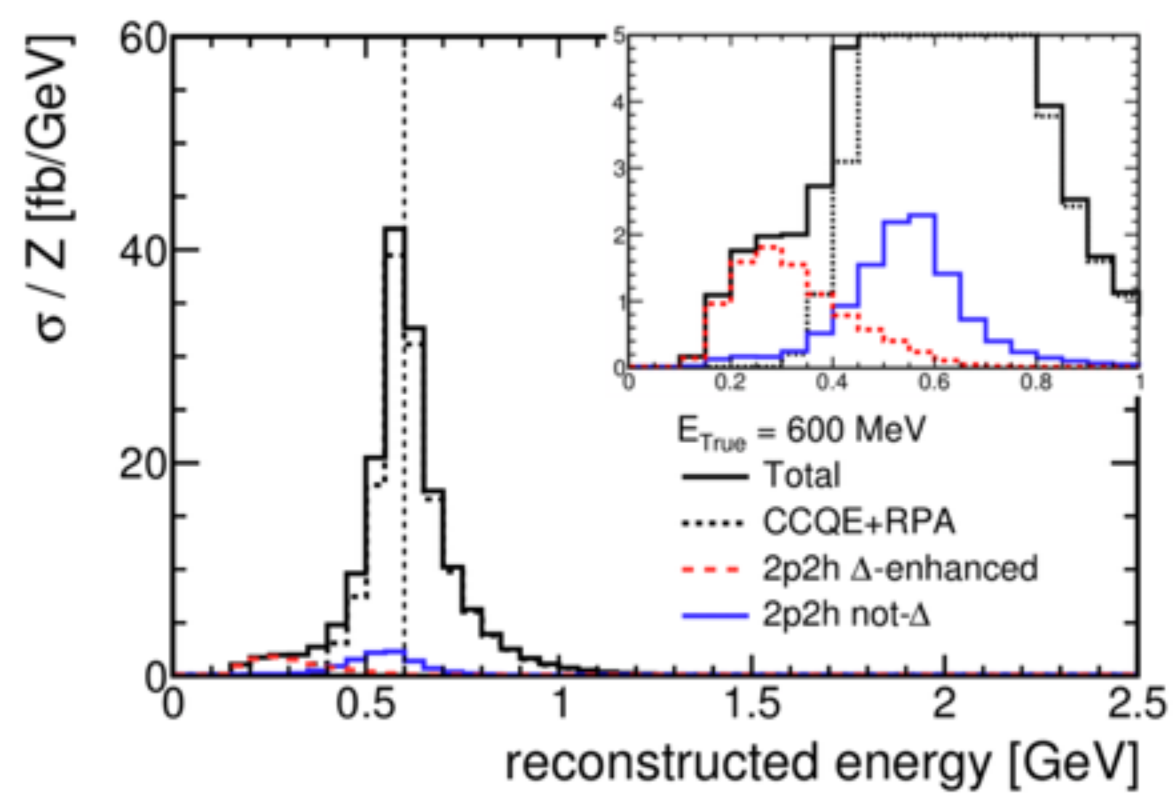
Protons usually below Cherenkov threshold
Neutrons can be counted but no energy measurement

For quasi-elastic interactions neutrino energy can be reconstructed from lepton kinematics

$$E_\nu^{\text{rec}} = \frac{m_p^2 - (m_n - E_b)^2 - m_e^2 + 2(m_n - E_b)E_e}{2(m_n - E_b - E_e + p_e \cos\theta_e)}$$

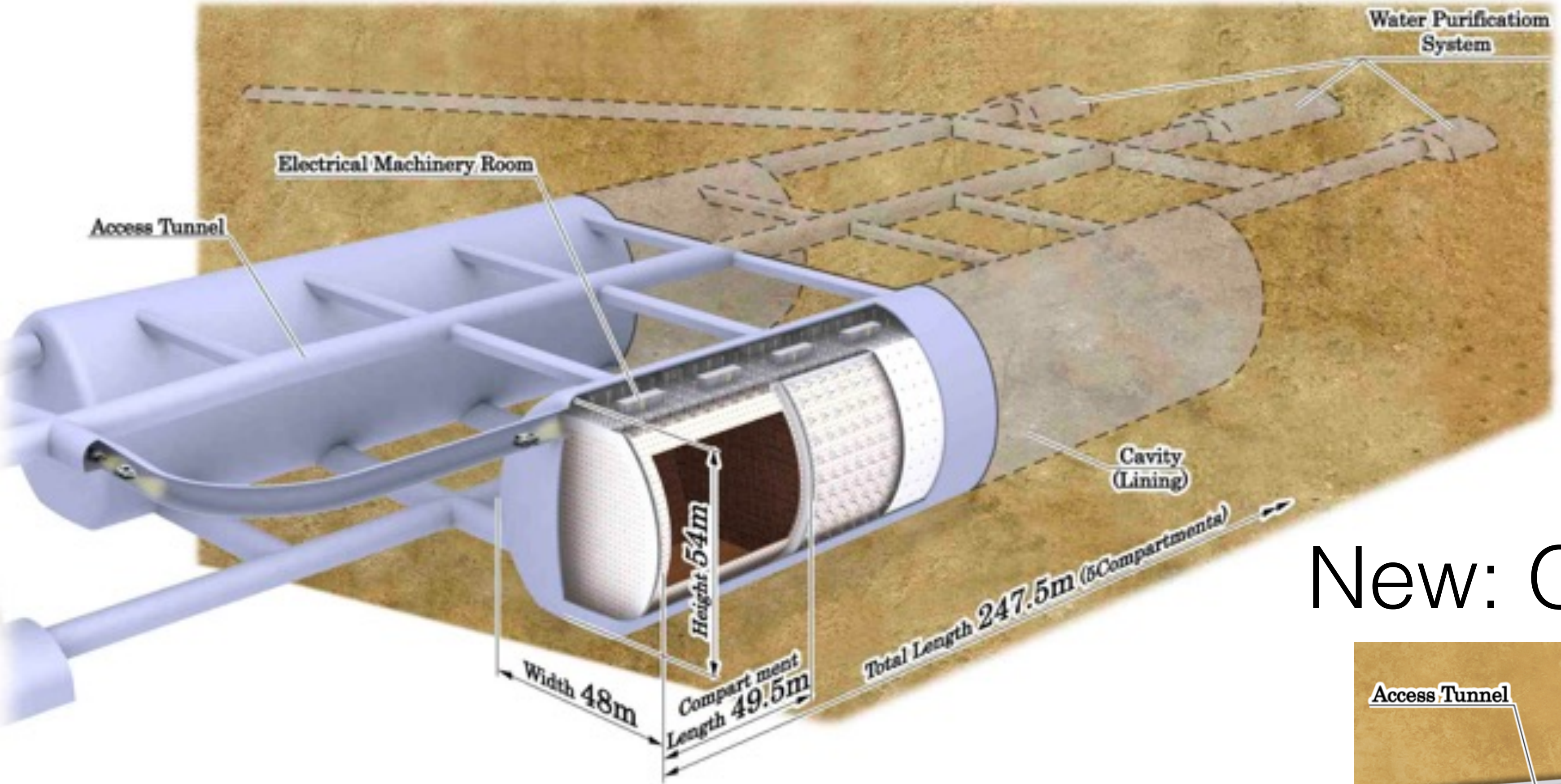
Background from inelastic scattering where energy is mis-measured

Interaction is on bound state
Nuclear effects are important

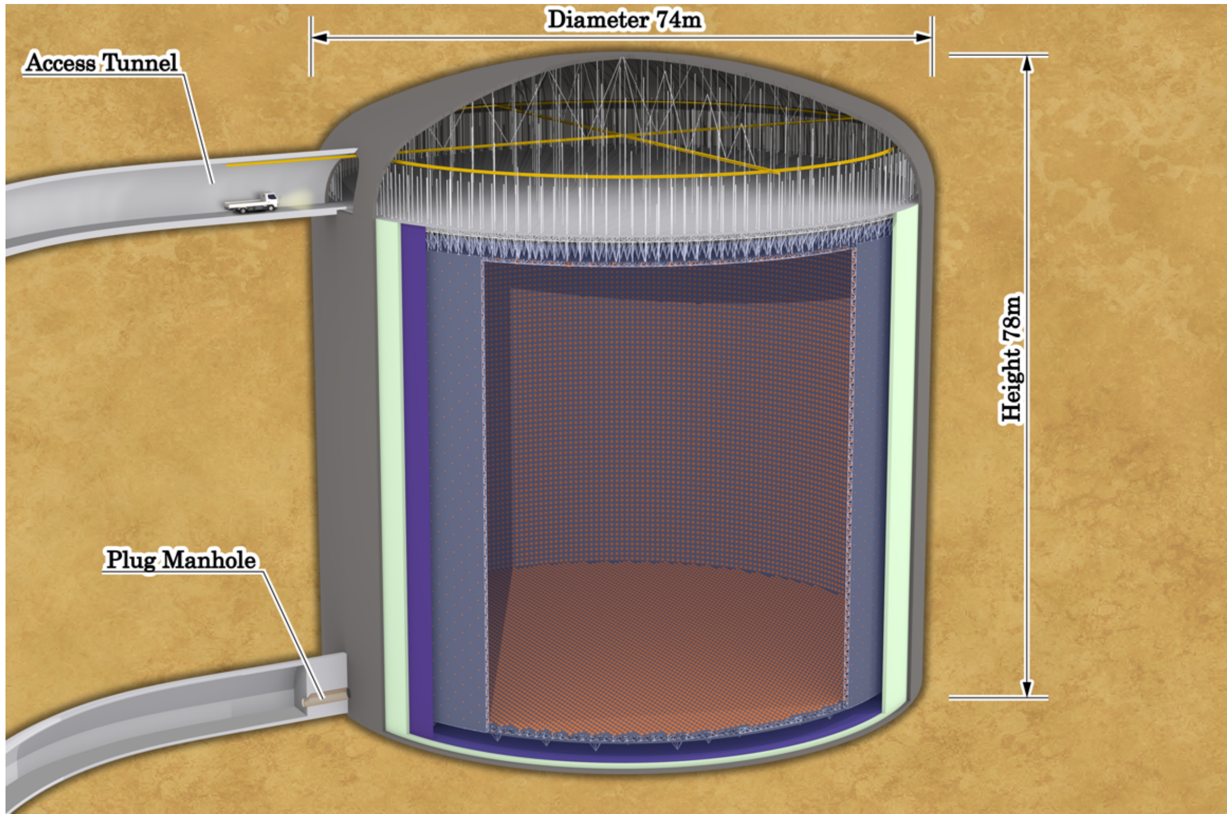


Tank Design

Old: Horizontal Egg-shaped Tank

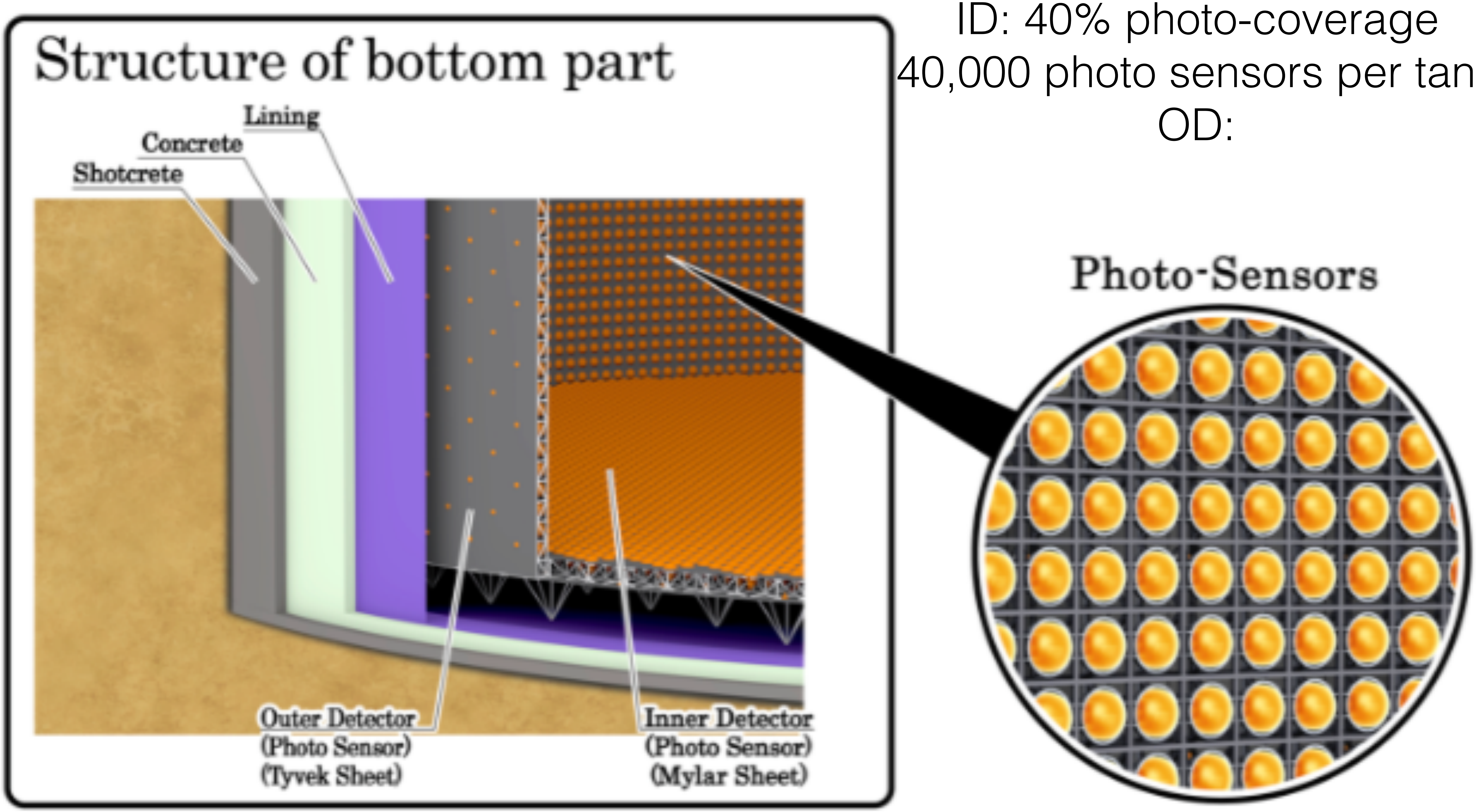


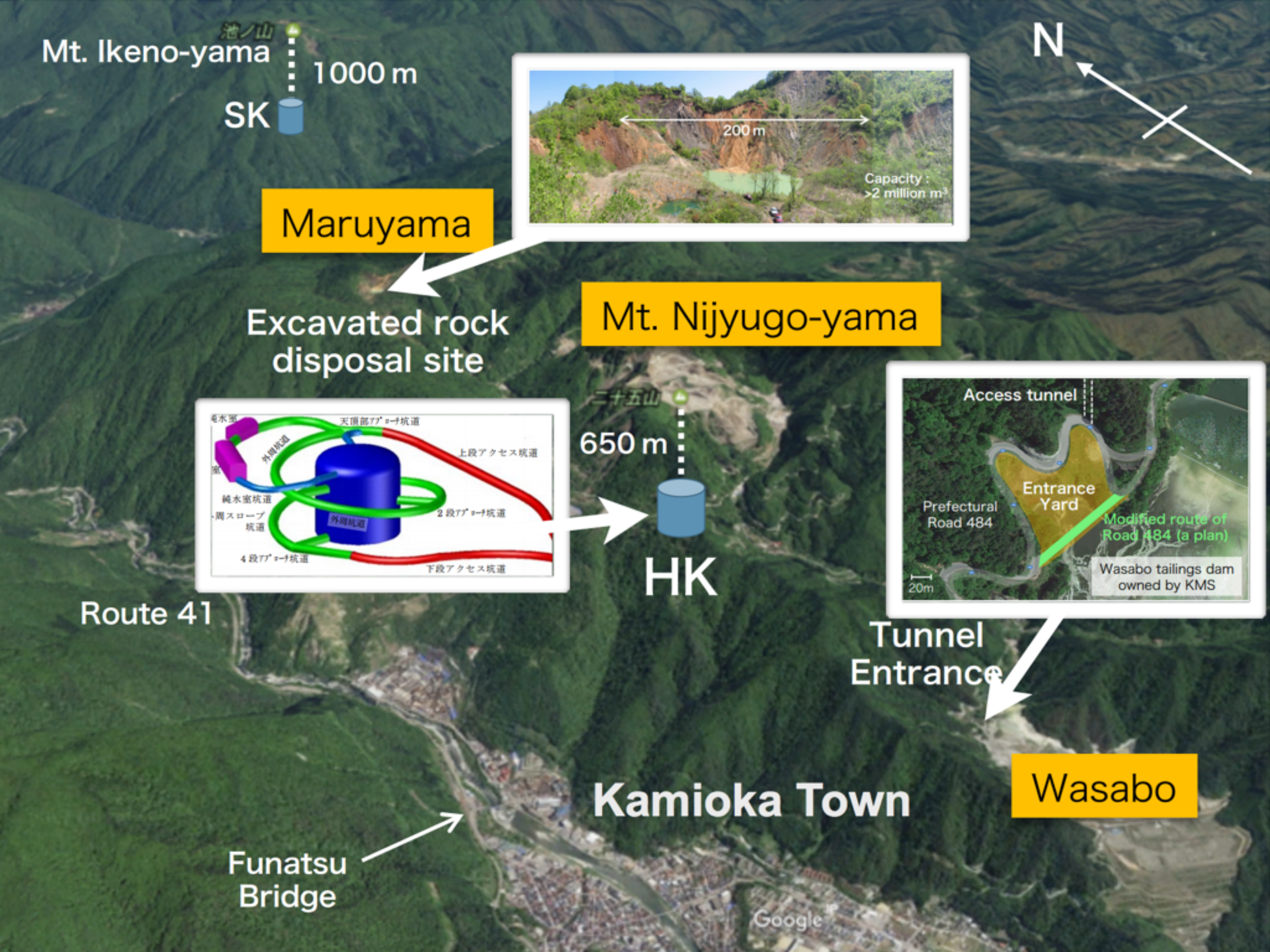
New: Optimised Vertical Tank



Tank Design

ID: 40% photo-coverage
40,000 photo sensors per tank
OD:

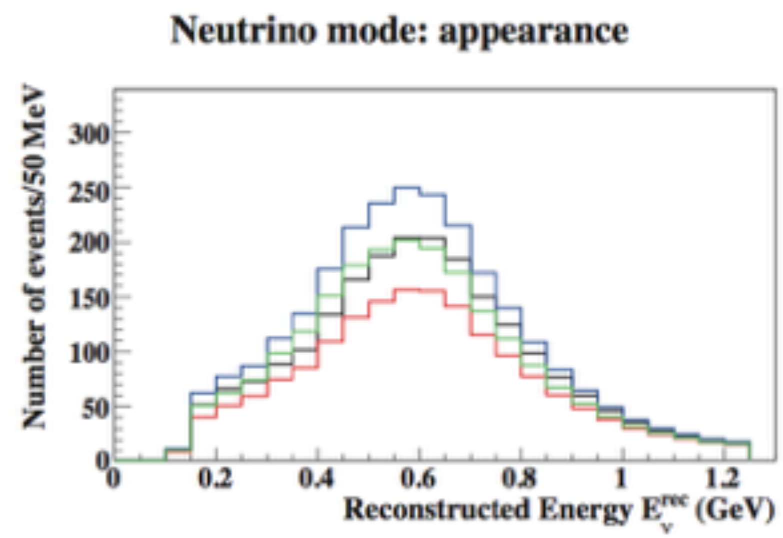


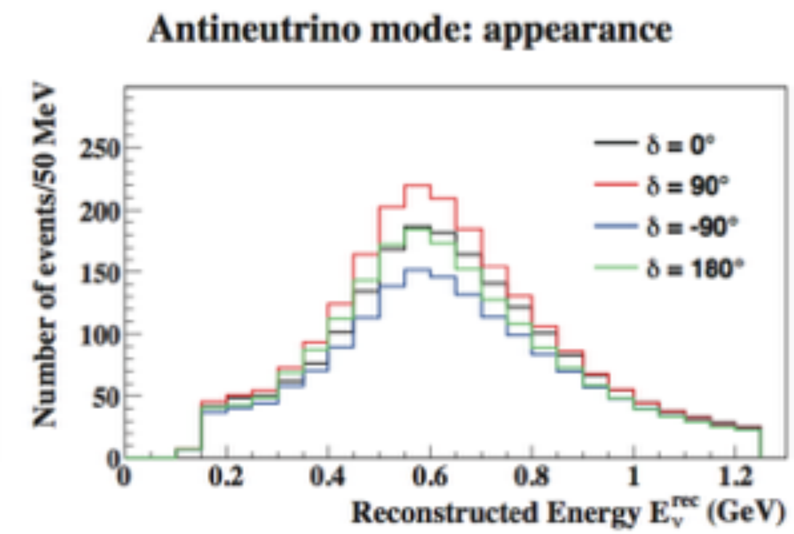


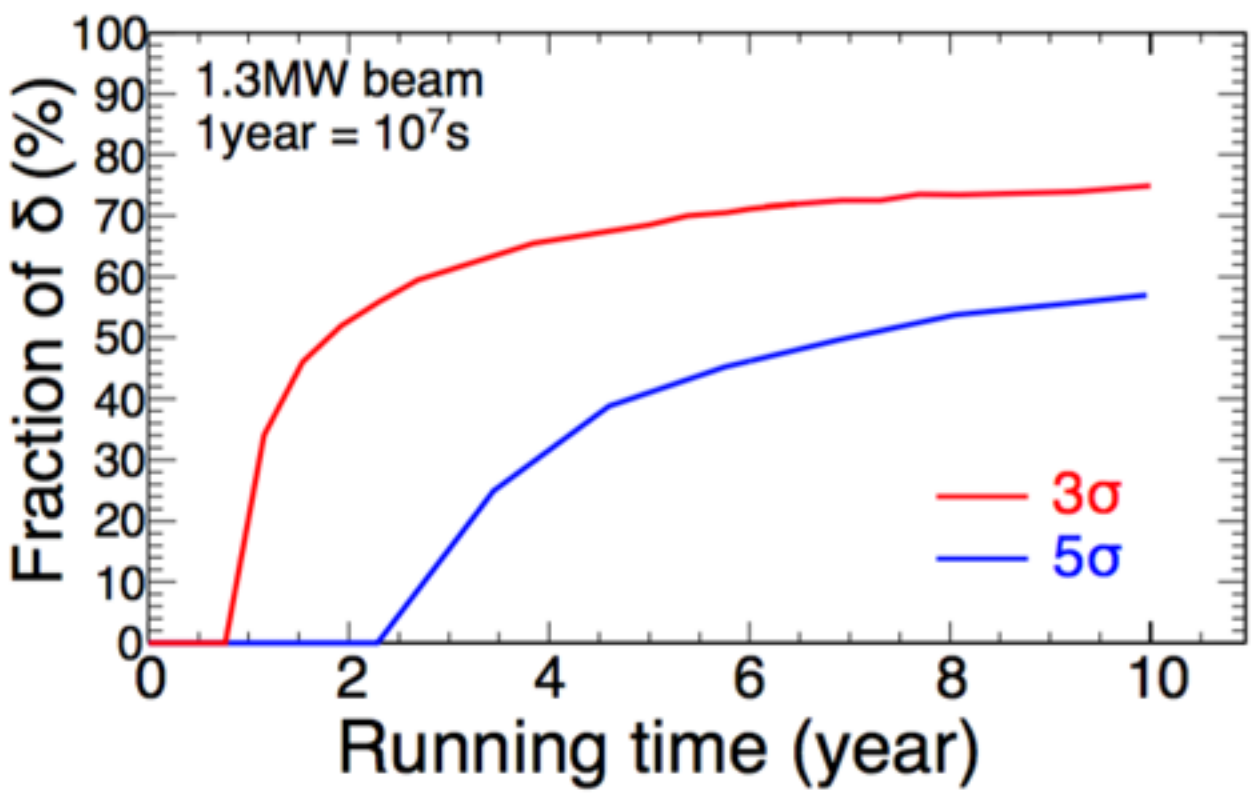
Mt. Ikeno-yama
1000 m
SK
N
200 m
Capacity :
>2 million m³
Maruyama
Excavated rock
disposal site
Mt. Nijyugo-yama
650 m
HK
Access tunnel
Entrance
Yard
Prefectural
Road 484
Modified route of
Road 484 (a plan)
Wasabo tailings dam
owned by KMS
20m
Route 41
Tunnel
Entrance
Wasabo
Kamioka Town
Funatsu
Bridge
Google

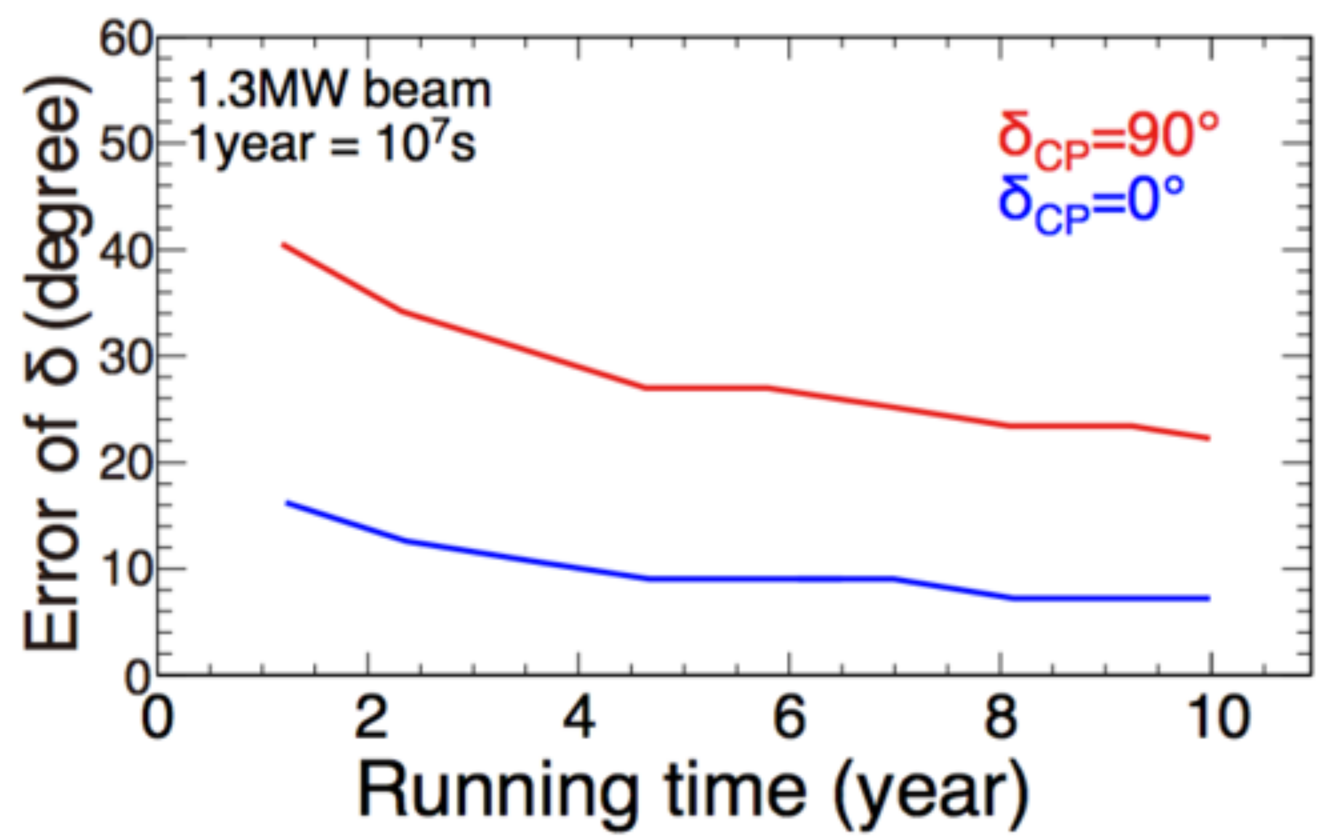
Hyper-K Projected Sensitivity

10 years x 1 tank x 1.3 MW
ν_e ~ 2058, $\bar{\nu}_e$ ~ 1906 events





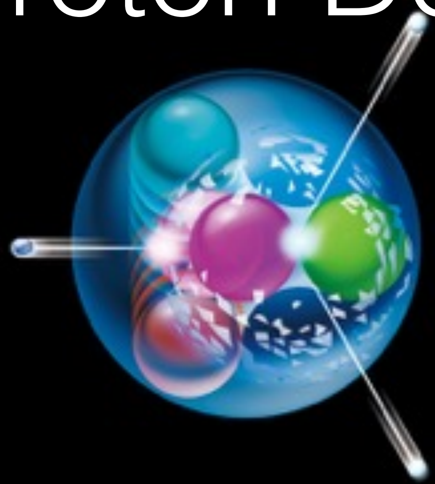


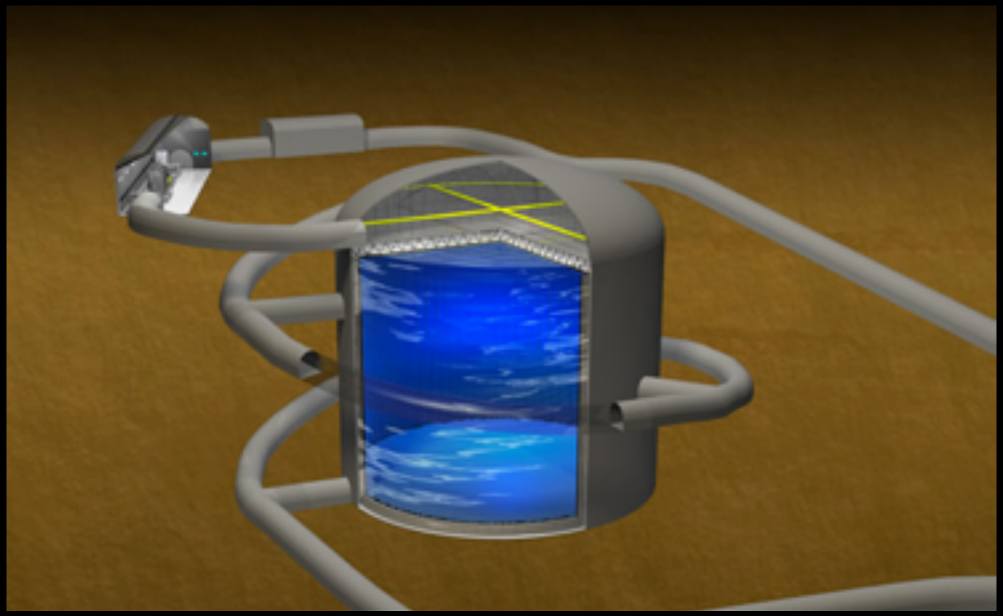


Assuming 3-4% systematic uncertainty (cf T2K present ~6%)

Physics at Hyper-K

Proton Decay

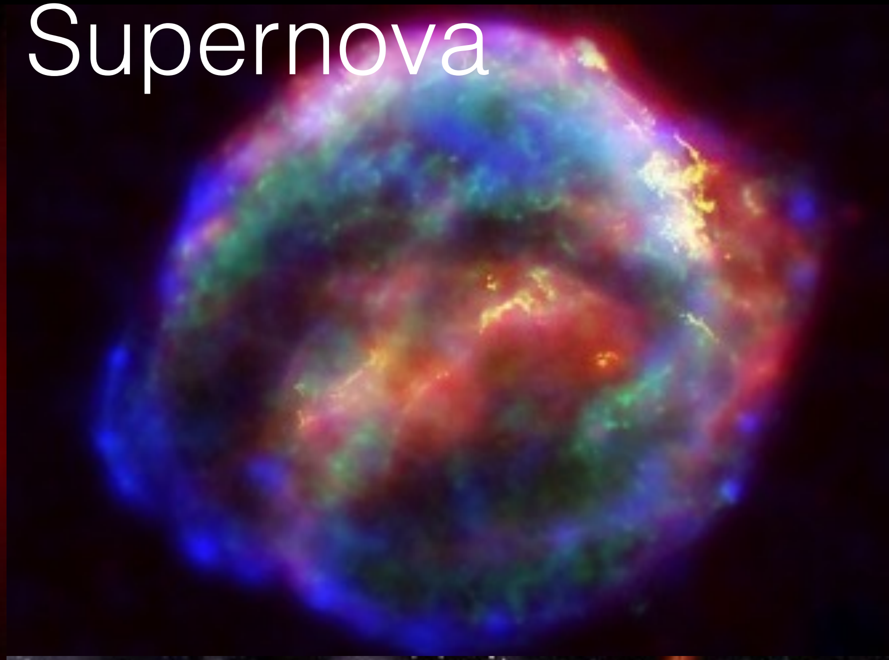




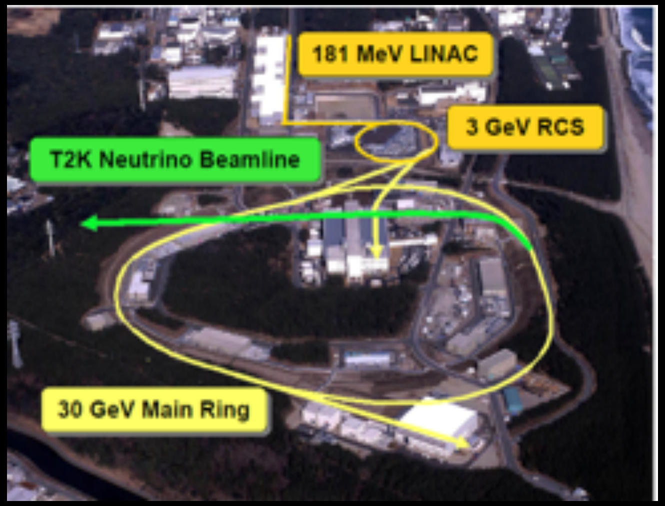
Neutrinos

Solar

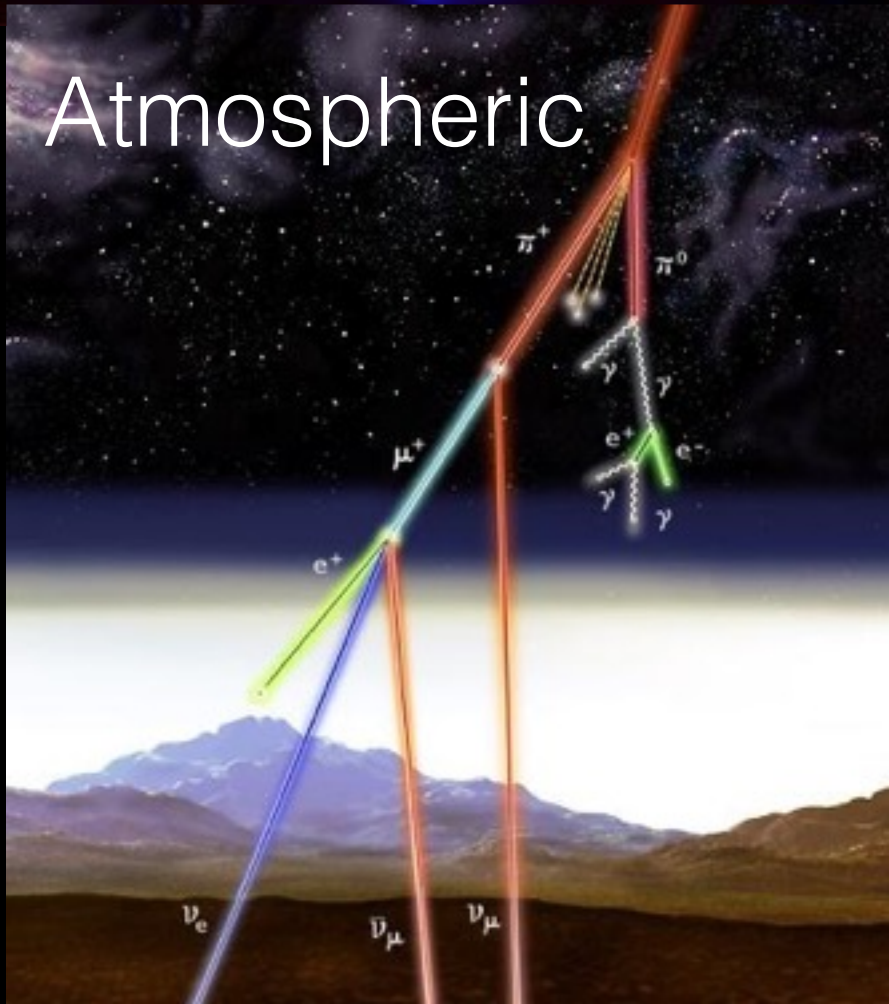
Supernova



Accelerator



Atmospheric



Broad physics programme.

Statistics

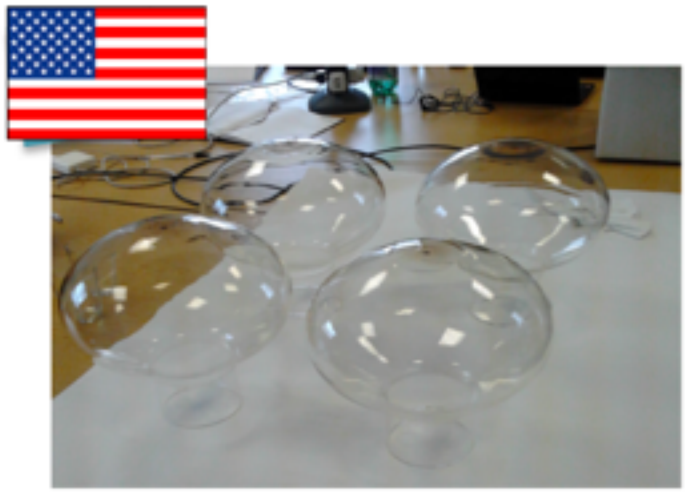
Experiment	$\nu_e + \bar{\nu}_e$	$1/\sqrt{N}$	Ref.
T2K (current)	74 + 7	12% + 40%	2.2×10^{21} POT
NOvA (current)	33	17%	FERMILAB-PUB-17-065-ND
NOvA (projected)	110 + 50	10% + 14%	arXiv:1409.7469 [hep-ex]
T2K-I (projected)	150 + 50	8% + 14%	7.8×10^{21} POT, arXiv:1409.7469 [hep-ex]
T2K-II	470 + 130	5% + 9%	20×10^{21} POT, arXiv1607.08004 [hep-ex]
Hyper-K	2058 + 1906	2% + 2%	10 yrs 1-tank 2017 Design Report TBR
DUNE	1200 + 350	3% + 5%	3.5+3.5 yrs x 40kt @ 1.07 MW arXiv:1512.06148 [physics.ins-det]

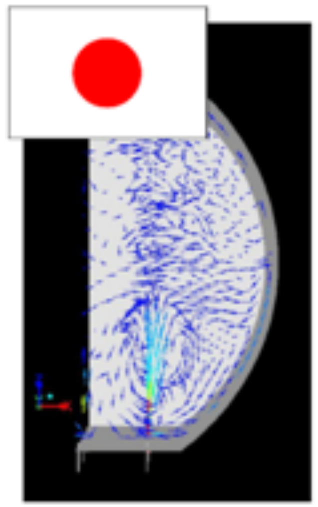
Current appearance measurements stats dominate

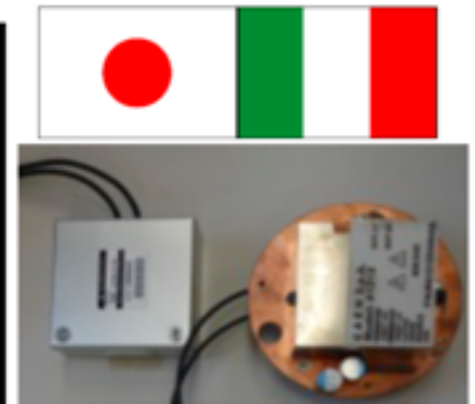
$O(10^3)$ ν_e at future experiments → demands ~2% systematics

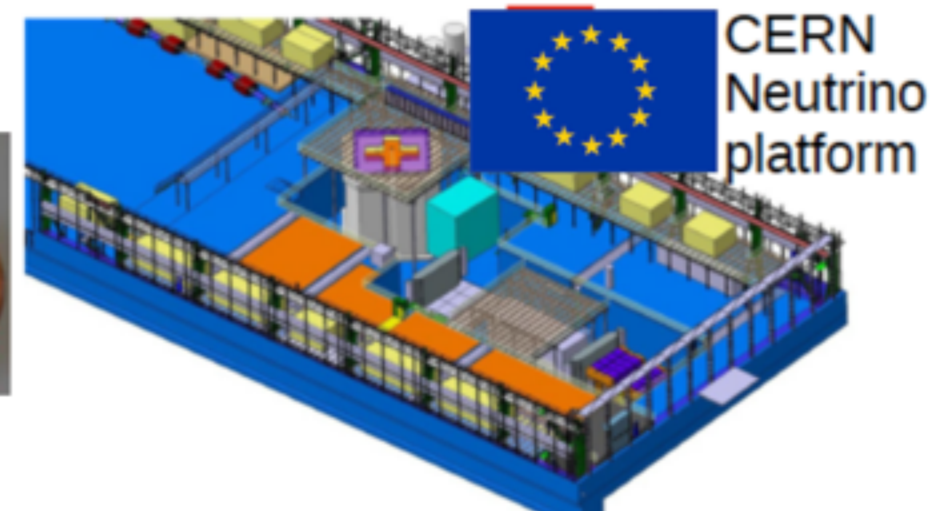
$O(10^4)$ ν_μ → need systematics as good as we can get!

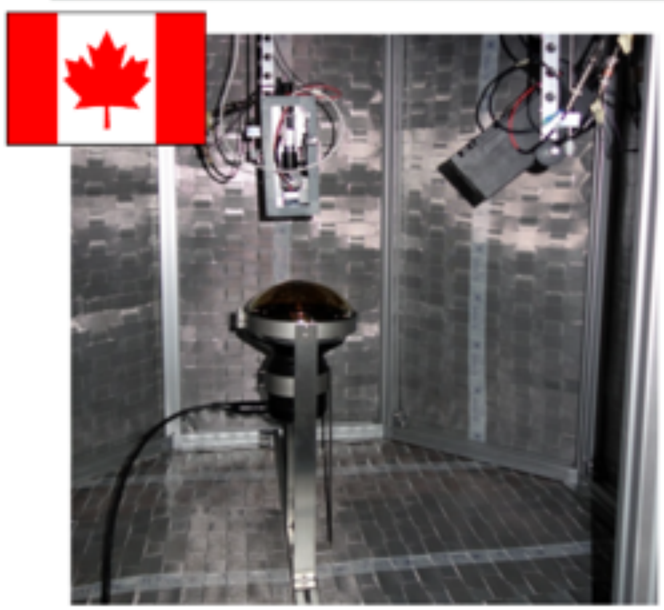
Worldwide R&D

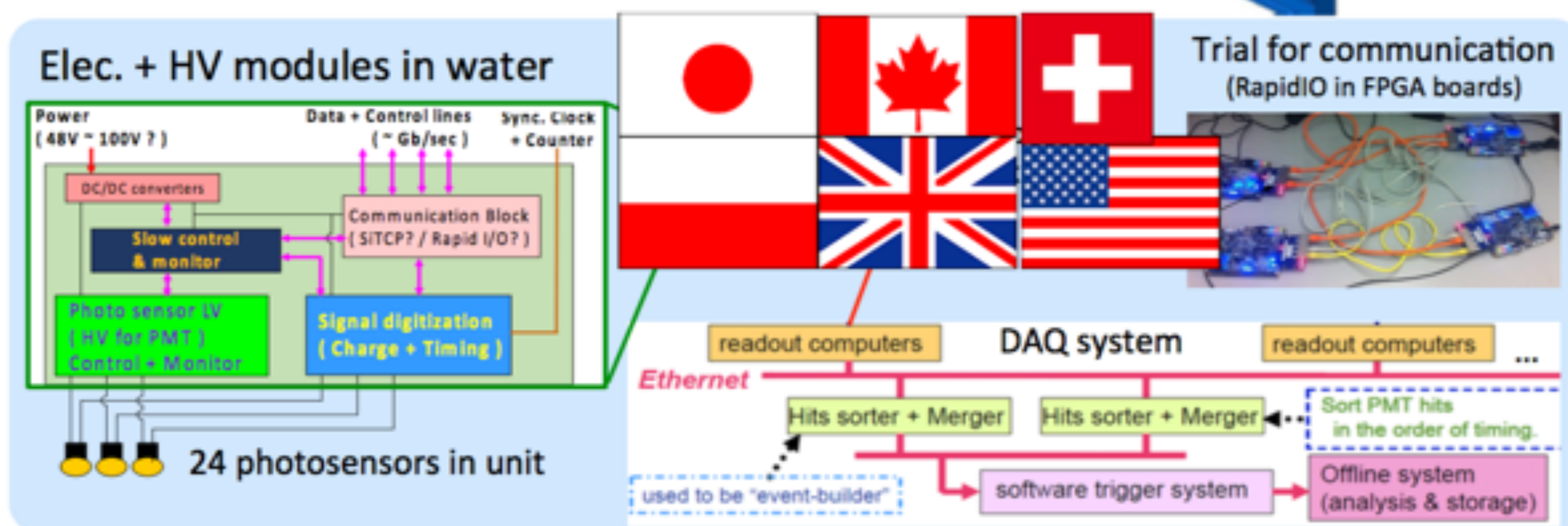


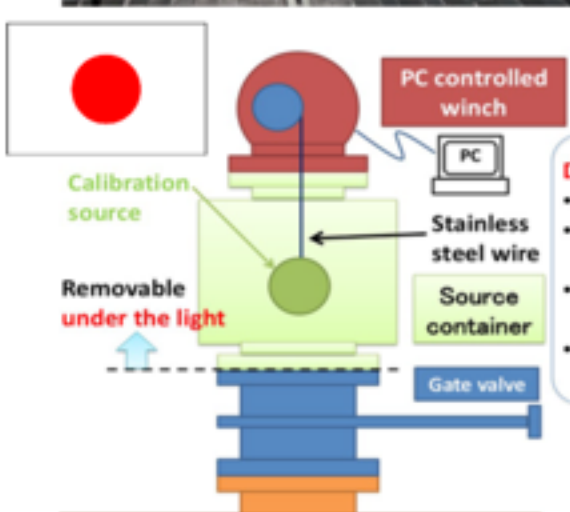






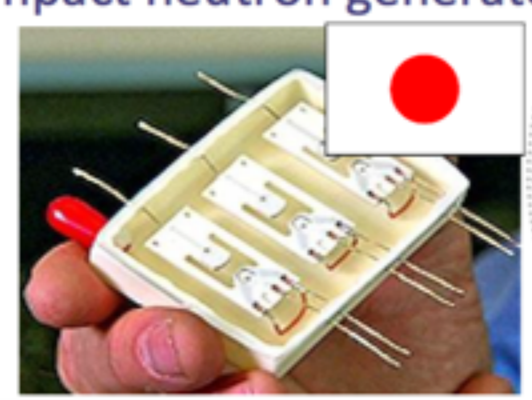






LED

Compact neutron generator



IEEE TRANSACTIONS ON PLASMA SCIENCE,
VOL. 40, NO. 9, SEPTEMBER 2012

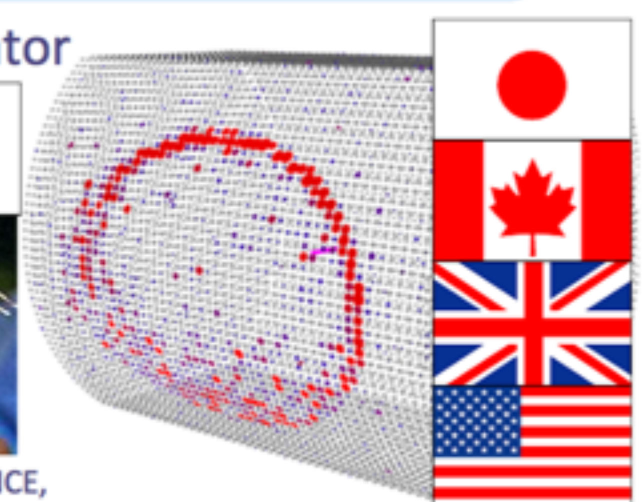
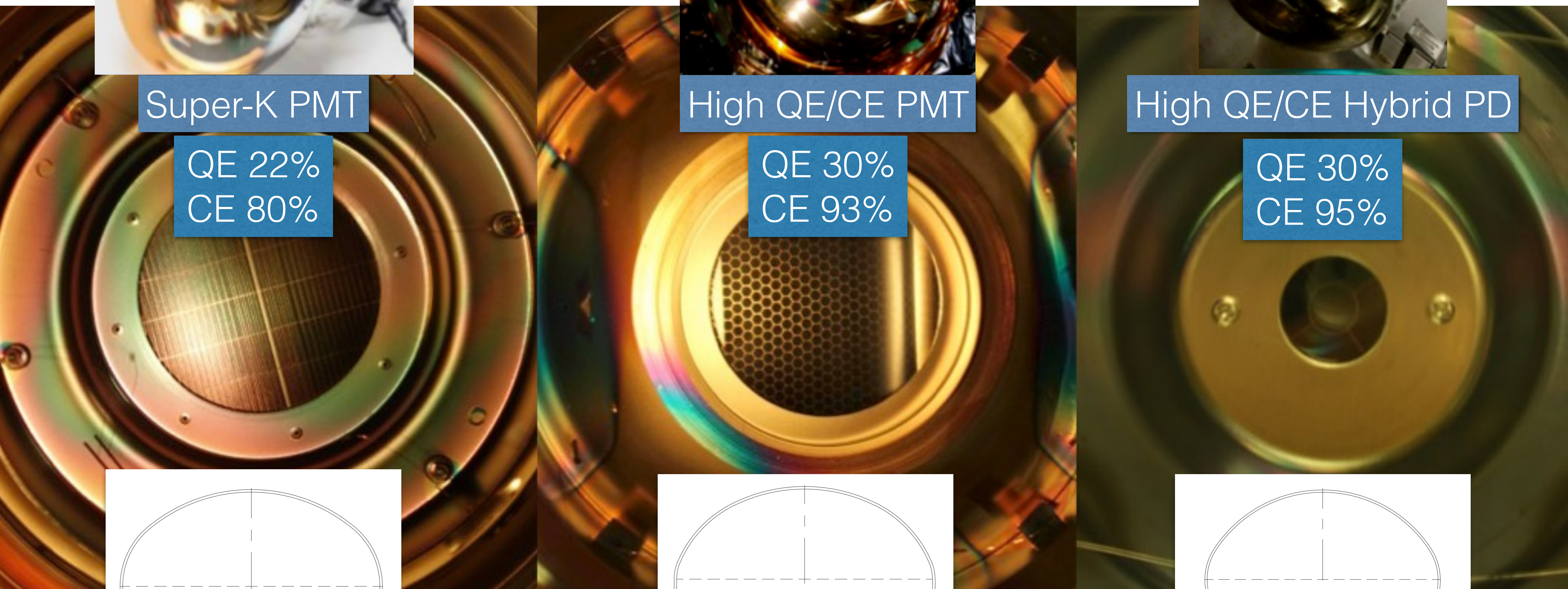
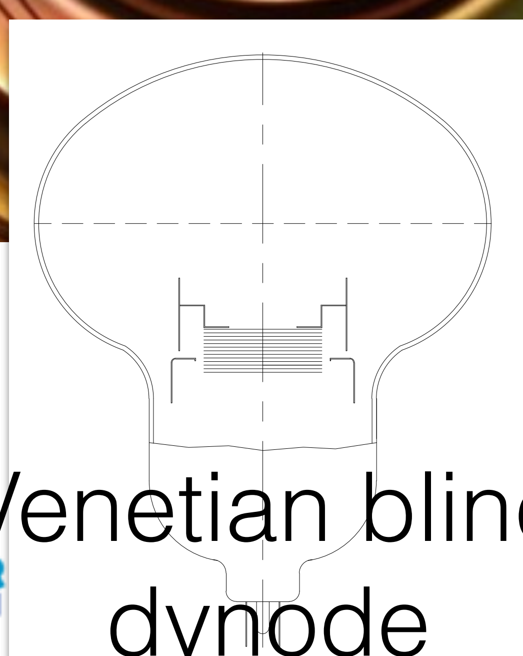
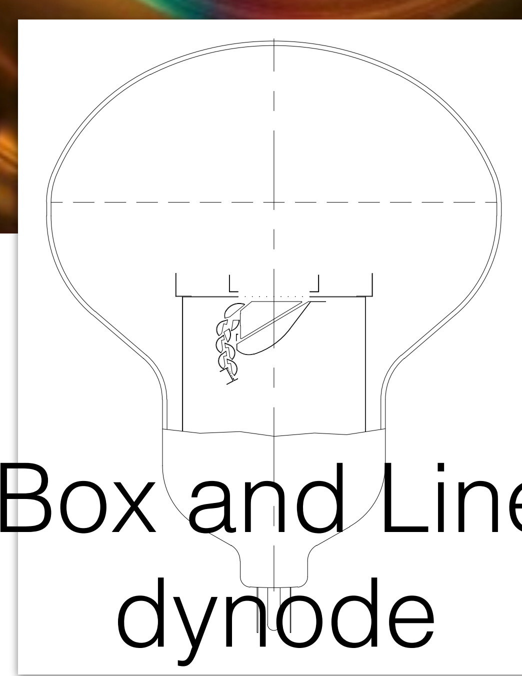


Photo Sensors





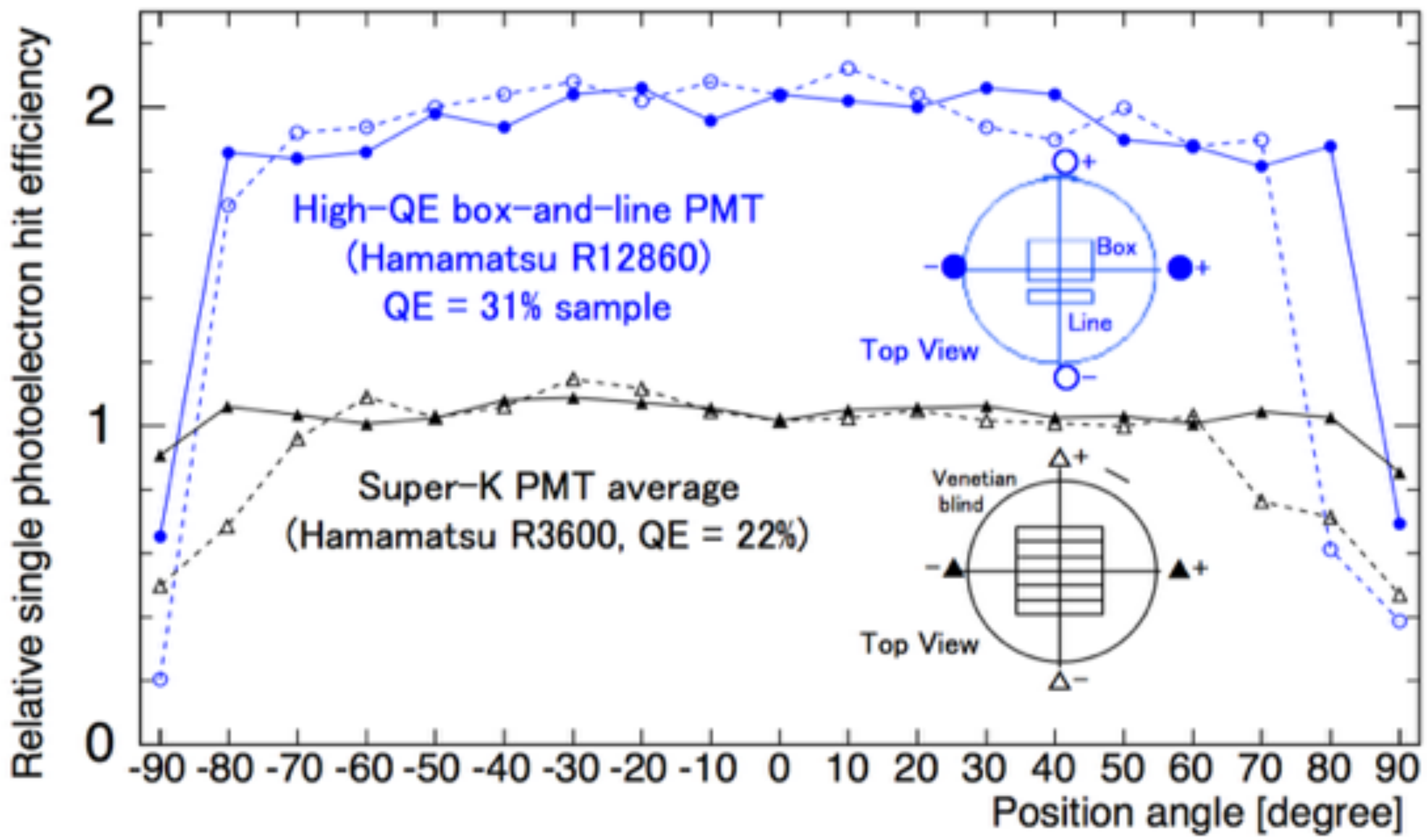
Venetian blind dynode

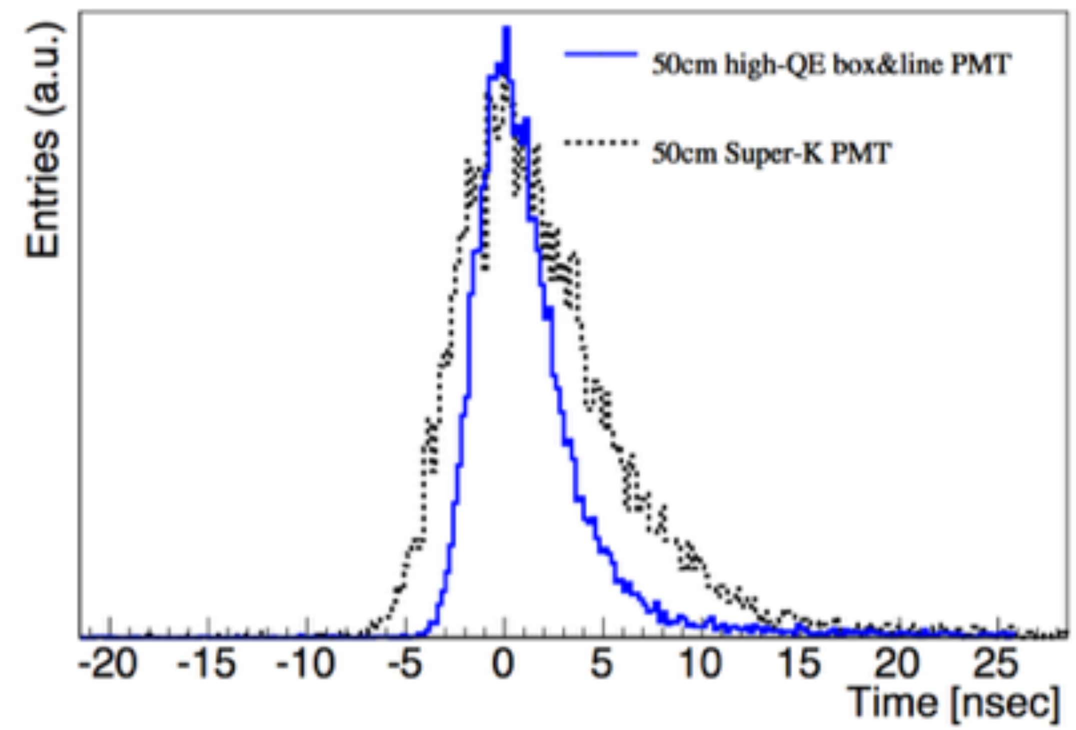


Box and Line dynode

Avalanche diode

Photo Sensors





2x improvement in photon detection efficiency

Better timing and charge resolution

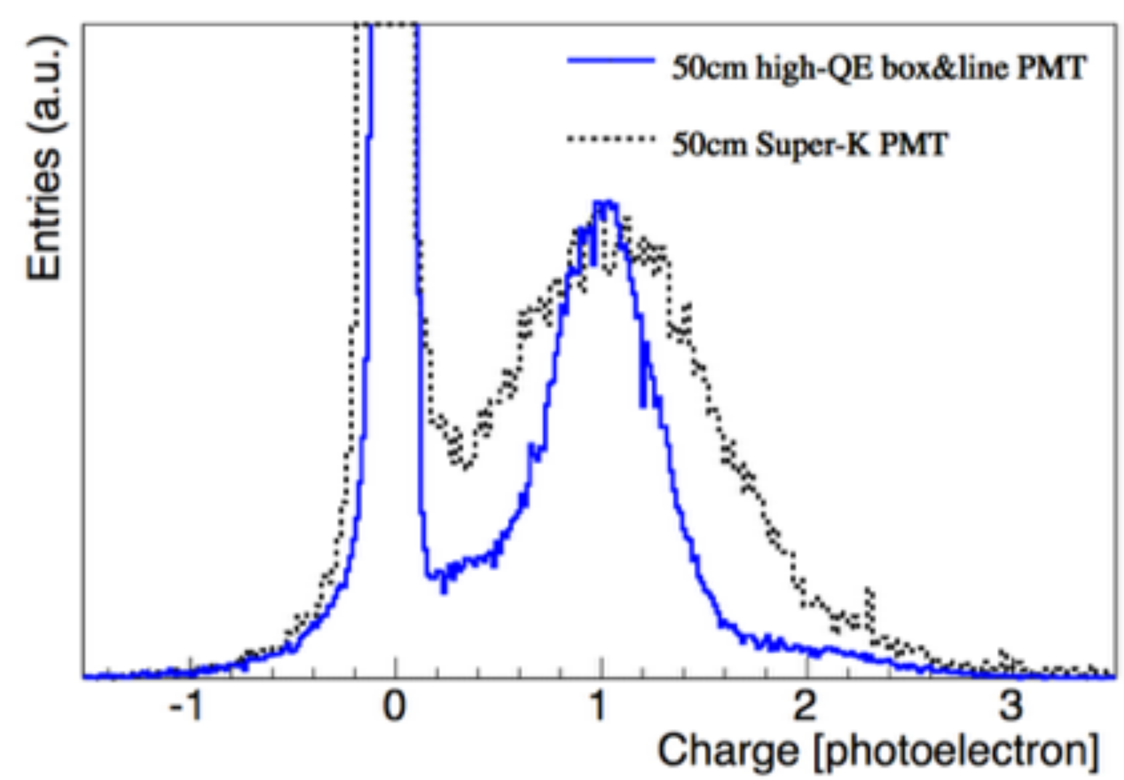
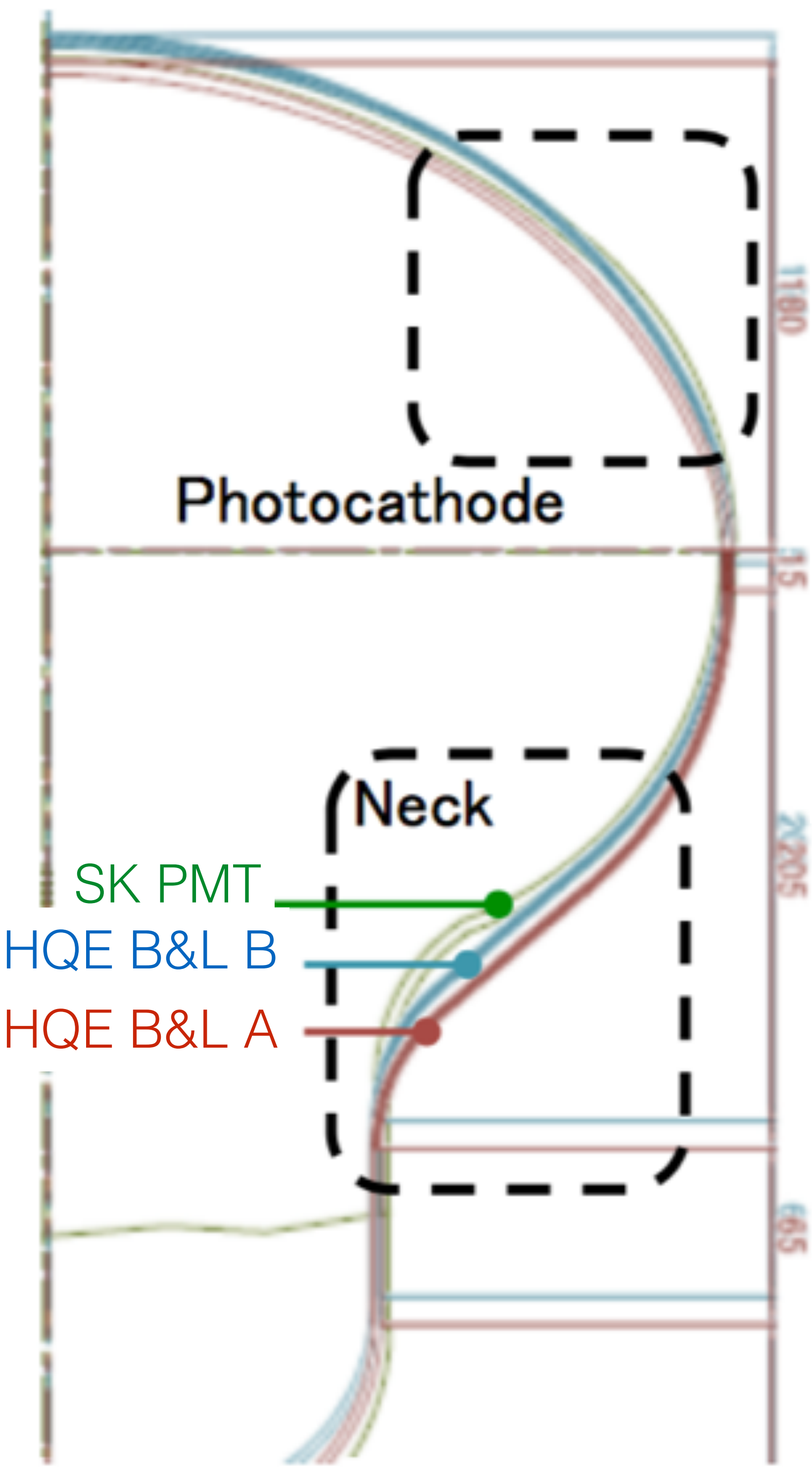
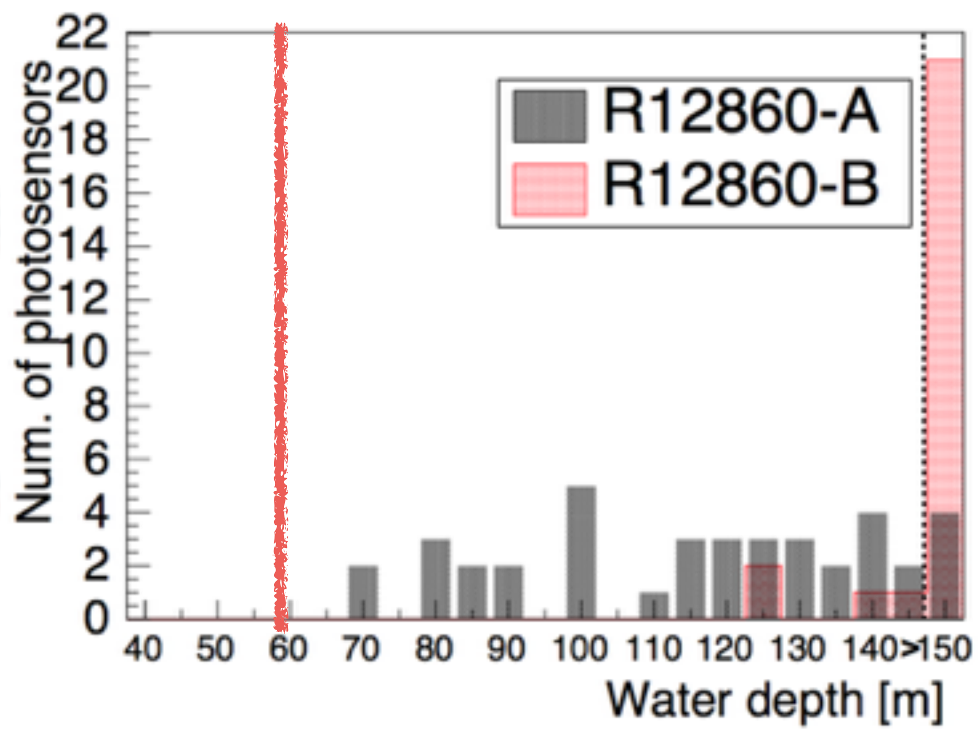
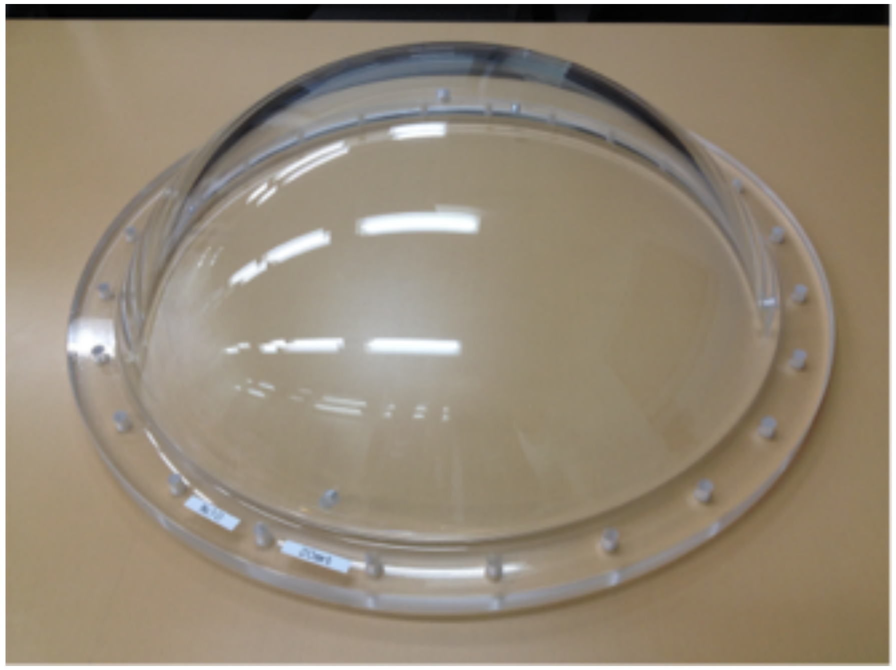


Photo Sensors



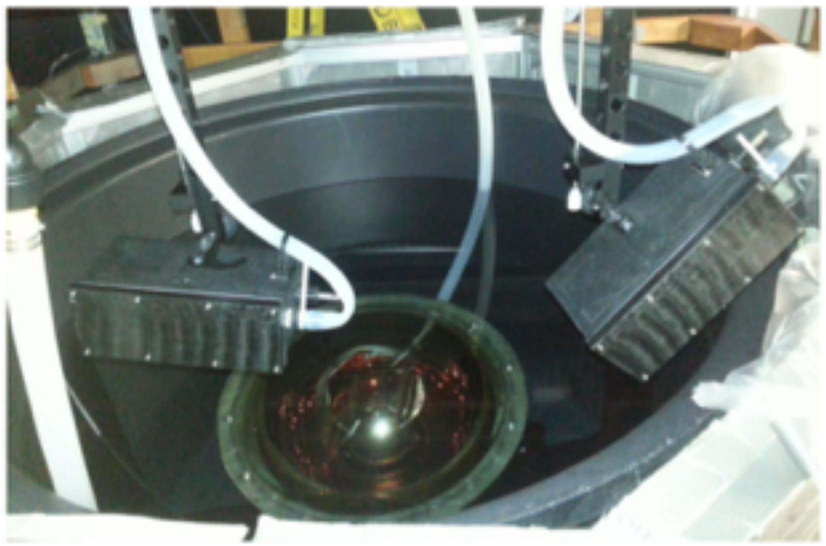


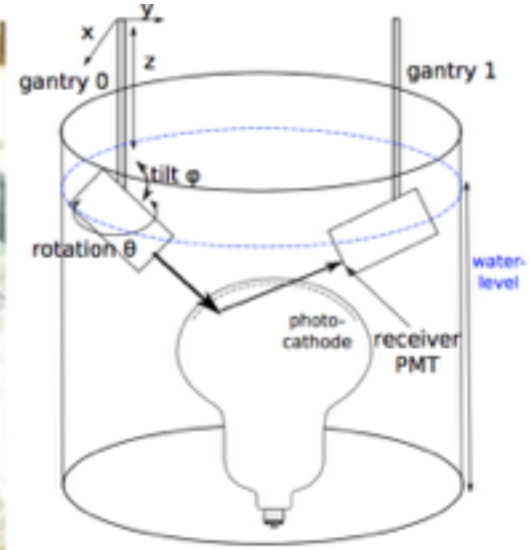


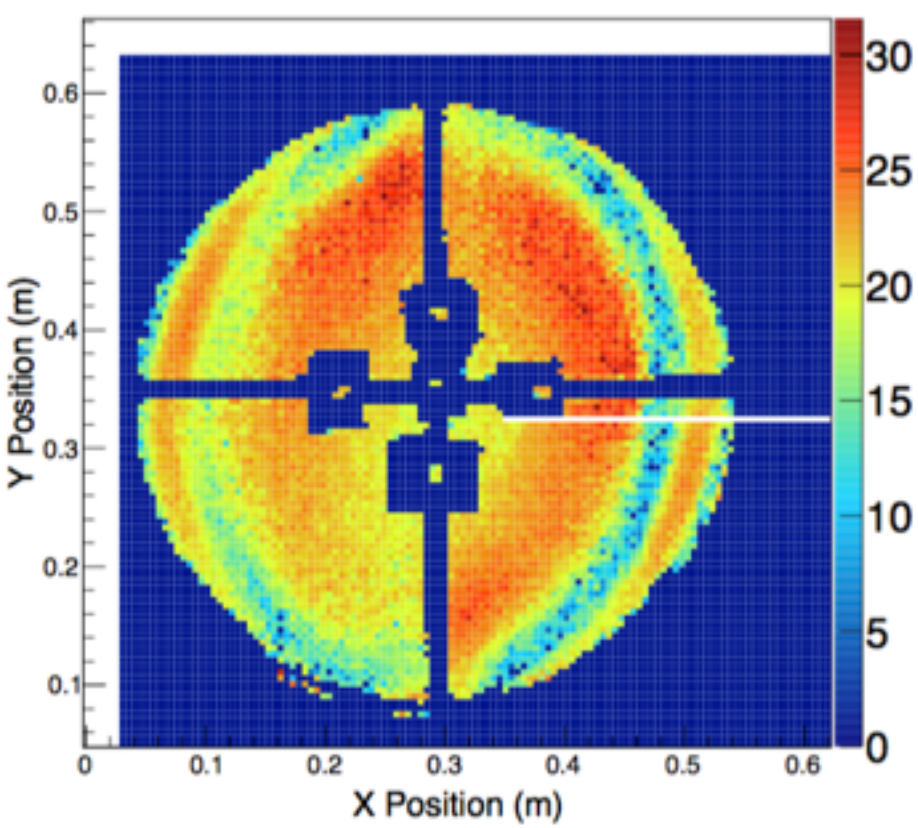
Optimised bulb design
High pressure and implosion tests show new PMTs safe for use in HK tank

Calibration

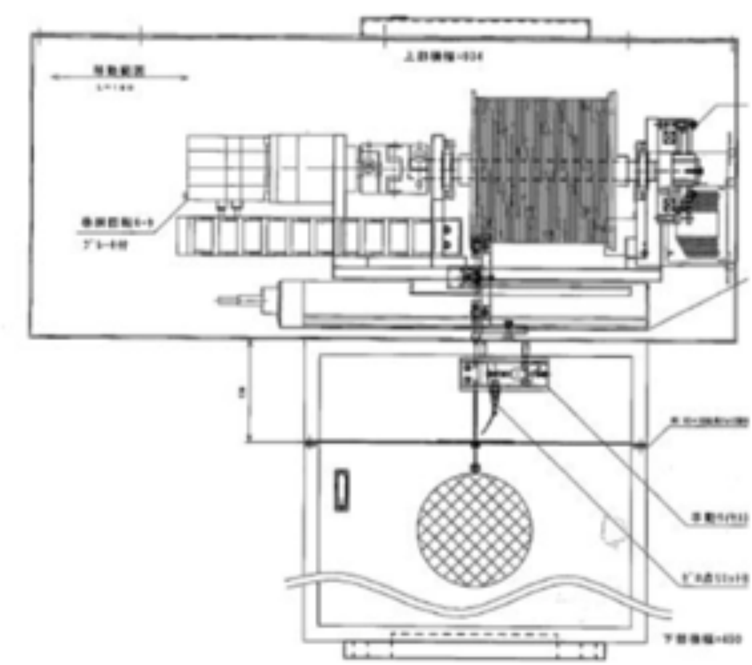
Precise PMT response testing



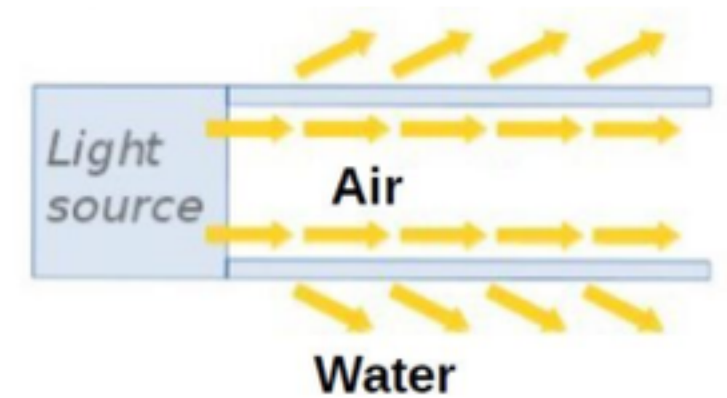




Automated source deployment

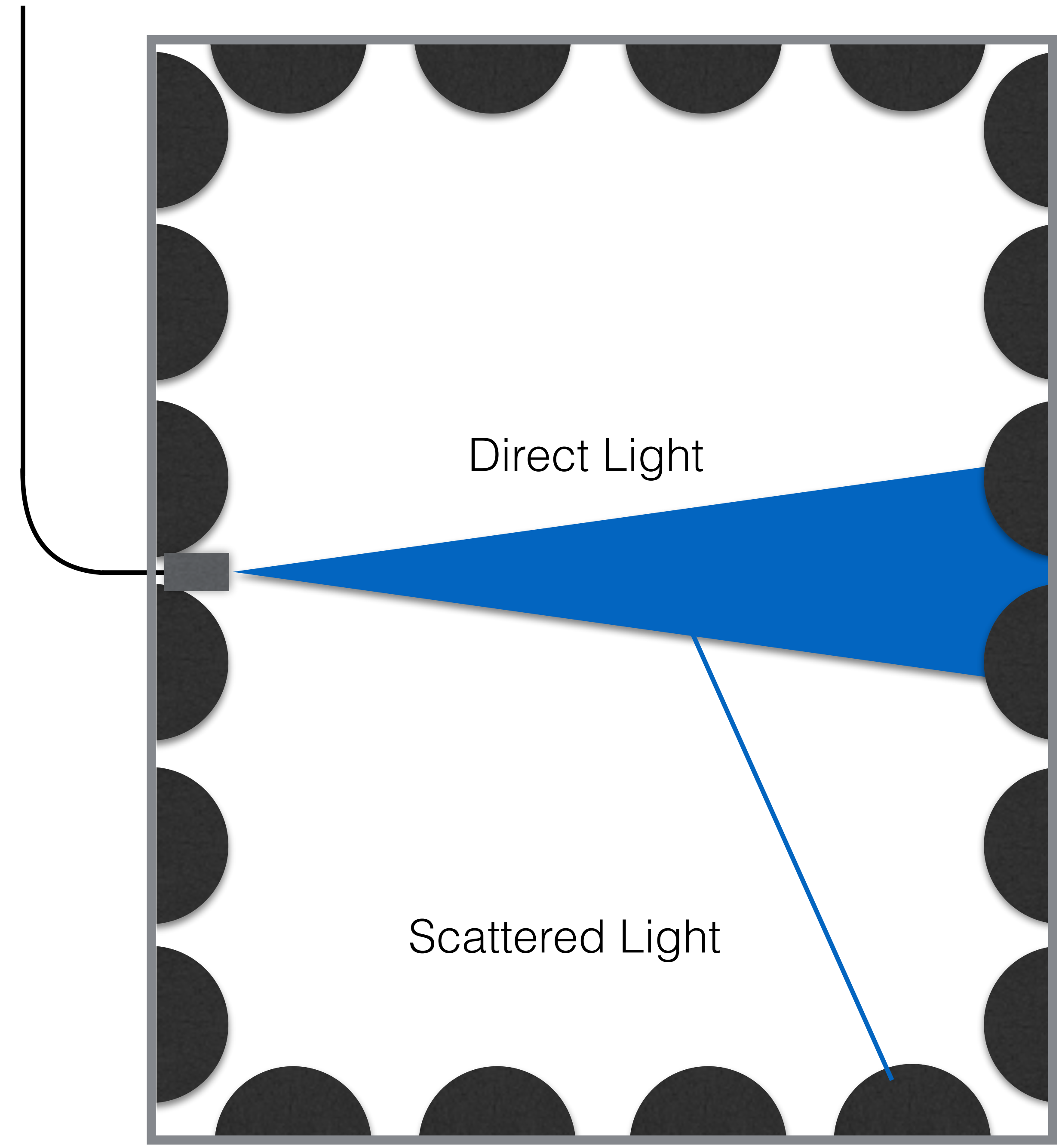


Fake muon source

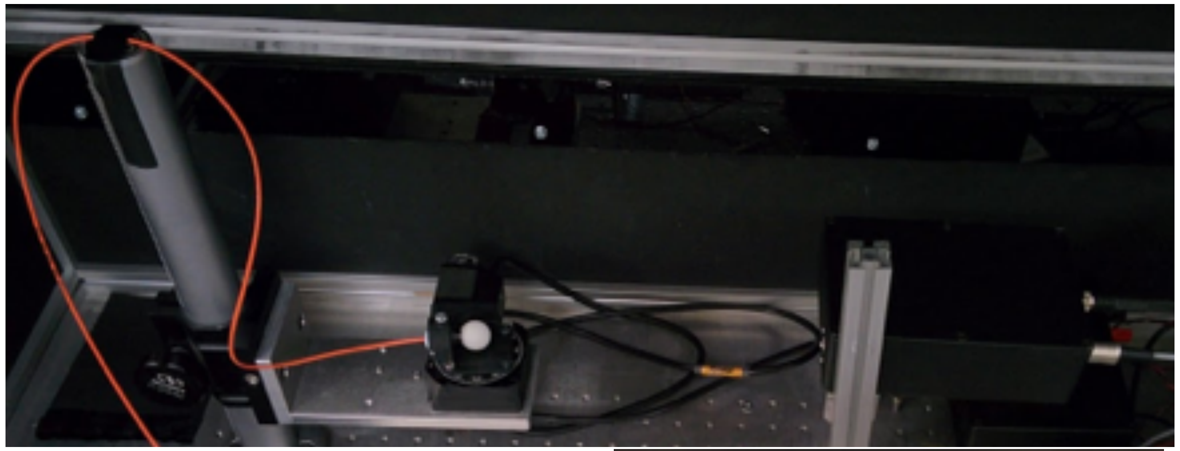


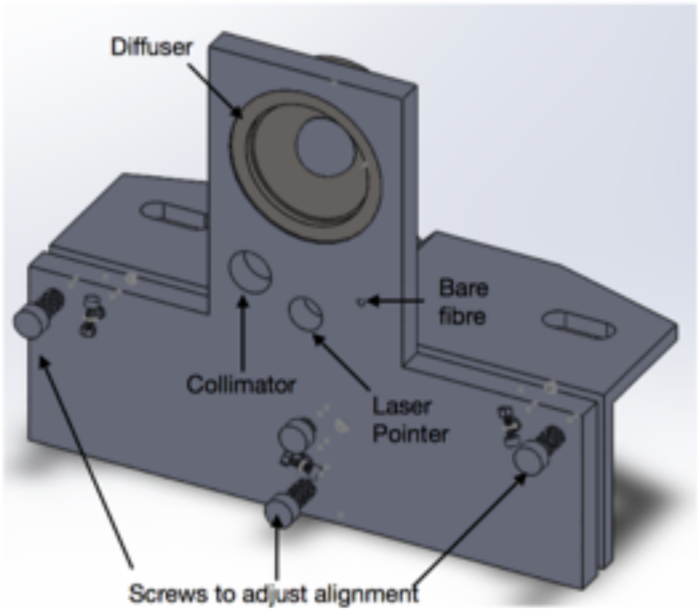
“Neutristor” Neutron Generator

Calibration



R&D for new optical calibration system in progress

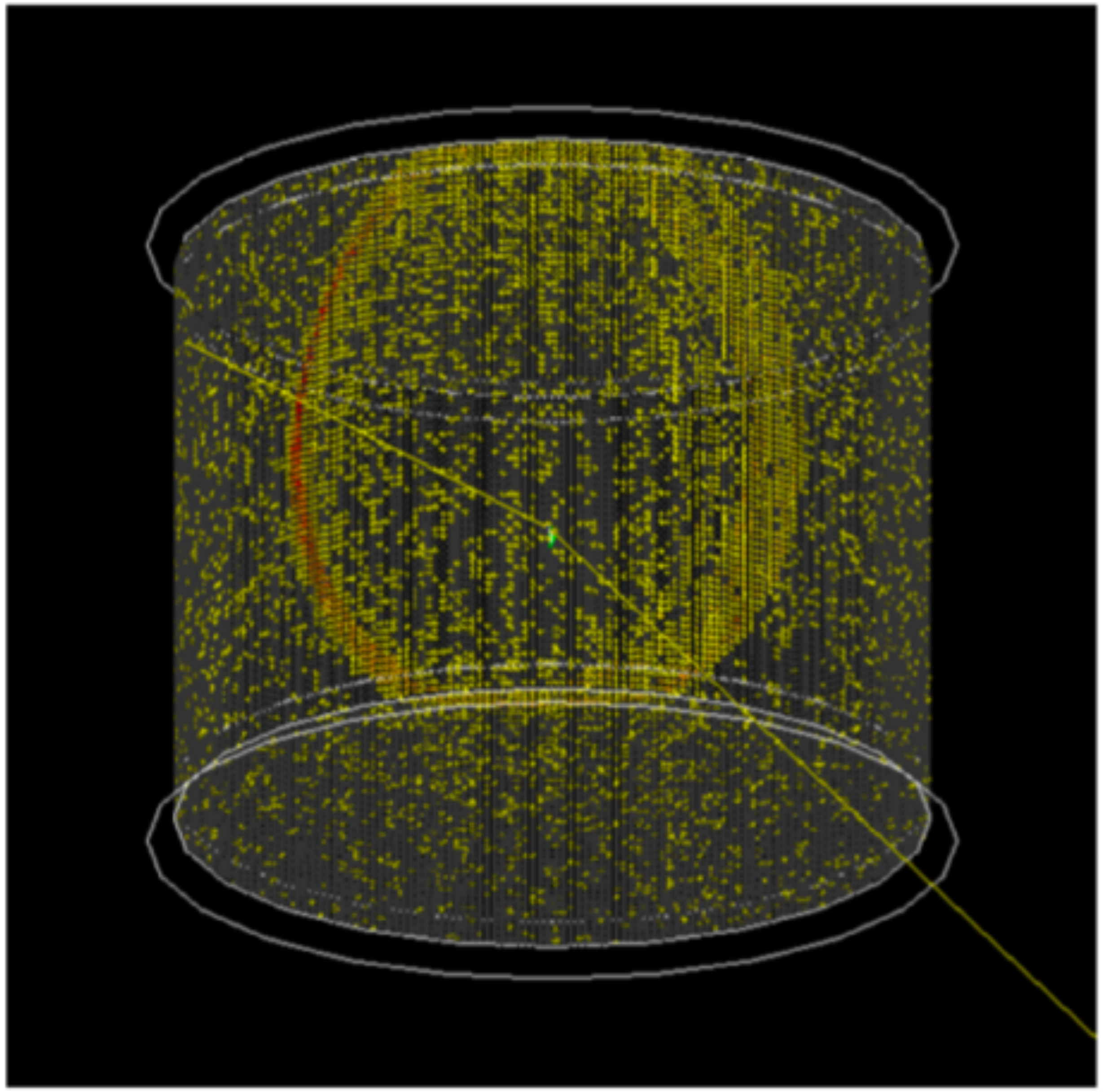




Using Super-K 2018 shutdown for direct testing of newly developed calibration systems for Hyper-K

Simulation

High fidelity calibration data is only as useful if it can be input into equally high fidelity simulation

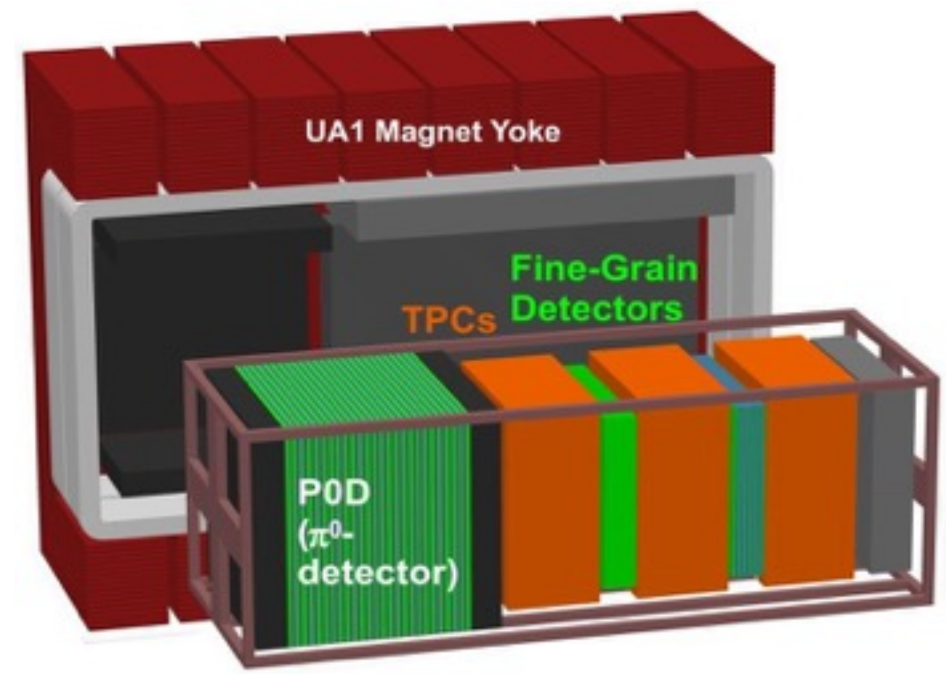


Near Detector Development

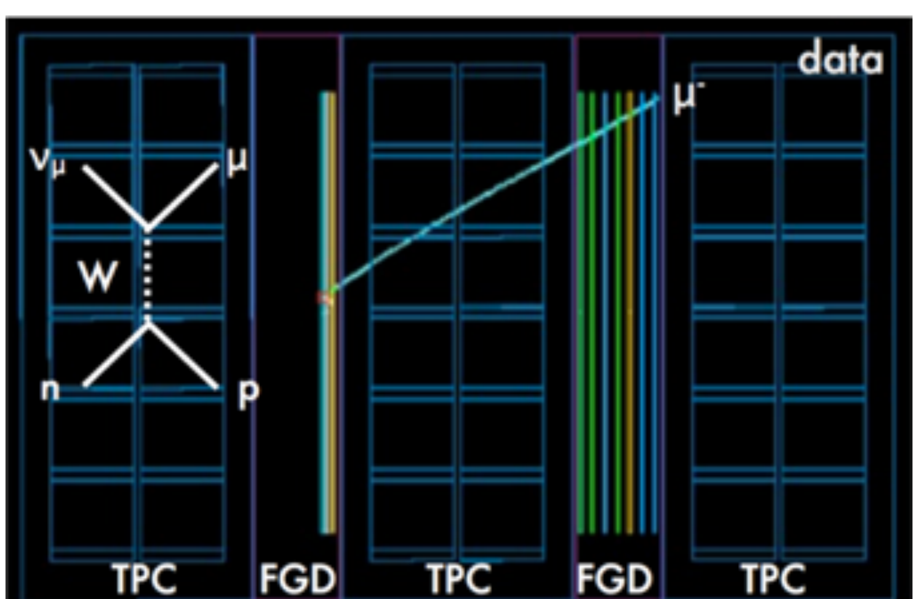
Carbon and Oxygen target materials

Acceptance differs from far detector

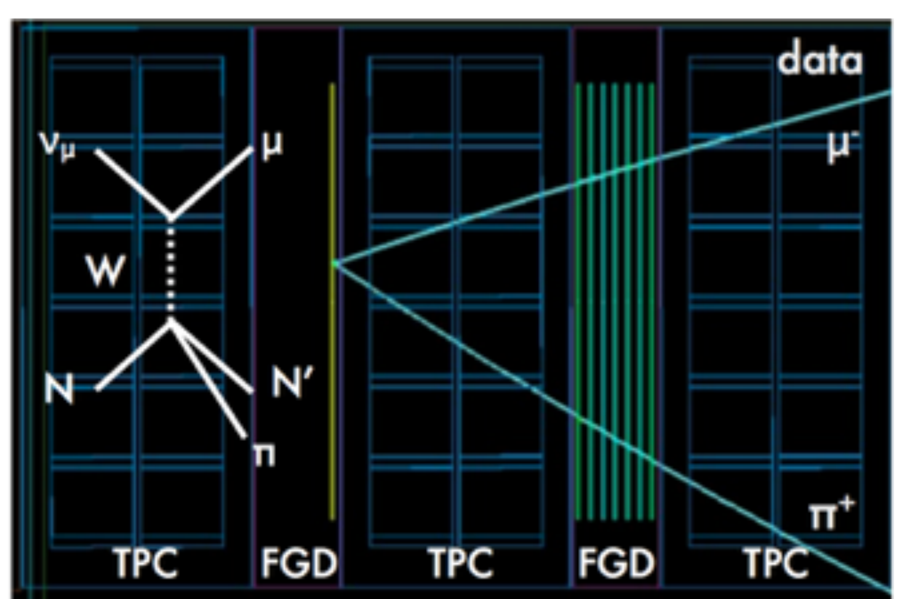
Magnetic field for sign selection



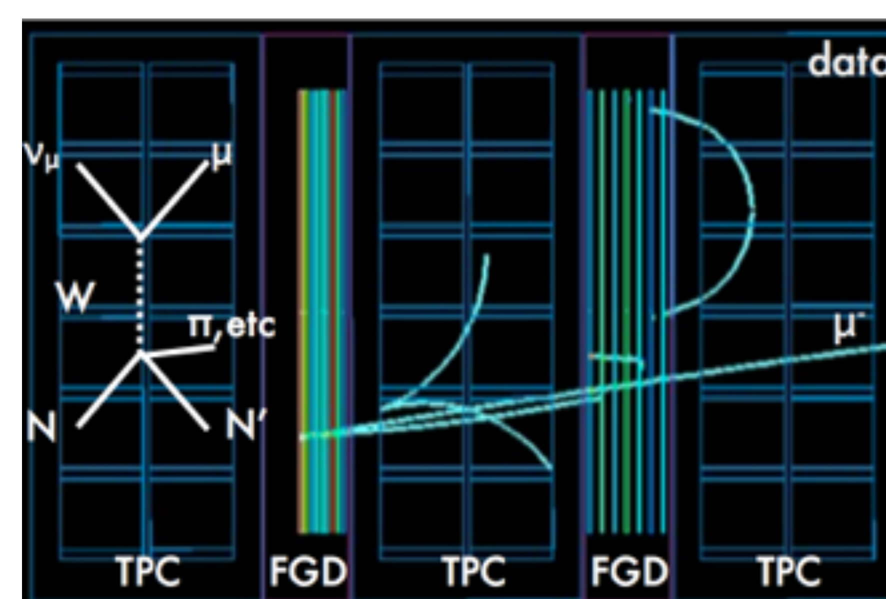
Near Detector (ND280)



CC 1μ + 0π + X



CC 1μ + 1π⁺ + X



CC other

Near Detector Development

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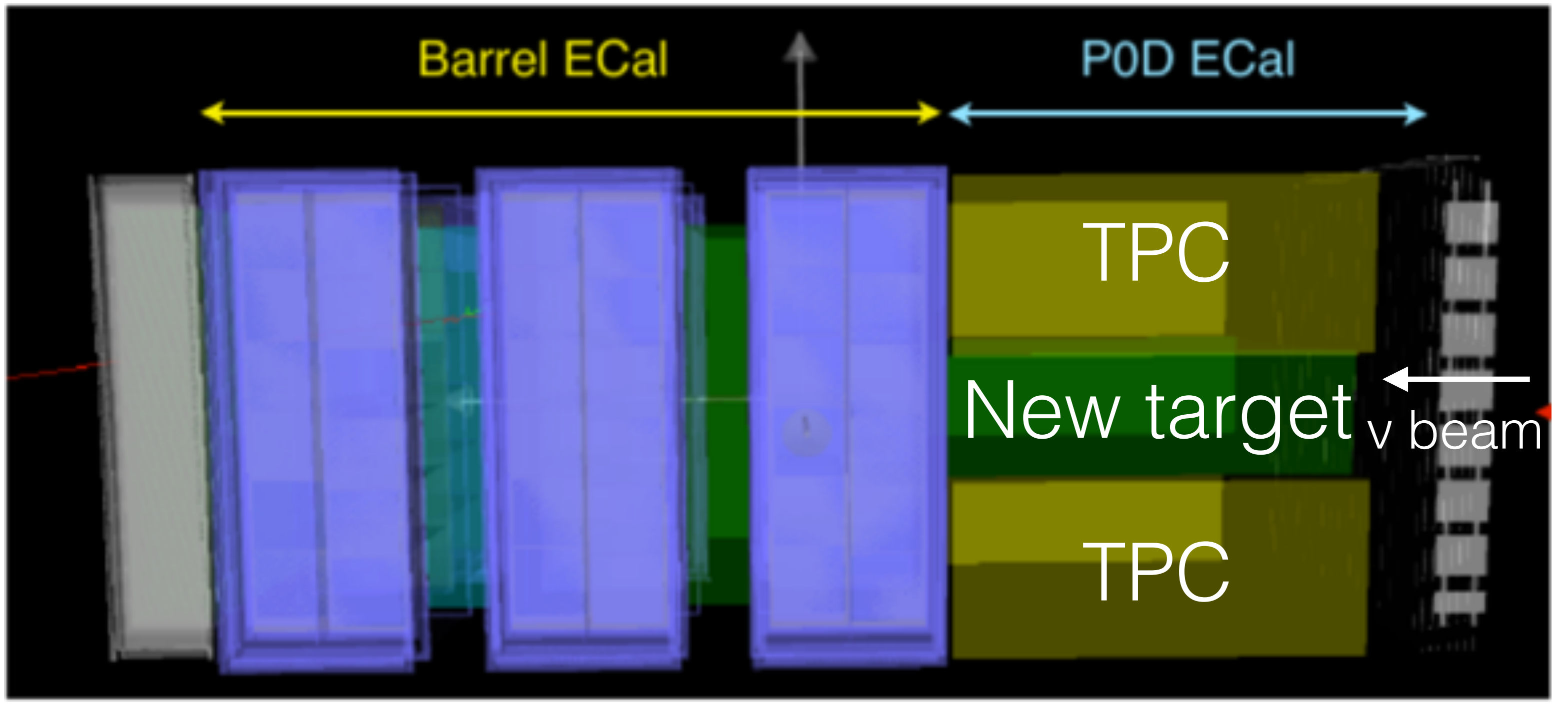
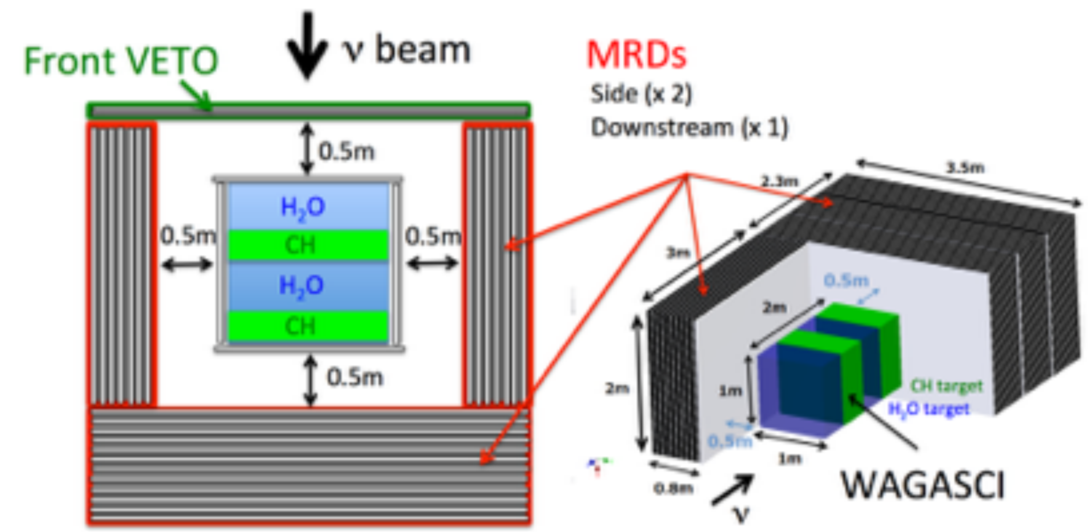
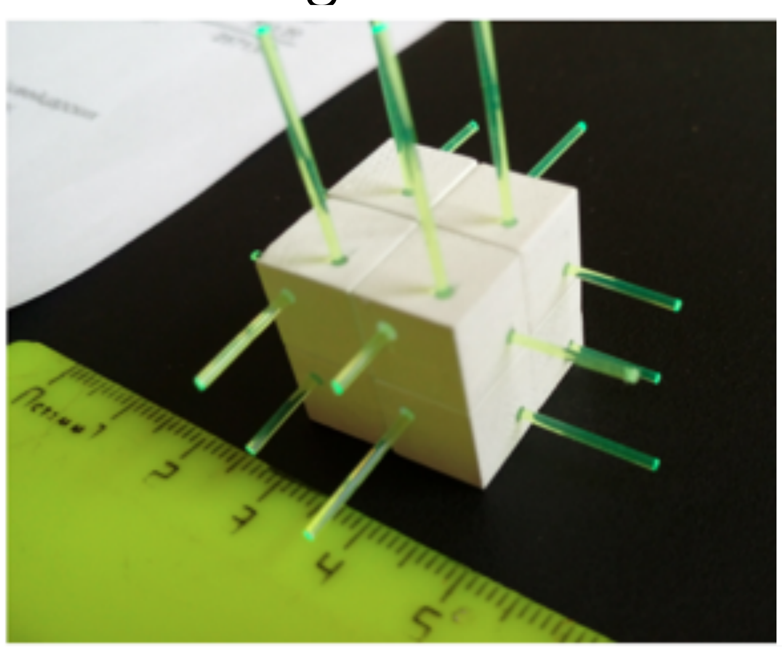
Planned ND280 Near Detector Upgrade

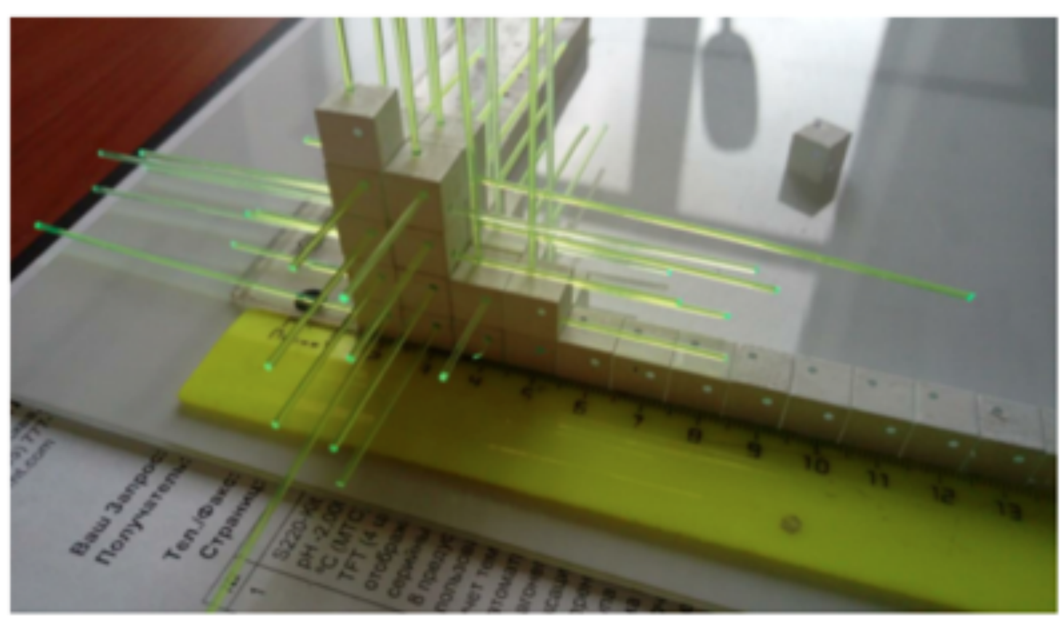


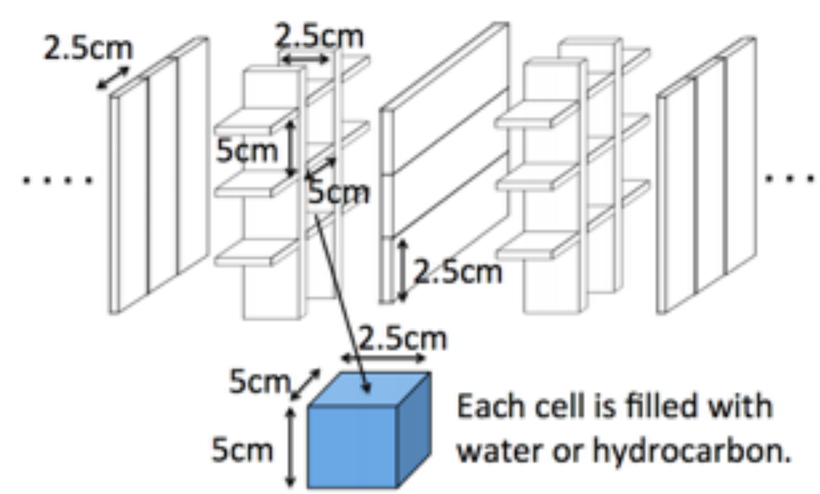


Near detector upgrades for T2K-II and T2HK era

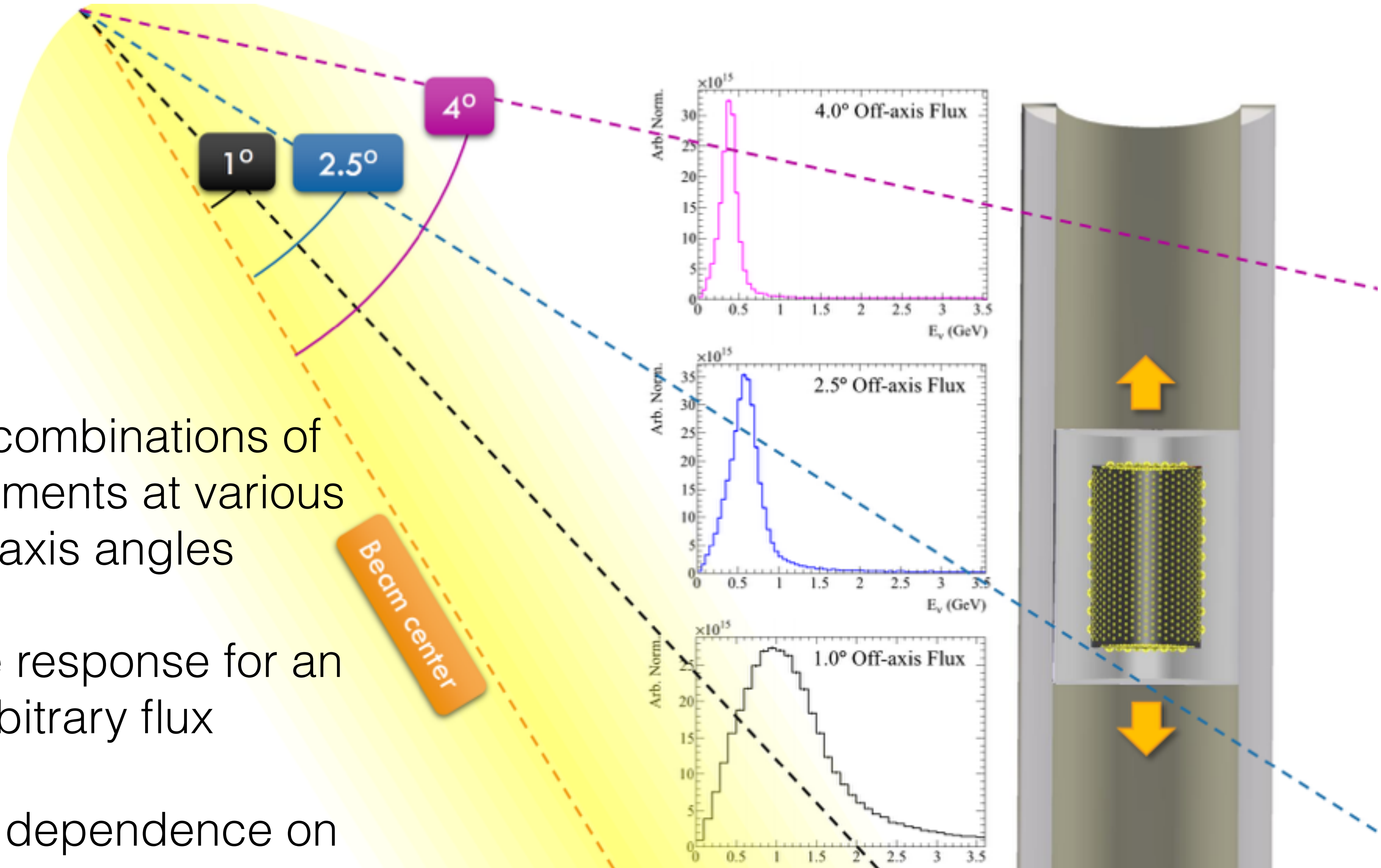
New target with increased angular acceptance







E61 Experiment



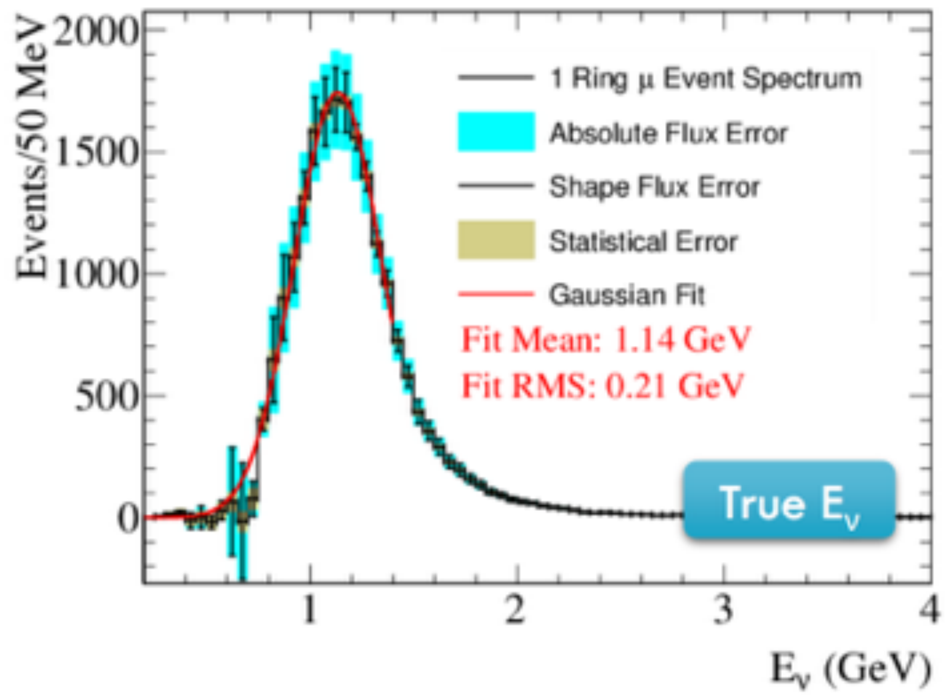
Linear combinations of measurements at various off-axis angles

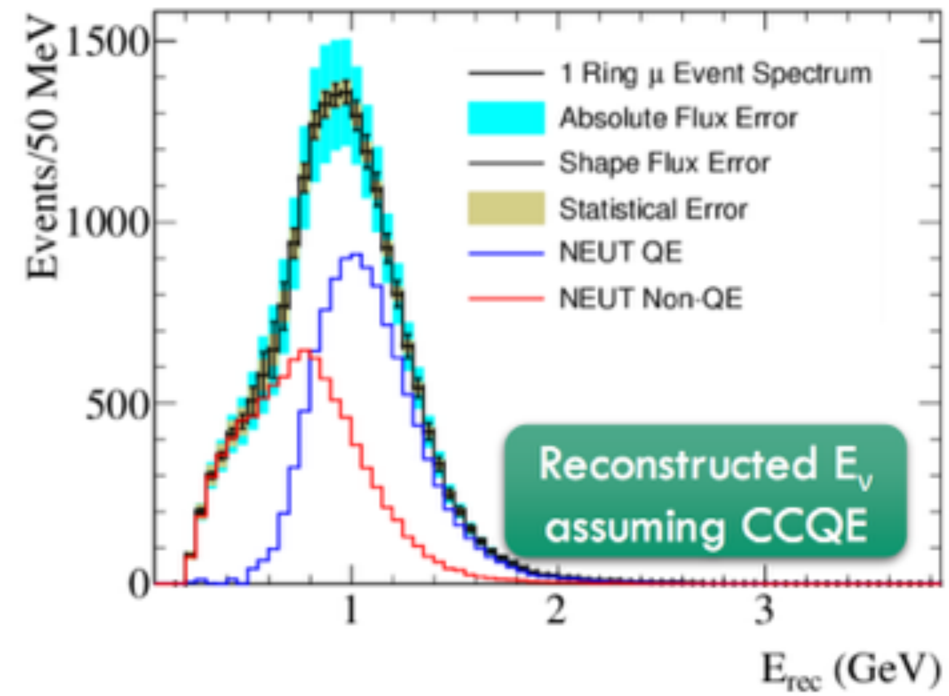
Measure response for an arbitrary flux

Reduce dependence on nuclear models

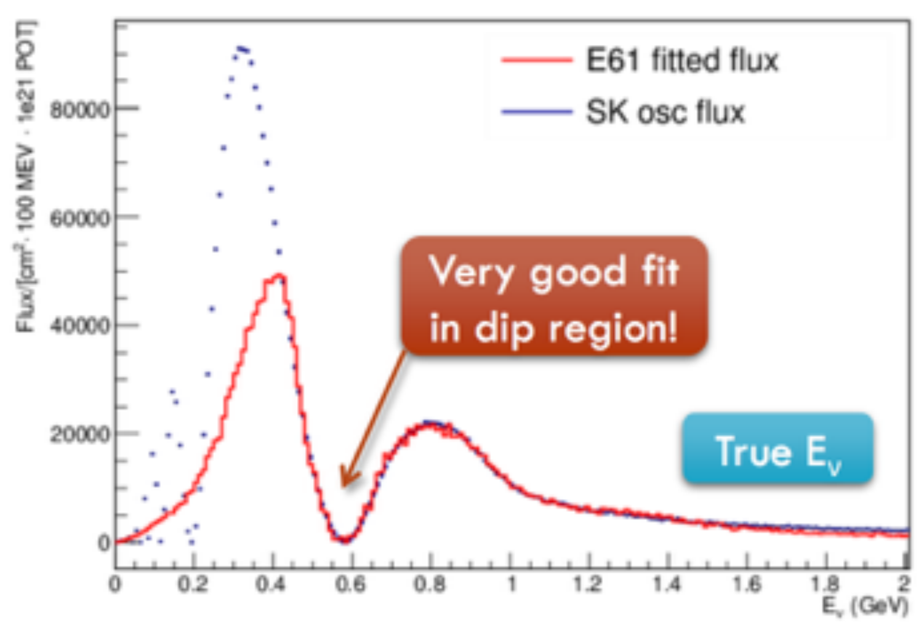
E61 Experiment

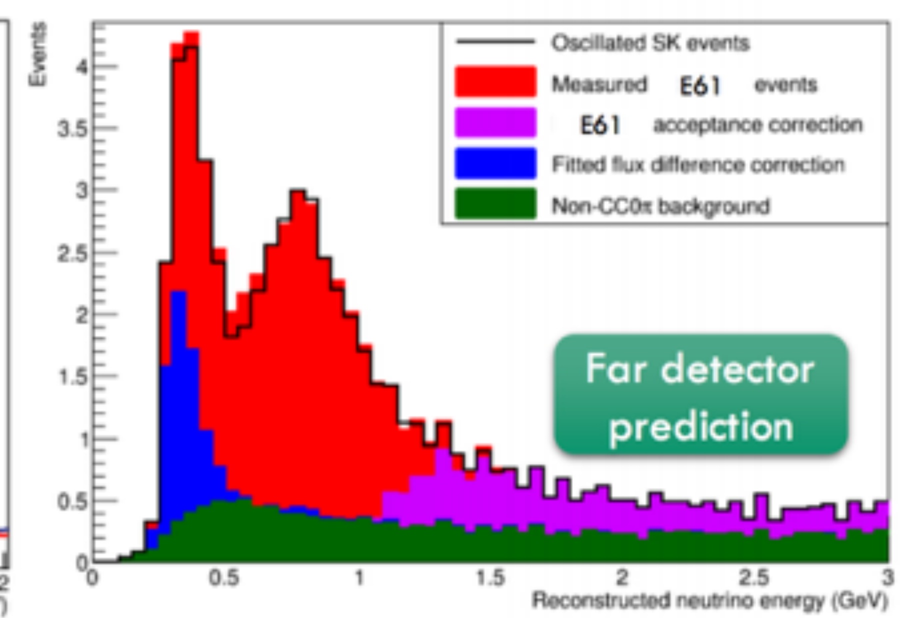
Pseudo-monochromatic beams





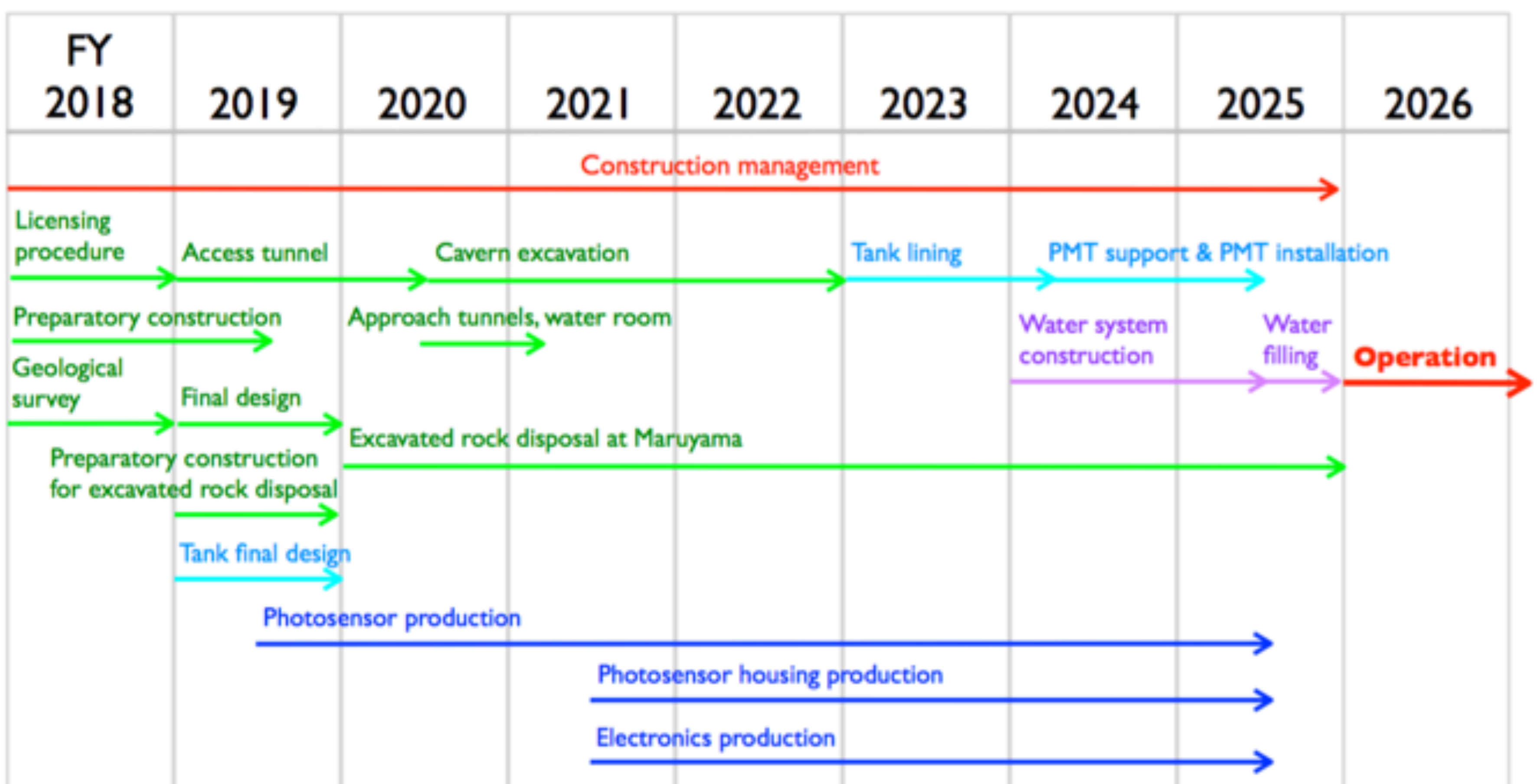
Far detector prediction for oscillated flux





Project Timeline

HK selected in “Master Plan” of Science Council in Japan
HK selected as highest-priority large-scale projects MEXT Roadmap 2017
Funding request in progress



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Summary

T2K established ν_e appearance
and sees hints of CP violation

Hyper-K well placed to build on the huge success
of Super-K and T2K experiments

Capable of world leading measurements in neutrino
oscillations, nucleon decay, neutrino astrophysics

Funding request in progress
If construction starts 2018, operation in 2026

References:

T2HKK White Paper, arXiv:1611.06118 [hep-ex]
HK Design Report, KEK Preprint 2016-21
HK Physics Sensitivity, PTEP (2015) 053C02

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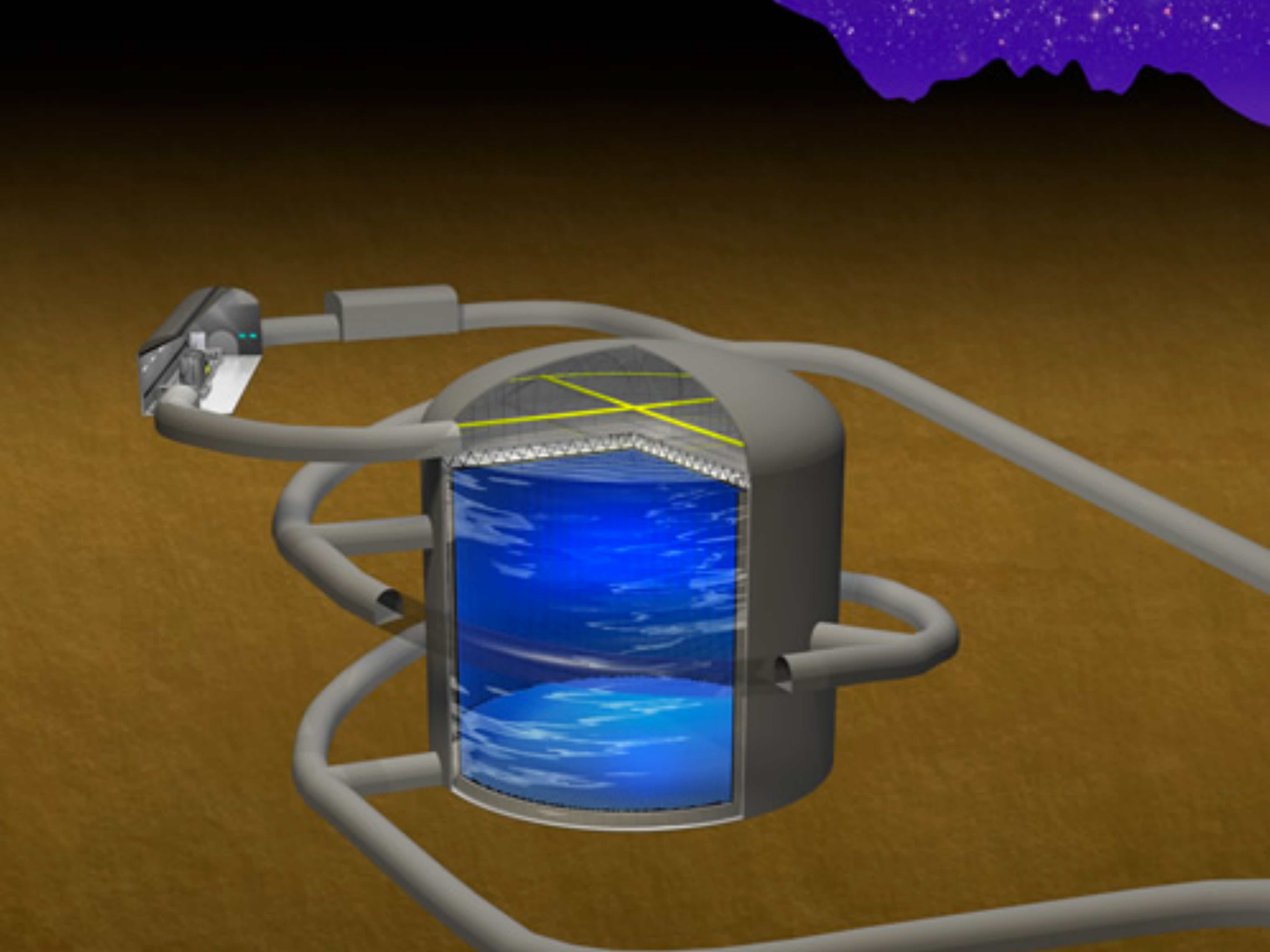
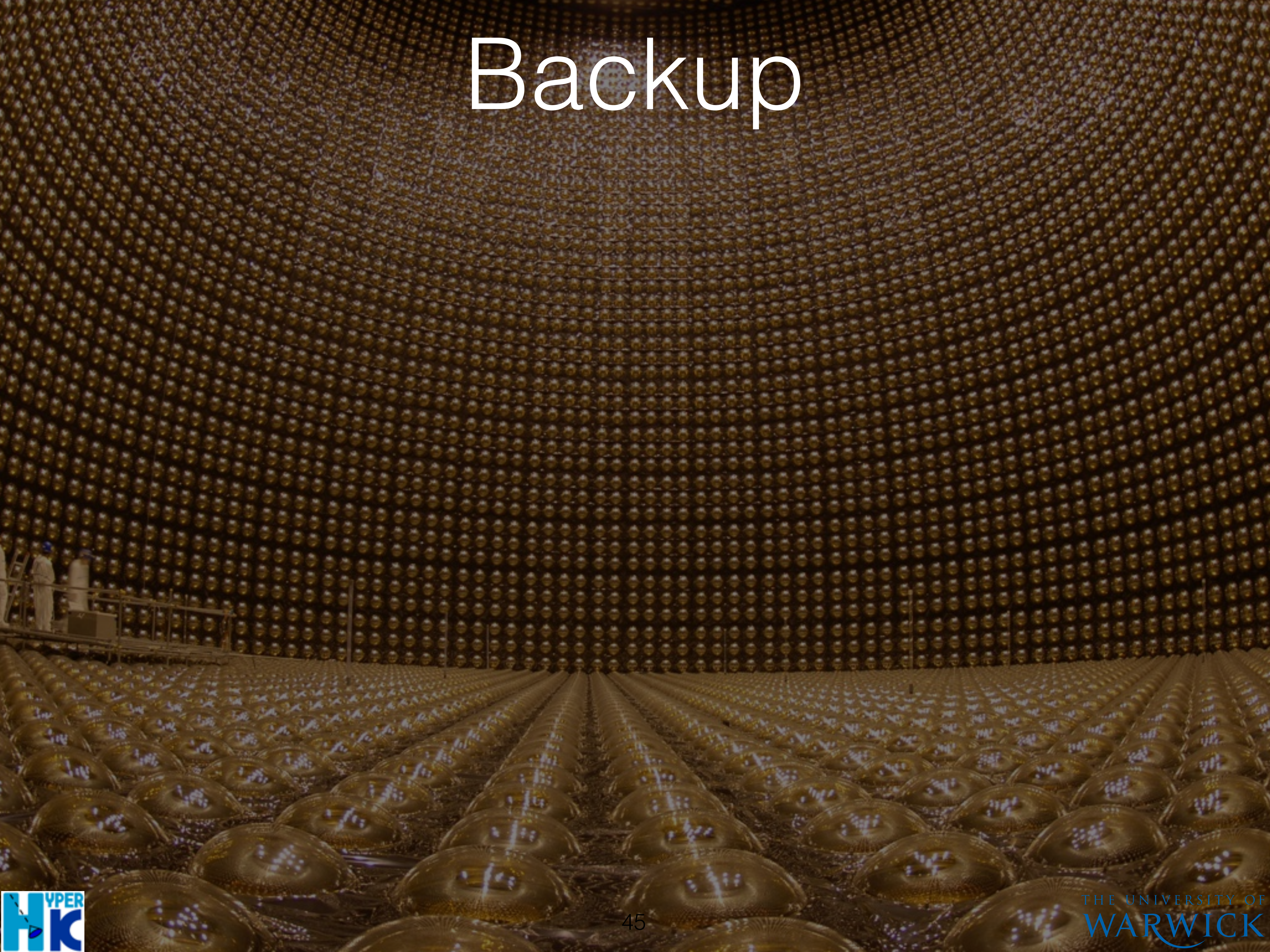
Hyper-K

David Hadley, University of Warwick



Backup



T2K Systematic Uncertainties

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Error Source	μ sample [%] ν	μ sample [%] $\bar{\nu}$	e sample [%] ν	e sample [%] $\bar{\nu}$
SK Detector	1.9	1.6	3.0	4.2
SK FSI+SI+PN	2.2	2.0	2.9	2.5
ND280 Constraint (Flux + Cross Section)	3.3	2.7	3.2	2.9
$\sigma(\nu_e)/\sigma(\nu_\mu)$	-	-	2.6	1.5
NC 1γ	-	-	1.1	2.6
NC other	0.3	0.1	0.1	0.3
Total Systematic	4.4	3.8	6.3	6.4
Statistical	6.5	12	12	40

T2K preliminary (final systematics pending)

Total systematic uncertainty
~4 - 6%
Smaller than stats. uncertainty
(for now!)

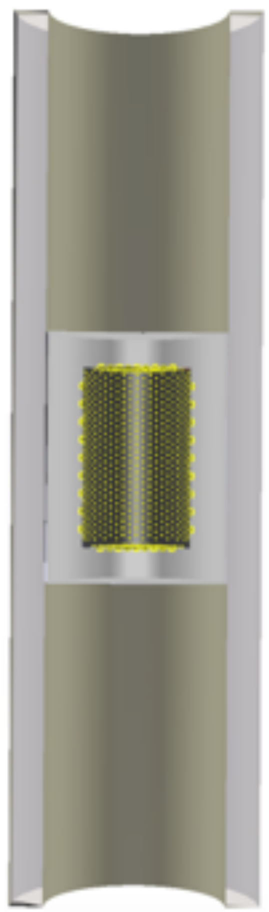
ND280 constraint
13%→3%

Pion Final State Interactions (FSI) and Secondary Interactions (SI) modelling important

Theoretical uncertainty
ν_e to ν_μ
Difficult to constrain with near detector

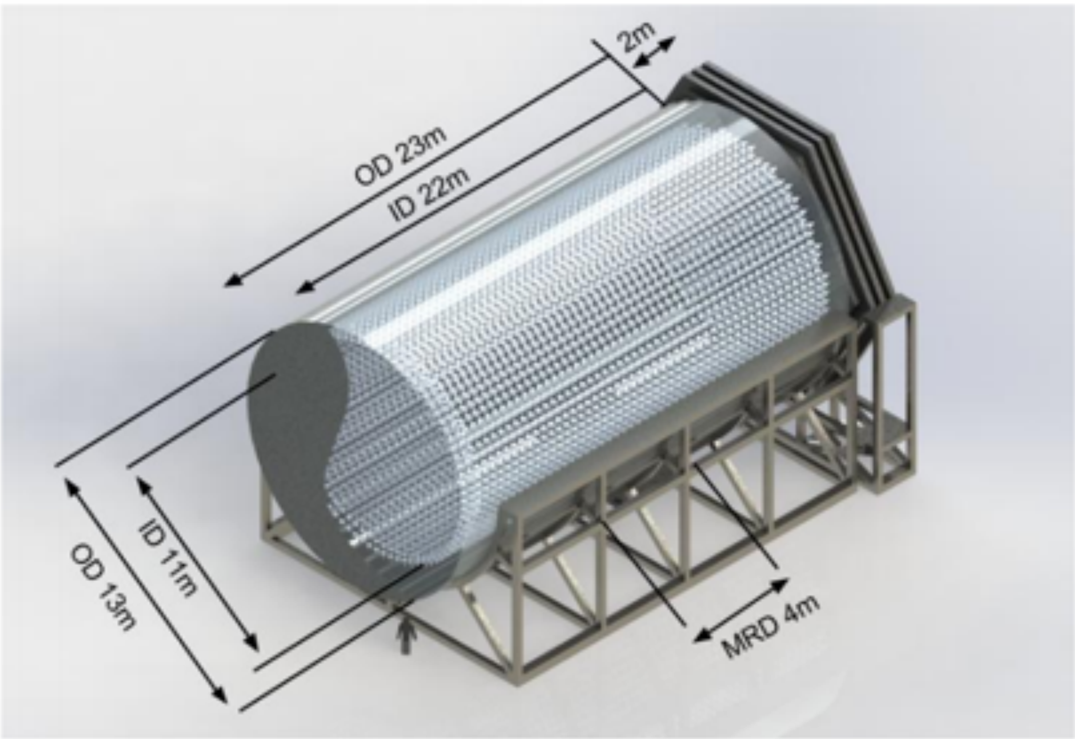
E61 Experiment

Two competing collaborations



nuPRISM

“Water elevator”
Measure ∫σ(E)ϕ(E)dE
as a function of theta
[arXiv:1412.3086]



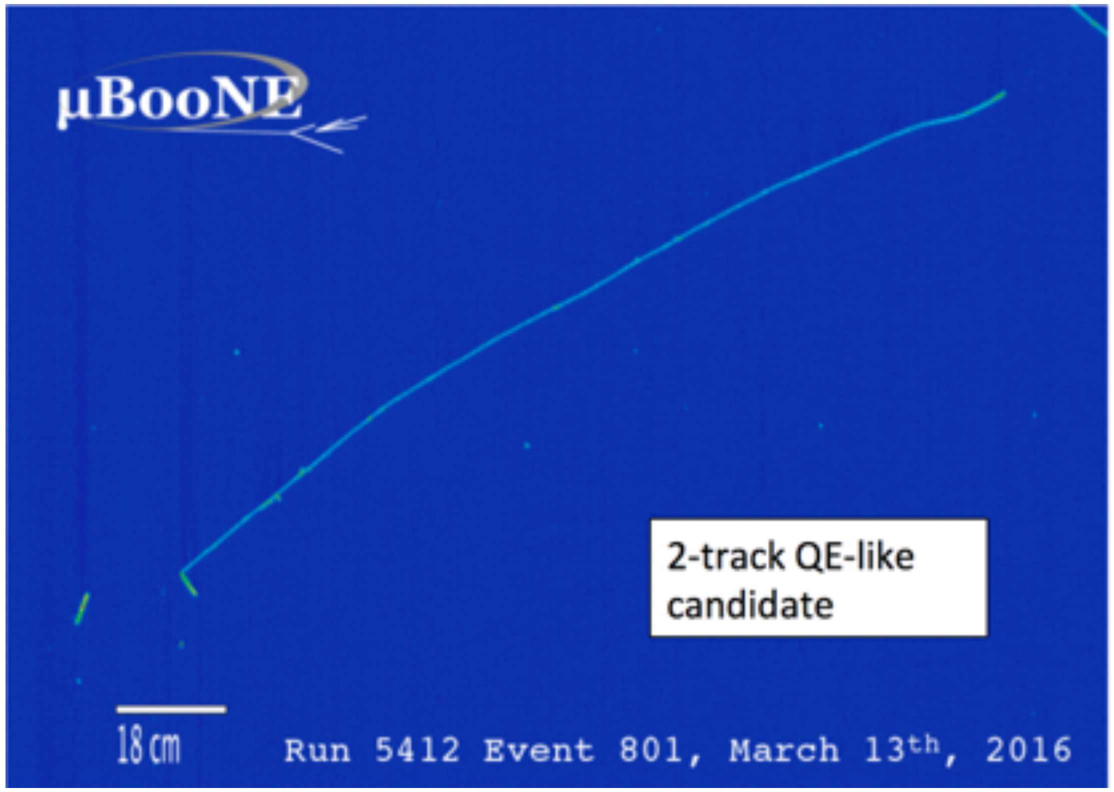
TITUS

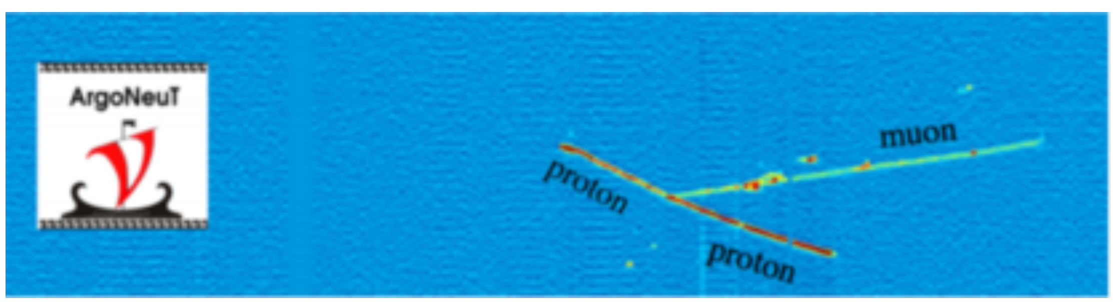
same off-axis angle far detector
Gd, muon range detector
[arXiv:1606.08114]

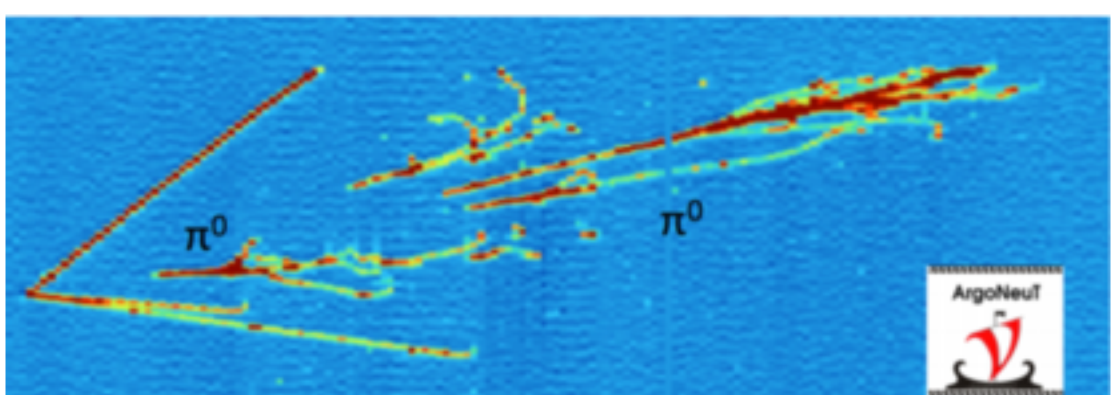
Merged into a single collaboration:
E61 Experiment

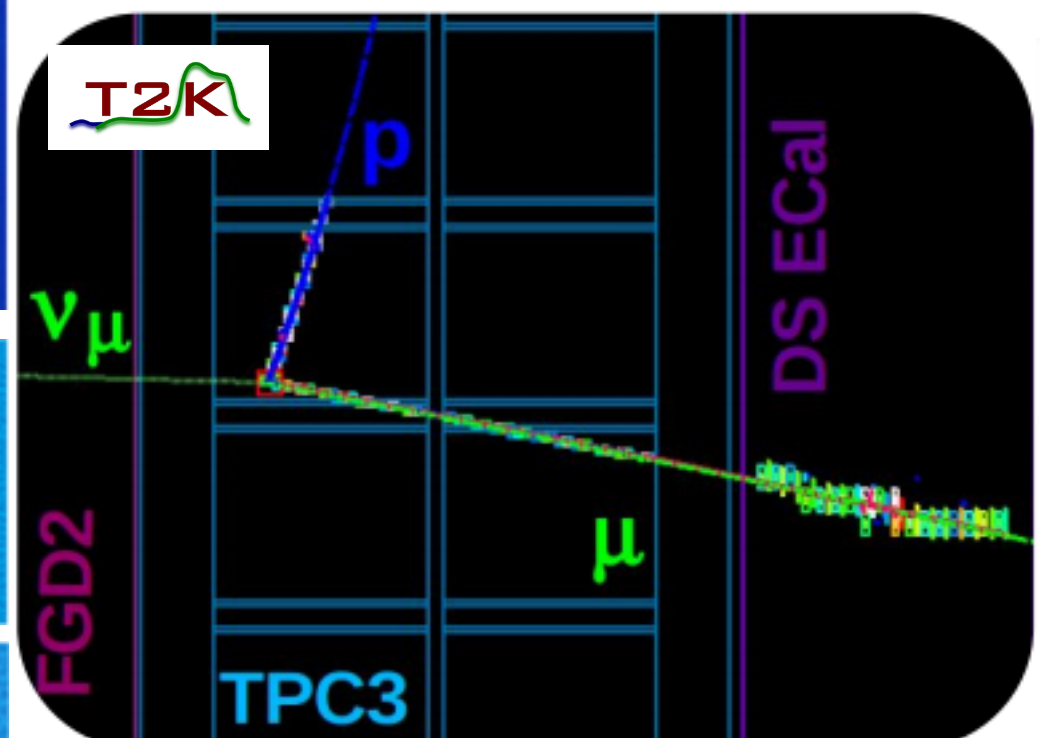
Near Detector Development

TPC measurements precisely image ν-nucleus interaction vertex
→ better constraints on models



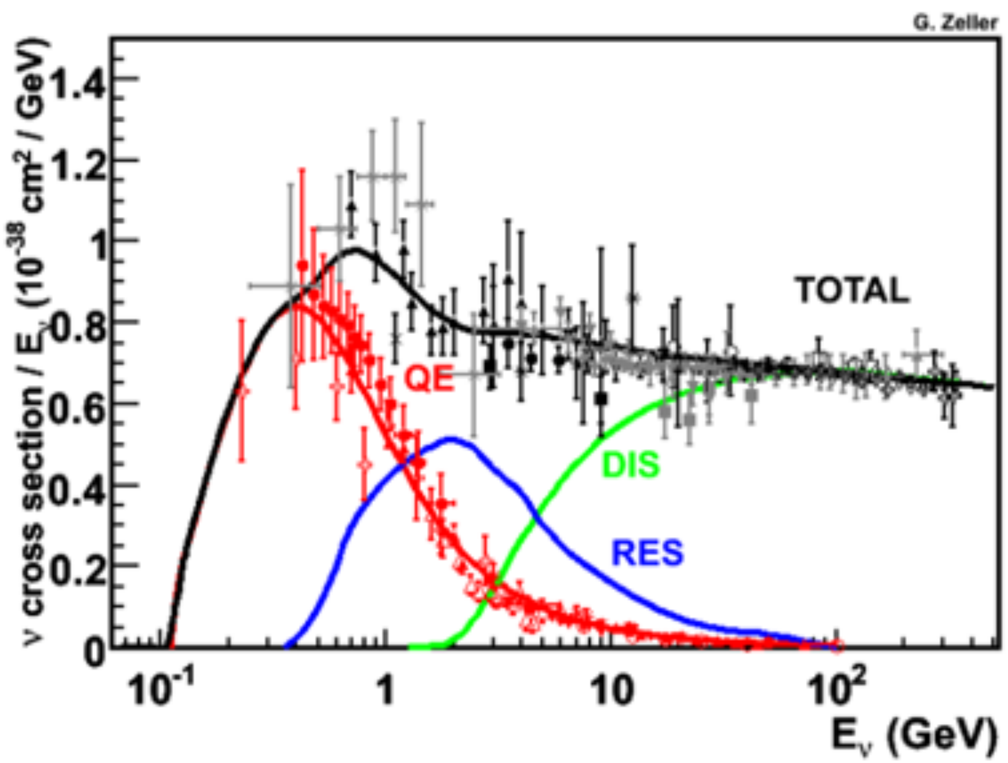


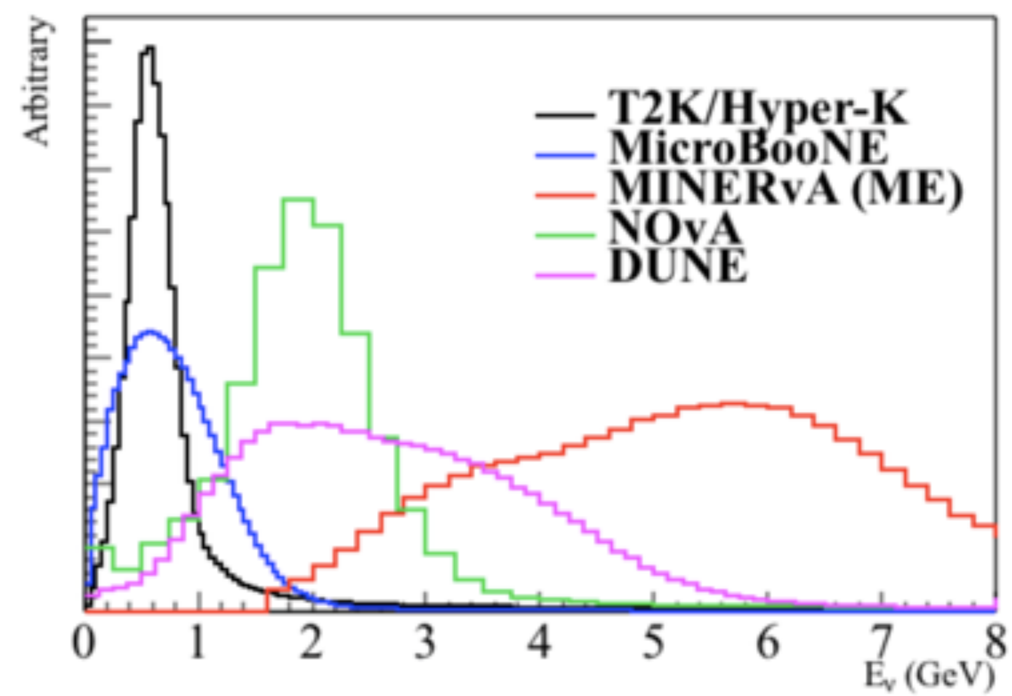




Ultra-low thresholds with gaseous TPC

Neutrino Interaction Model Uncertainties





Wide range of processes need to be simulated
Require both lepton and hadronic side of the interaction

Nuclear effects important in the relevant energy regime

Experiments rely on MC generators
for $E_{visible} \rightarrow E_\nu$ extrapolation

Model parameter uncertainties from fits to external datasets
Sometimes parameter error must be inflated or ad-hoc parameters to account for discrepancies between model and data or known flaws in the model

T2K Cross-Section Model

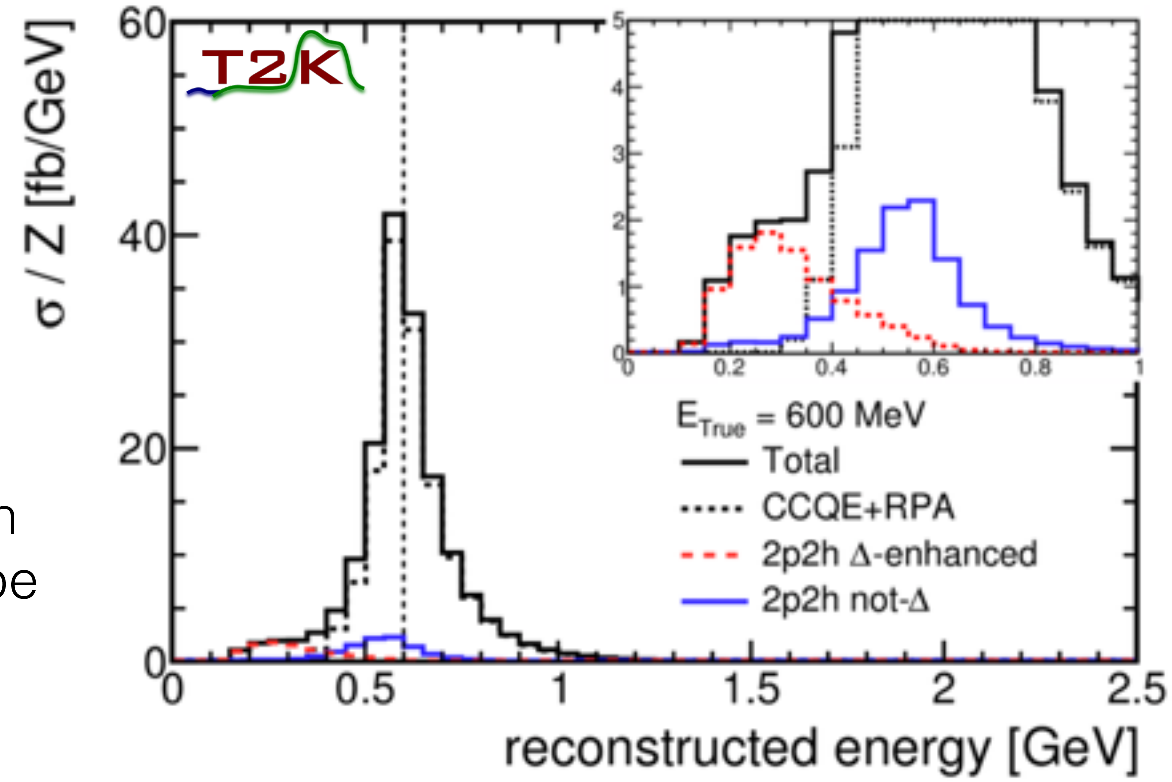
Implemented in NEUT MC generator

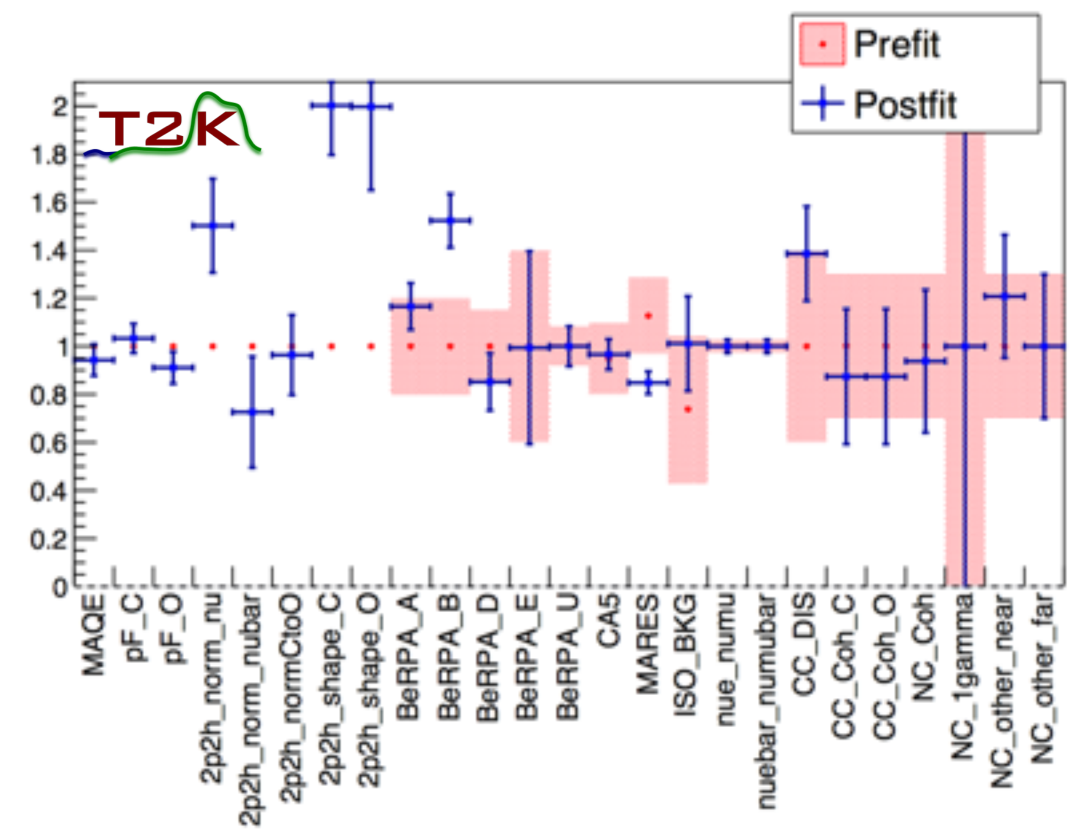
Quasi-elastic scattering most important process at T2K energies

- Valencia 2p-2h model Phys. Rev. C83 (2011) 045501
- Long-range effects with Random Phase Approximation
- Parameters introduced to vary normalisation and shape
- Relativistic Fermi Gas (RFG) nuclear model
- Uncertainties from RFG ↔ Local Fermi Gas
- Final state interactions with cascade model

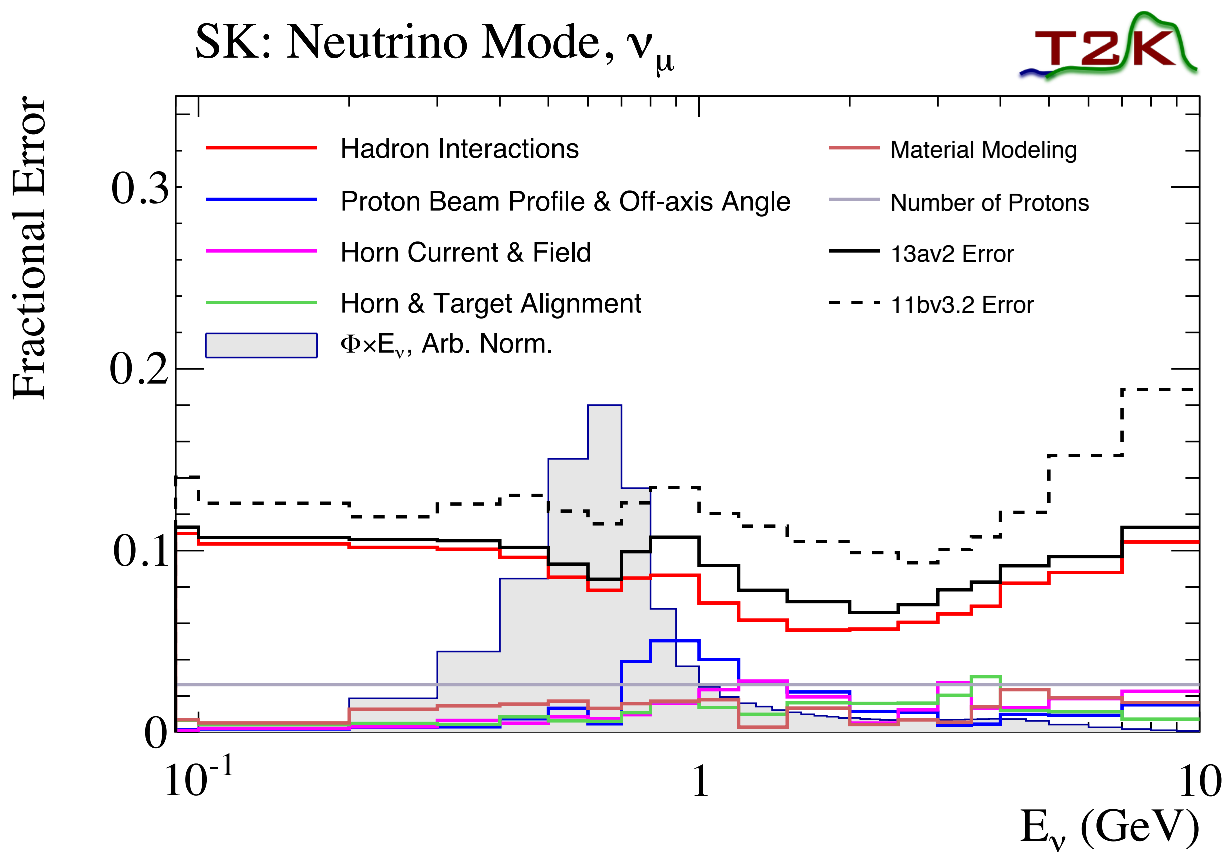
No priors on most CCQE parameters
Constraint from near detector

Impact of alternative models not implemented in oscillation analysis evaluated with fake data studies





Flux Uncertainties



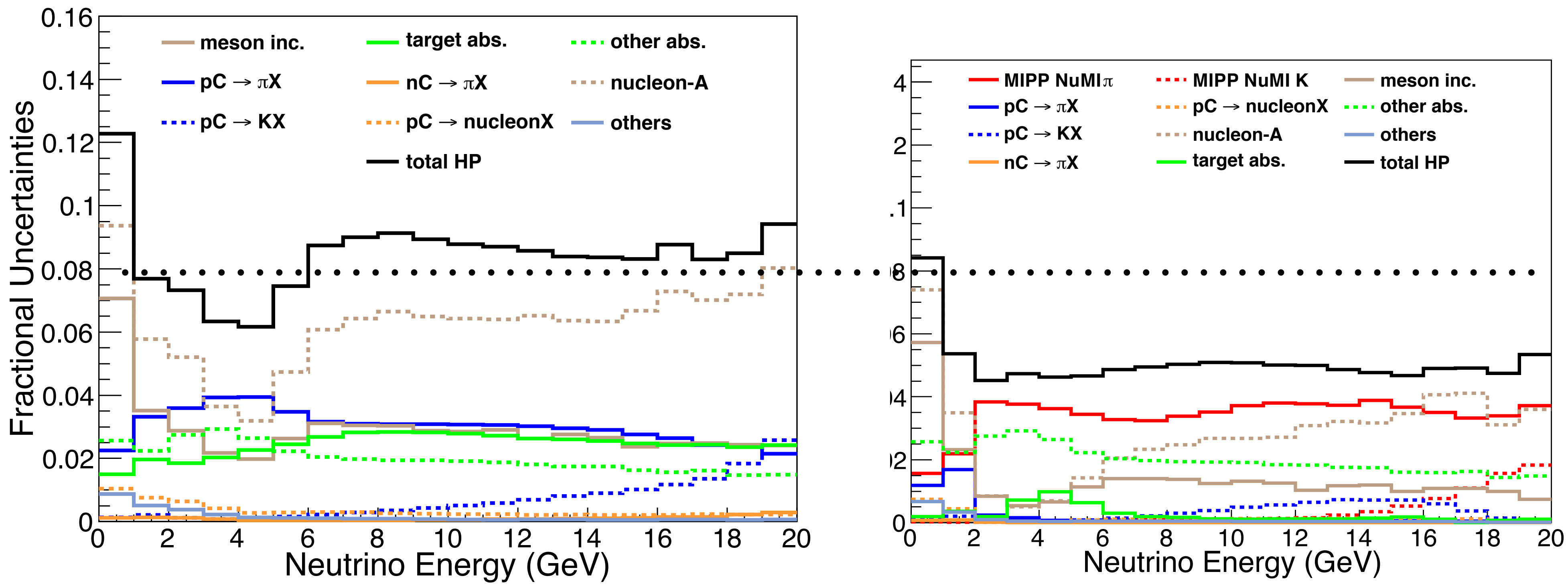
T2K ~ 8-12% (based on thin target tuning)

Dominated by hadron interaction modelling

Alignment/focussing uncertainties are also important (especially for near to far extrapolation)

Flux Uncertainties

Thin Target → Thick Target



MINERvA Low E NuMI Flux Uncertainties, Phys. Rev. D 95, 039903 (2017)

Significant reductions from thick/replica target

If high power beam requires different target material/geometry new dedicated hadron production measurements will be necessary

J-PARC Beam Upgrades

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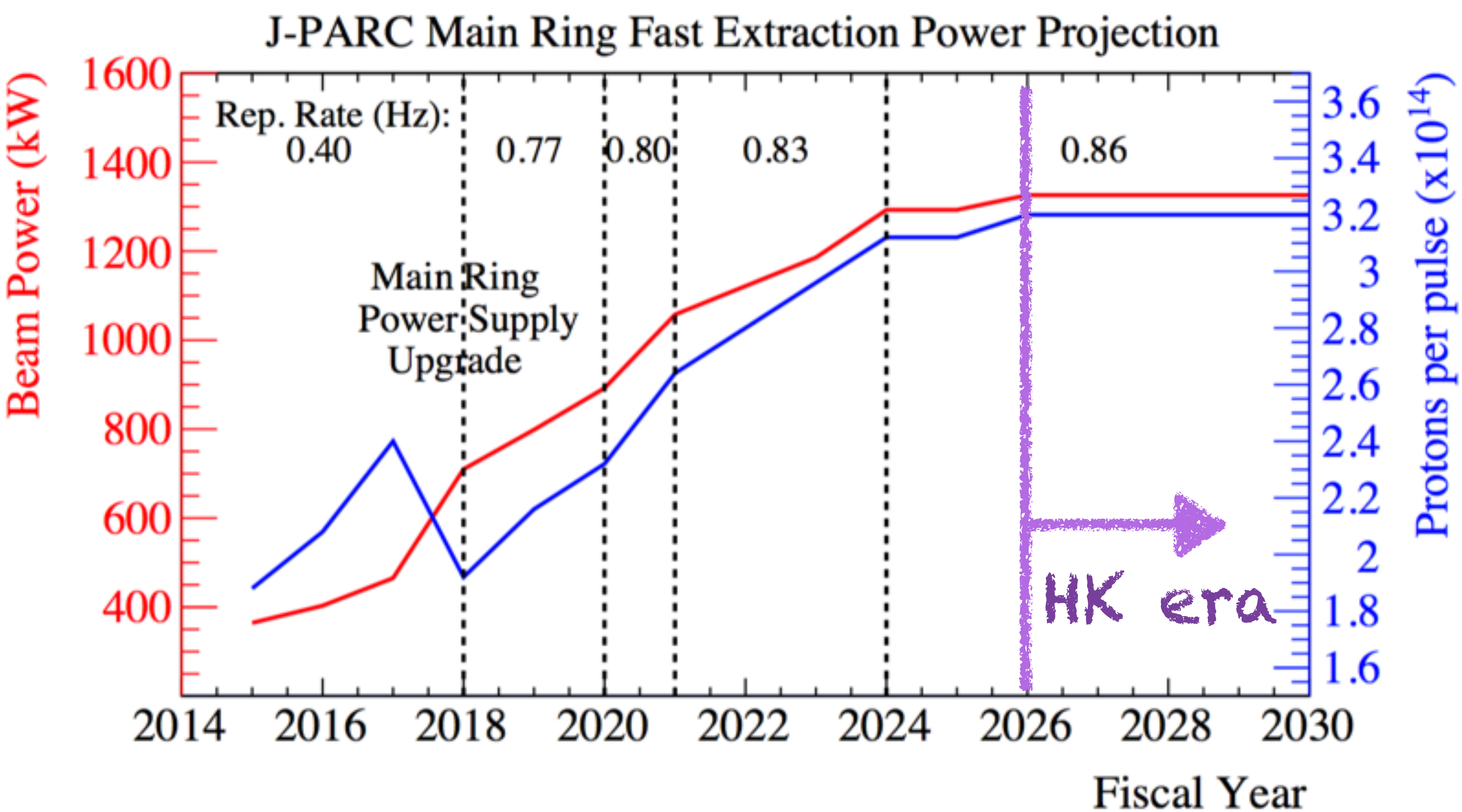
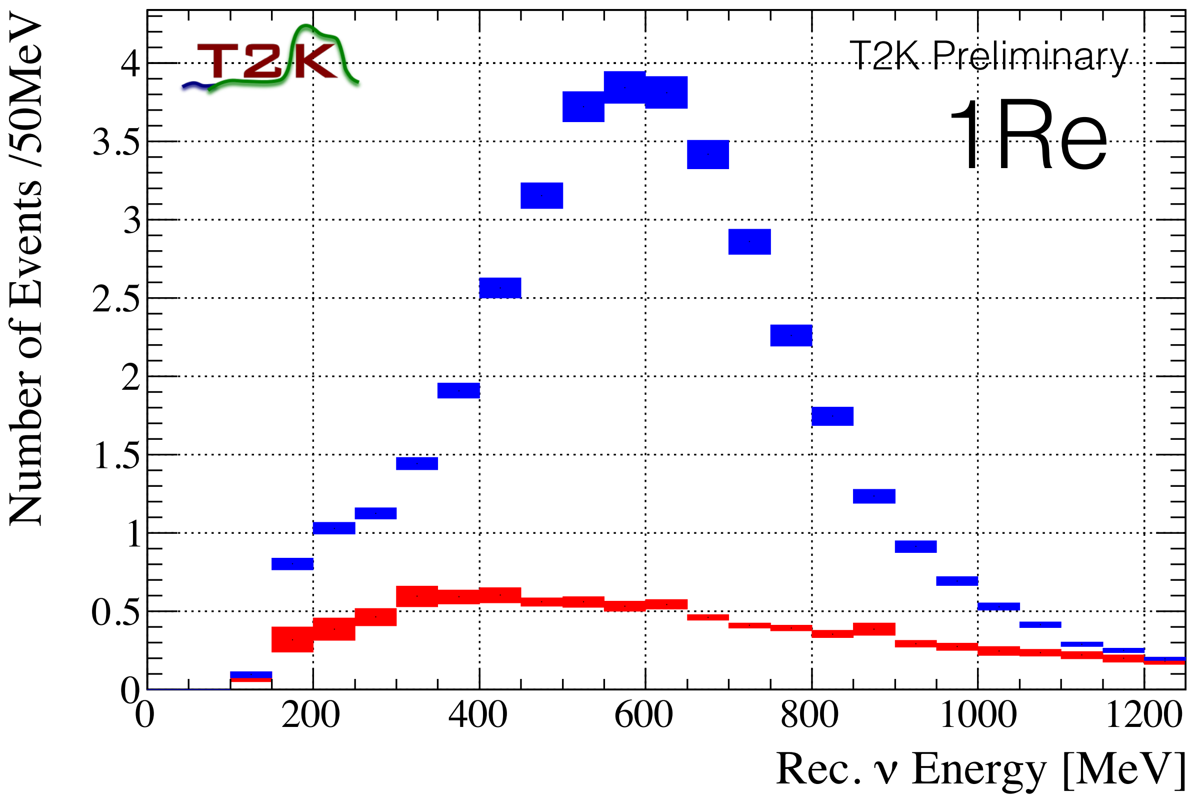


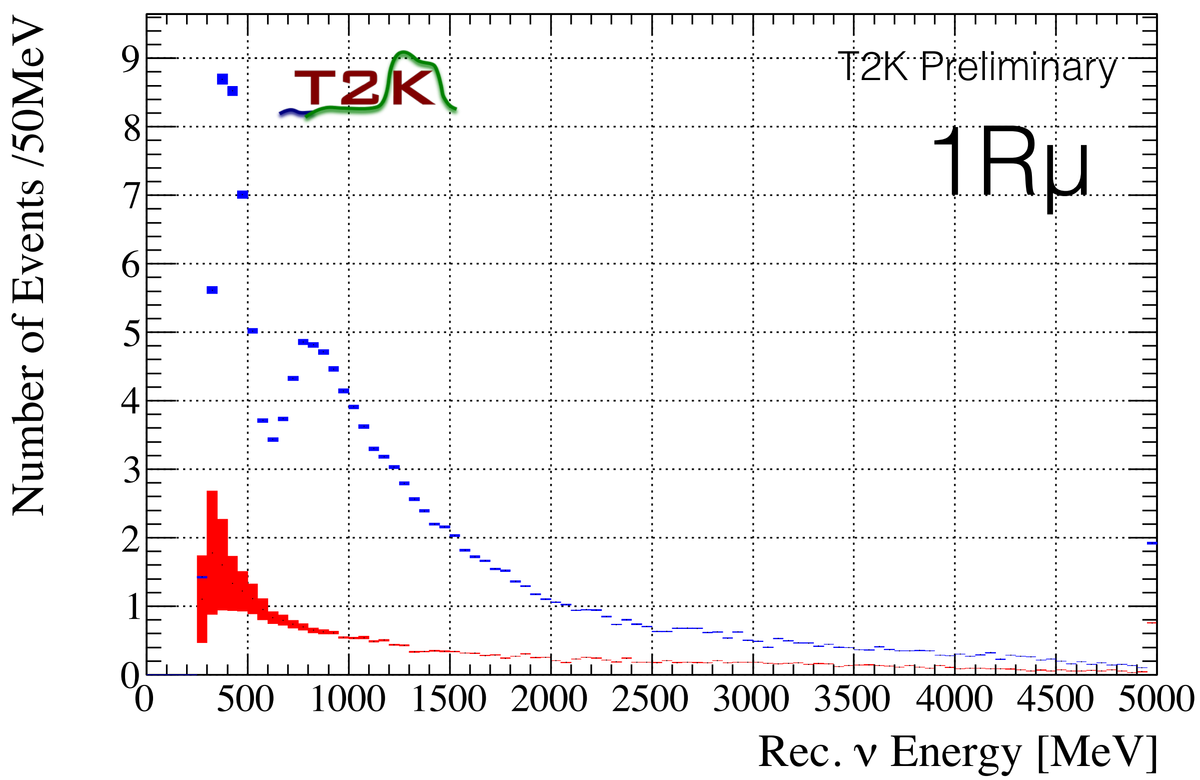
Current: ~470 kW

Short-term: 750 kW after 2018 long shutdown

Goal: 1.3 MW operation at HK operation

Detector Modelling Uncertainties



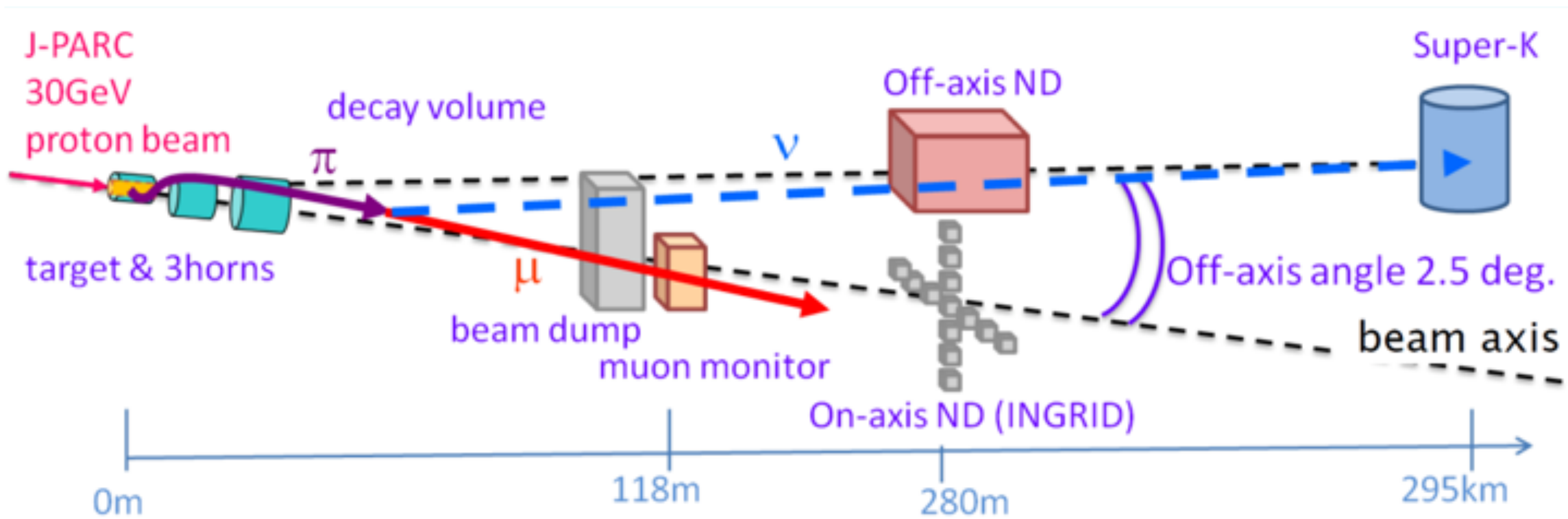


SK detector response evaluated with data-MC comparisons in atmospheric sample

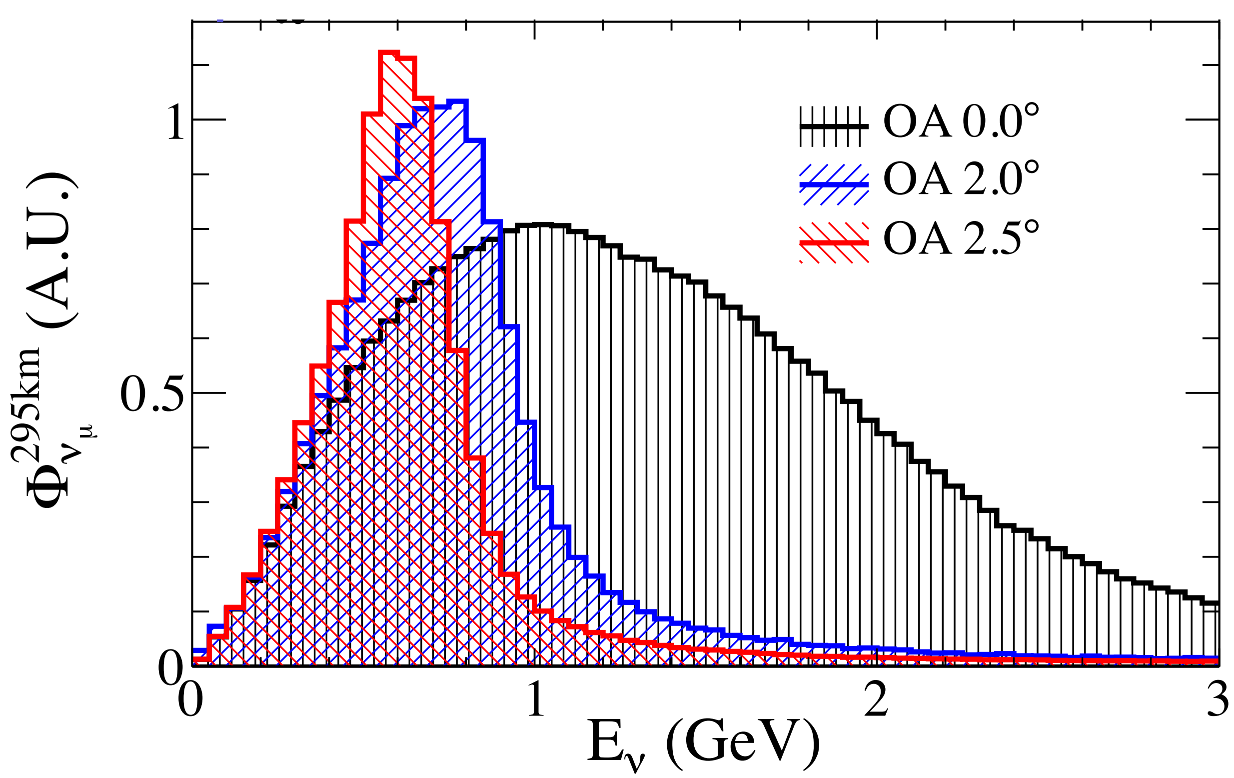
May be limited by control sample statistics

Possible to move toward bottom-up detector systematic uncertainty

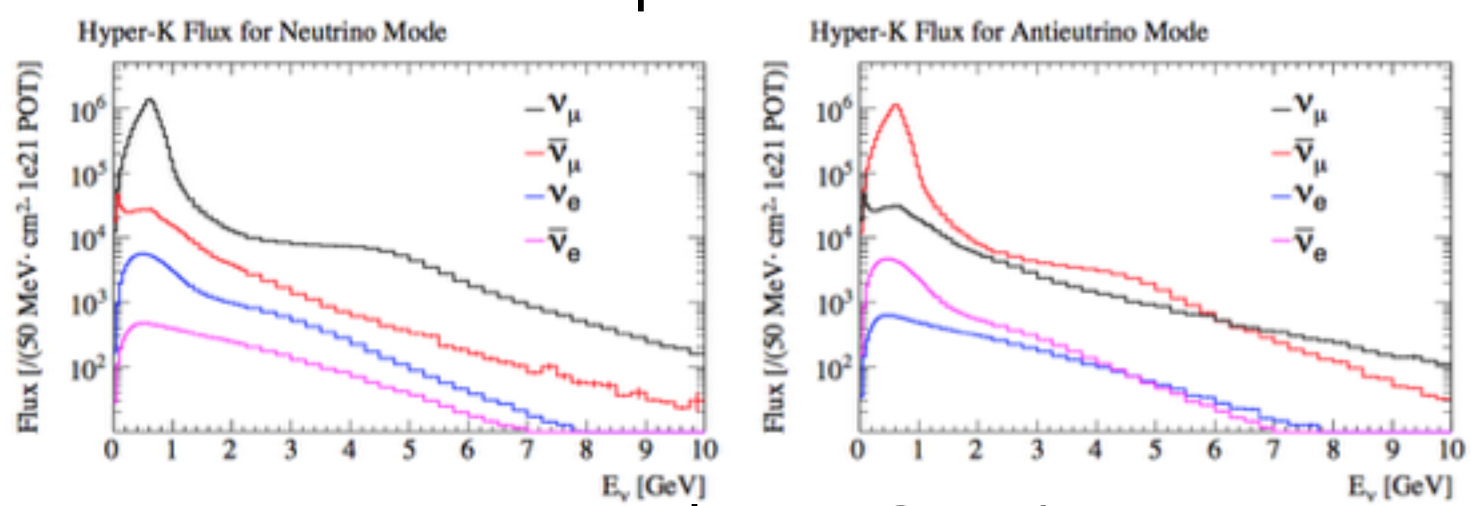
T2K / Hyper-K Flux



Narrow band beam off-axis



Flavour composition



nu-mode: ~94% ν_μ

anti-nu mode: ~92% $\bar{\nu}_\mu$

(for E < 1.25 GeV)

Physics at Hyper-K

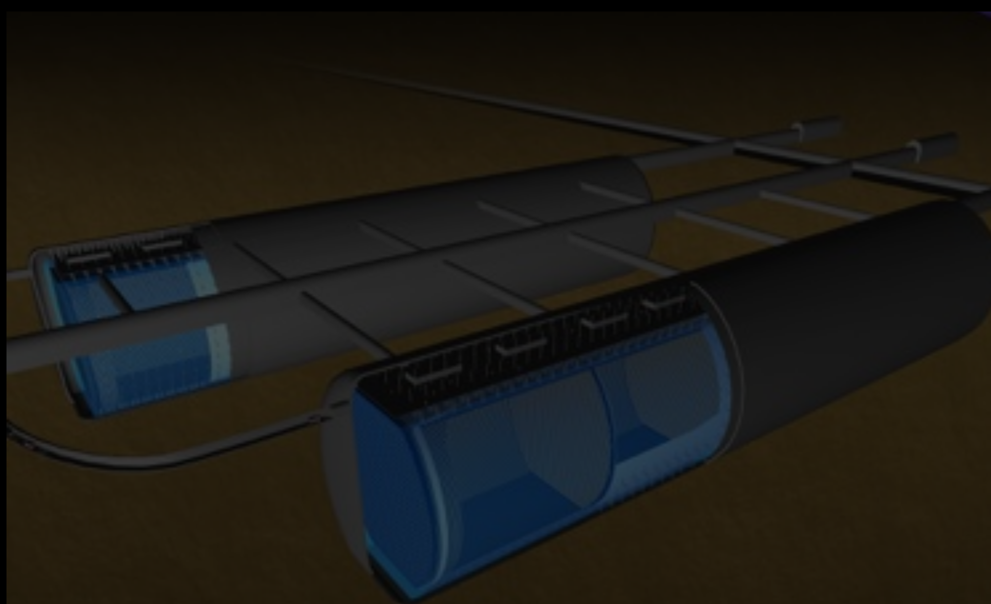
Proton Decay

$p \rightarrow e^+ + \pi^0$

$>1.3\times10^{35}$ years 90% CL

$p \rightarrow \bar{\nu} + K^+$

$>3.2\times10^{34}$ years 90% CL



Neutrinos

Solar

Supernova

SN ~200,000 @ 10kPC

SN ~30-50 @ M31

200 solar ν per day

Indirect dark matter search

Accelerator

Atmospheric

Leptonic CP violation

(see following slides)

Mass Hierarchy determination

$>3\sigma$

θ_{23} octant determination

3σ for $\sin^2\theta_{23} > 0.56$ or $\sin^2\theta_{23} < 0.46$

Broad physics programme.

Lots of Physics with Hyper-K

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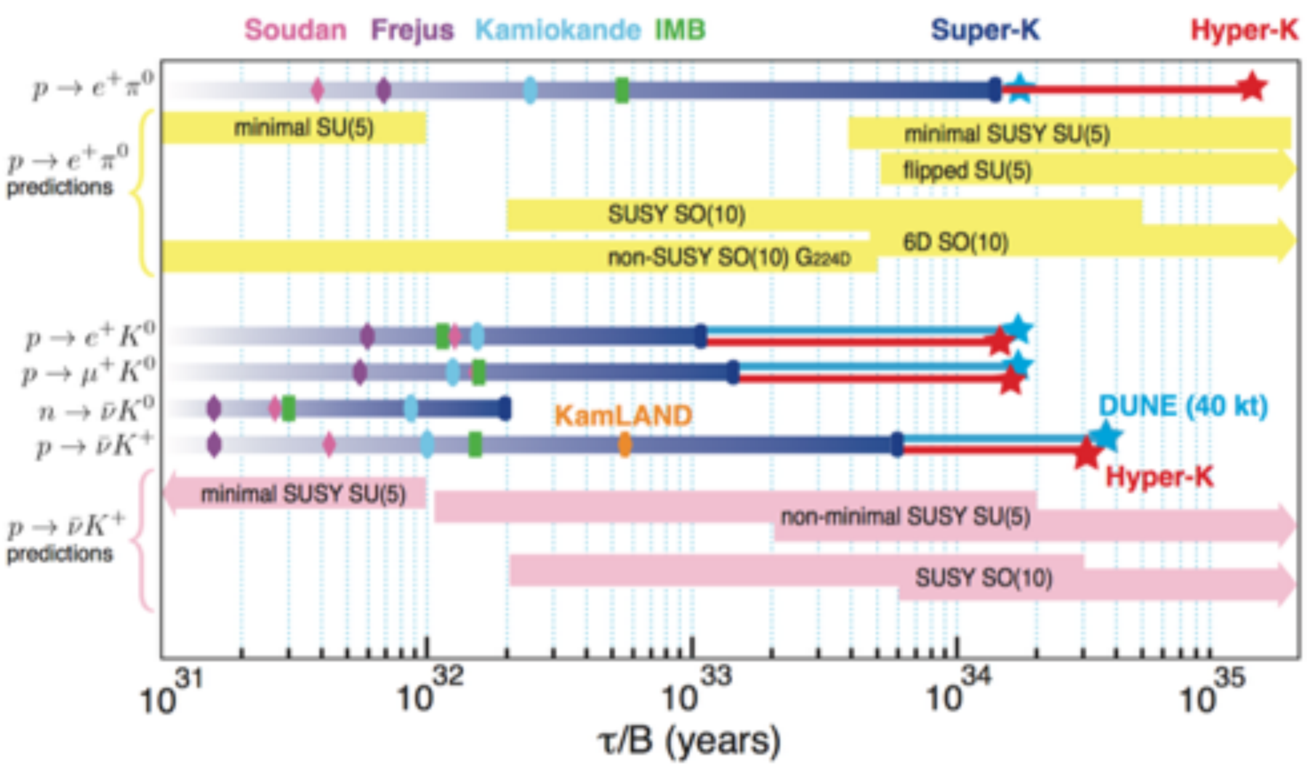
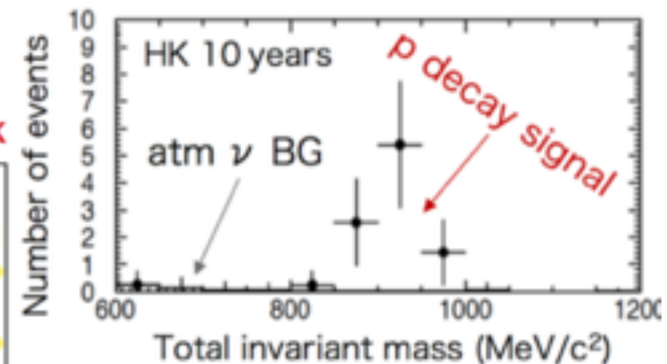
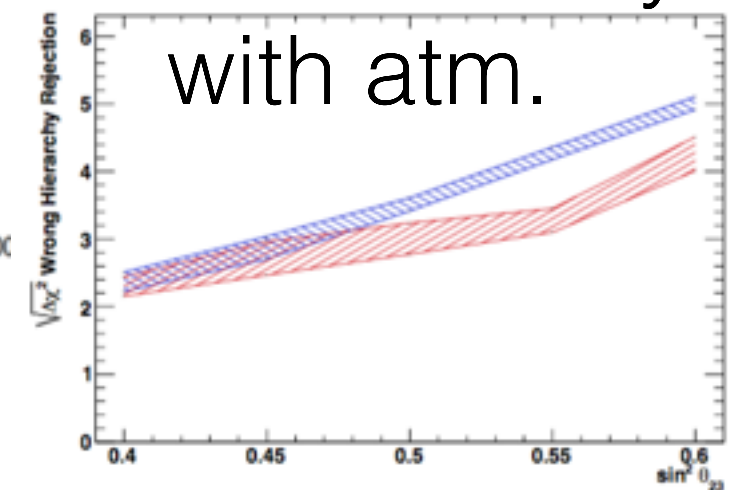
Proton Decay

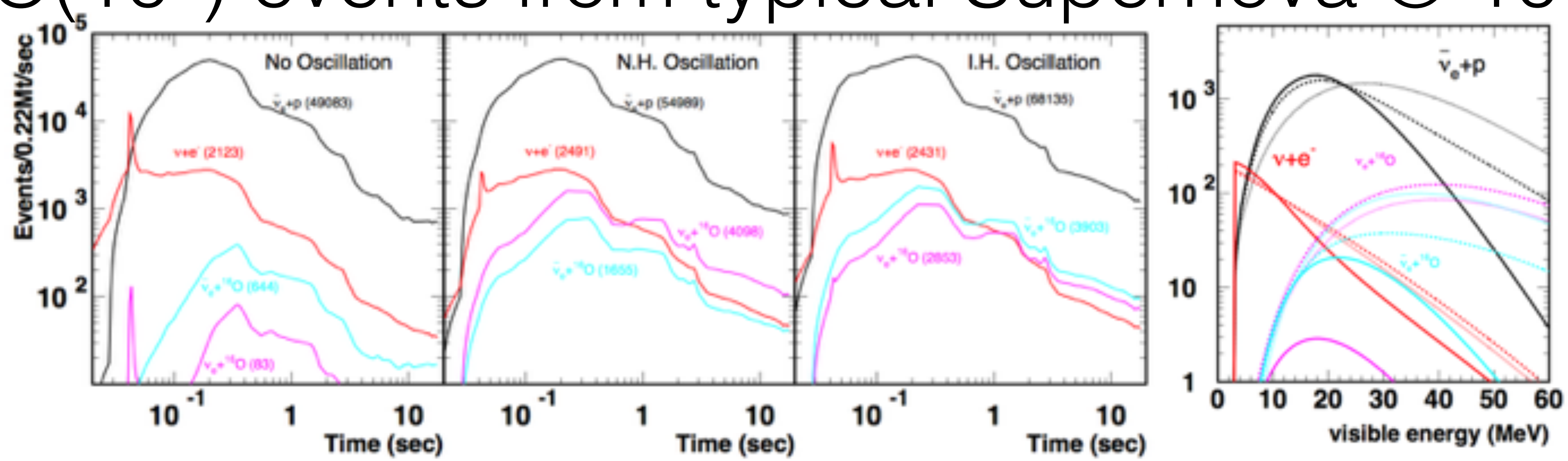




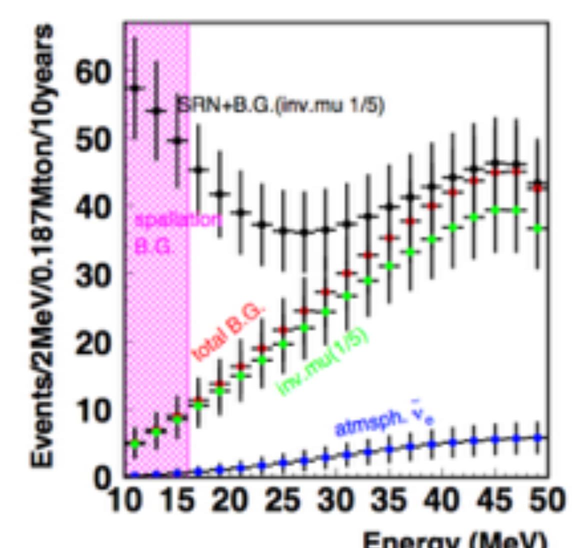
Mass hierarchy with atm.



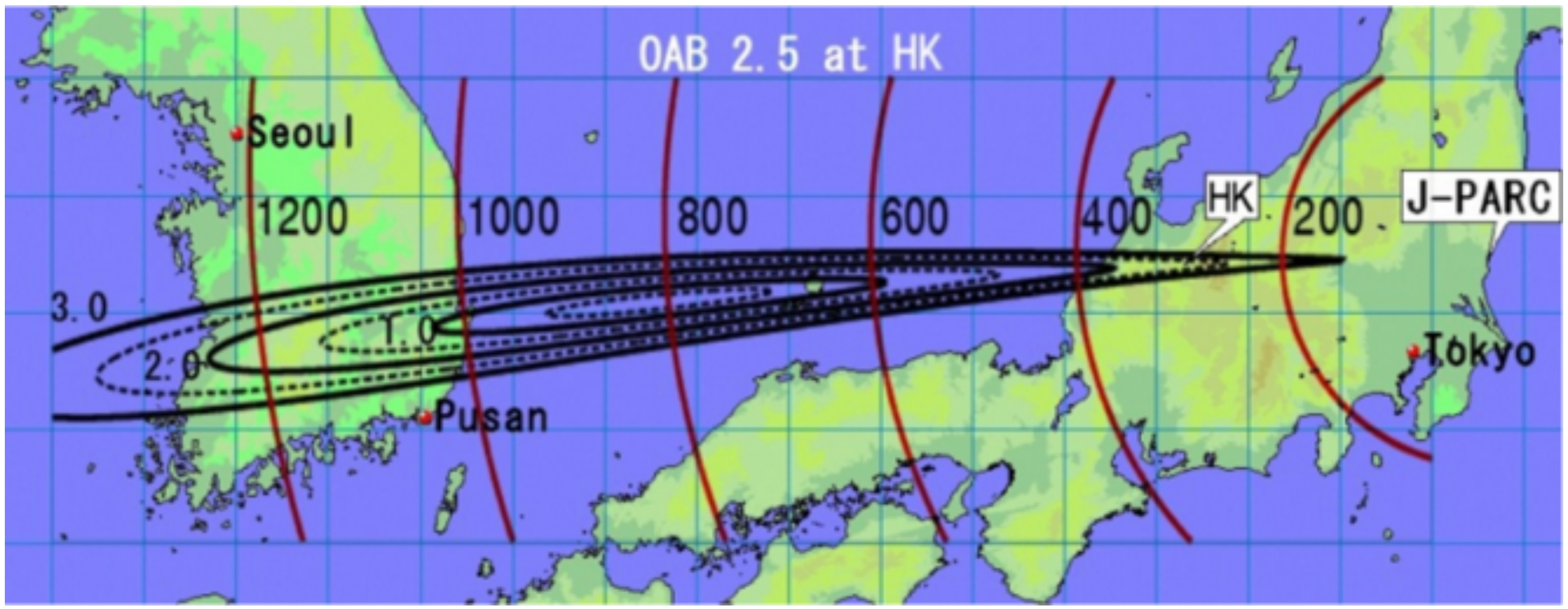
O(10⁵) events from typical Supernova @ 10 kpc

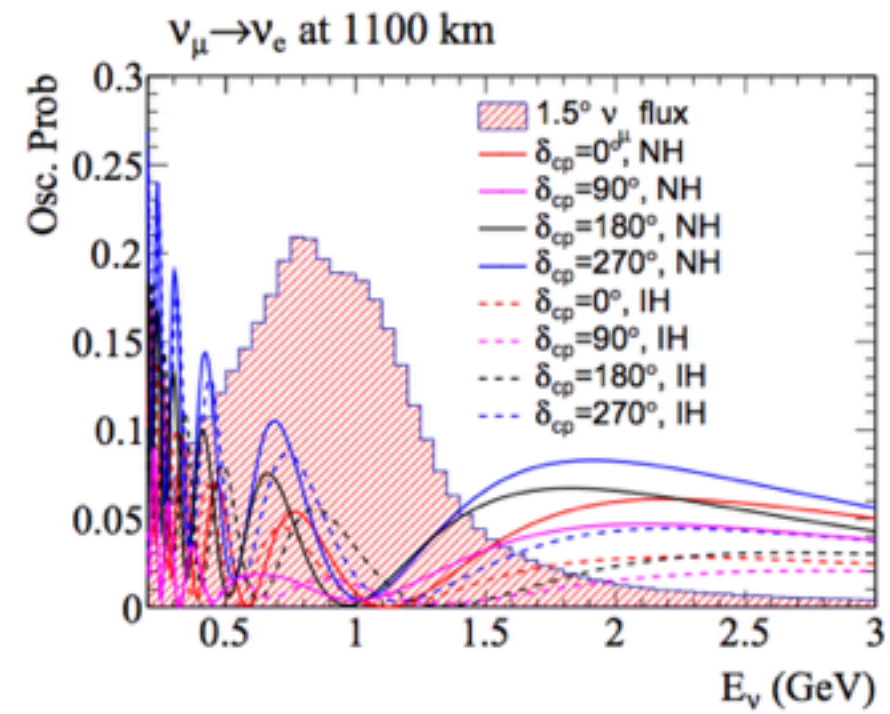


SRN



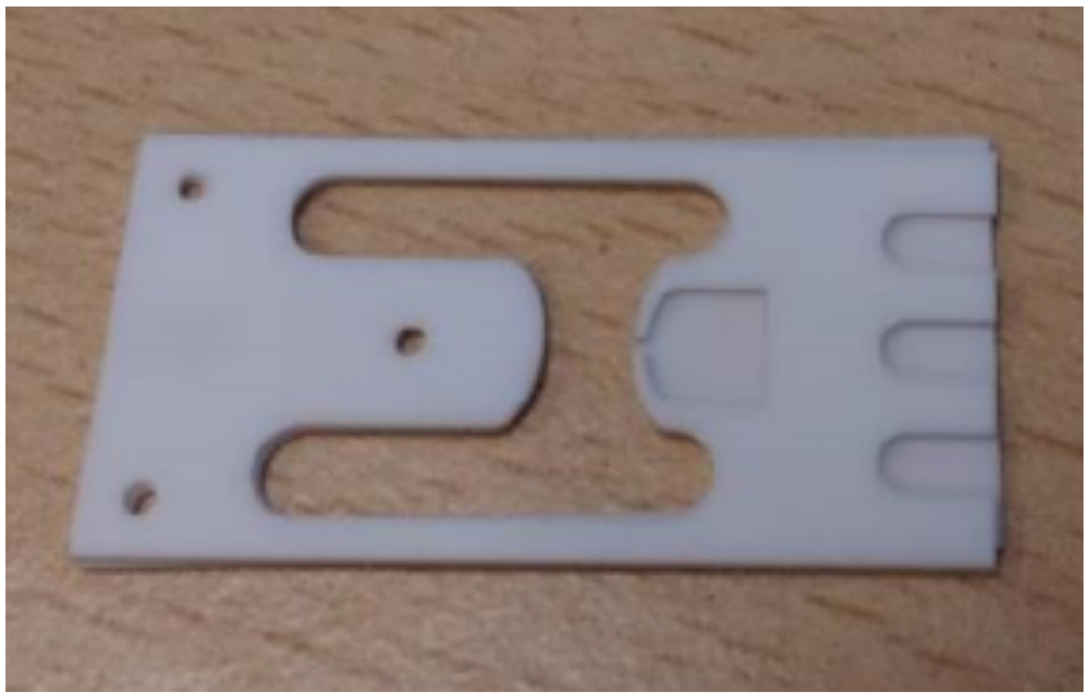
Korean Tank

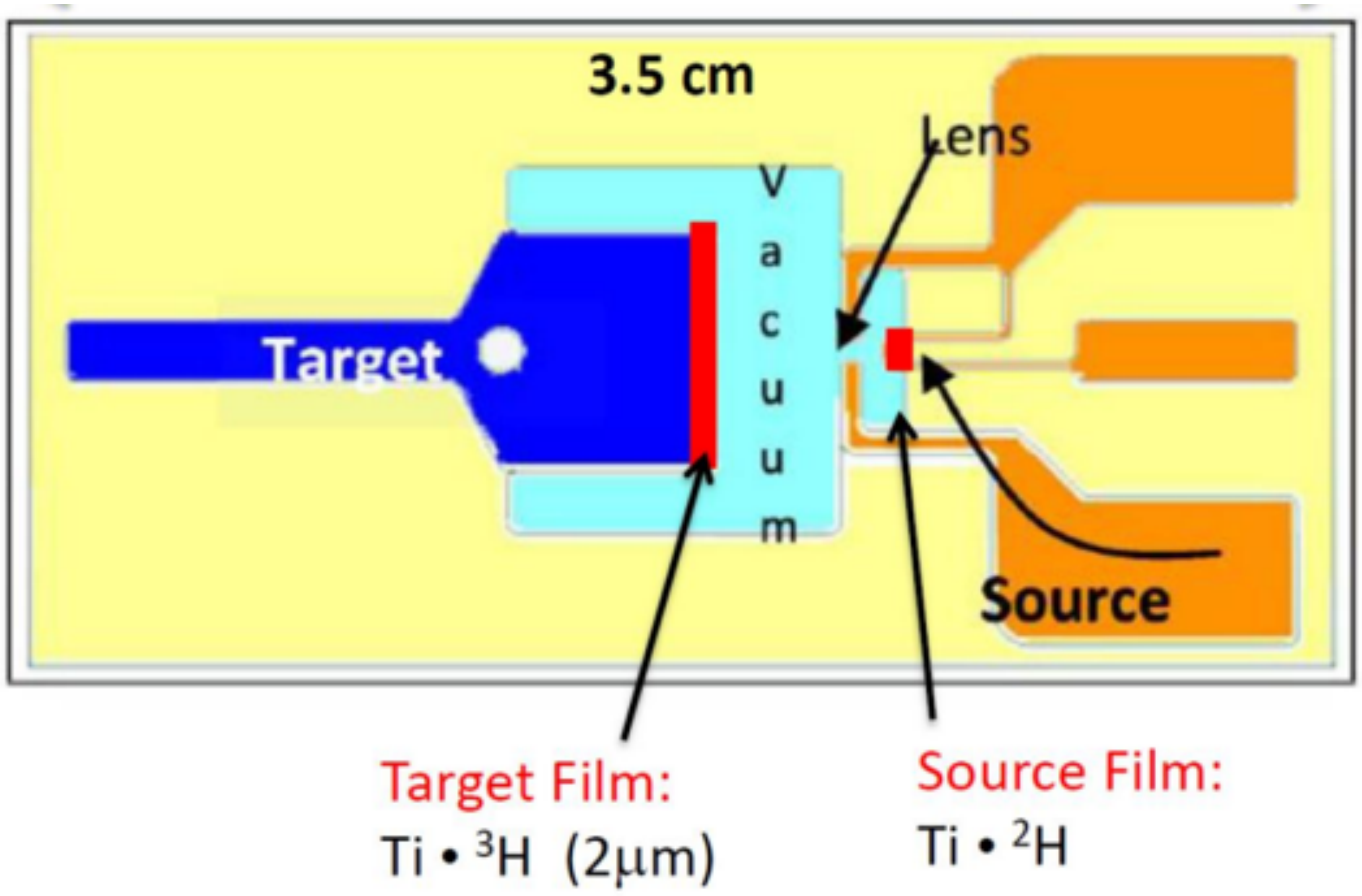




Stronger CP effect at the second oscillation maximum

A second tank in Korea would be be able to measure this effect



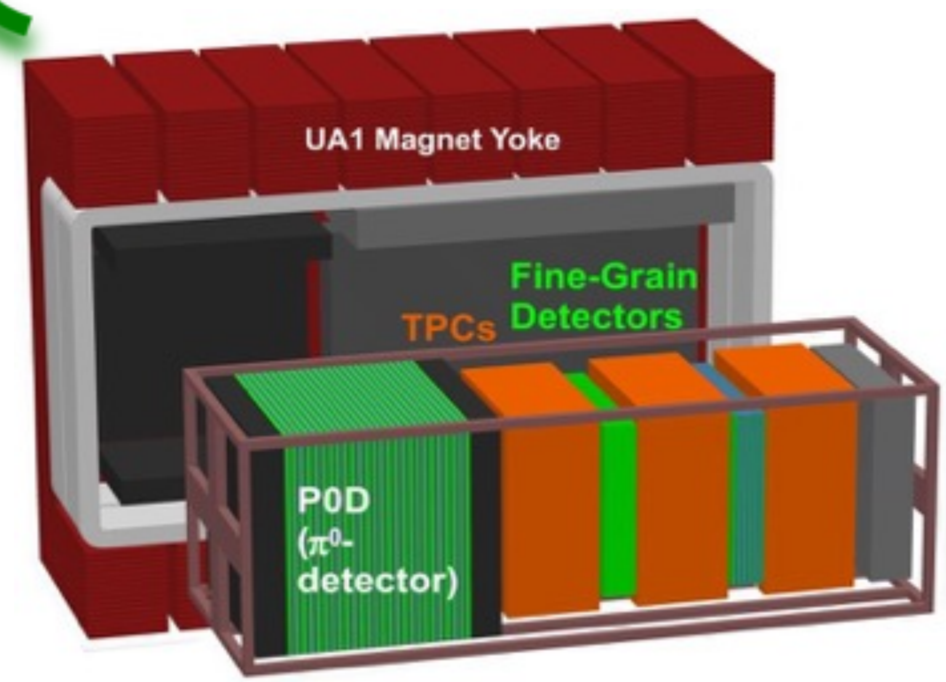


3.5 cm
Lens
V
a
c
u
u
m
Target
Source
Target Film:
Ti • ^{3}H (2μm)
Source Film:
Ti • ^{2}H

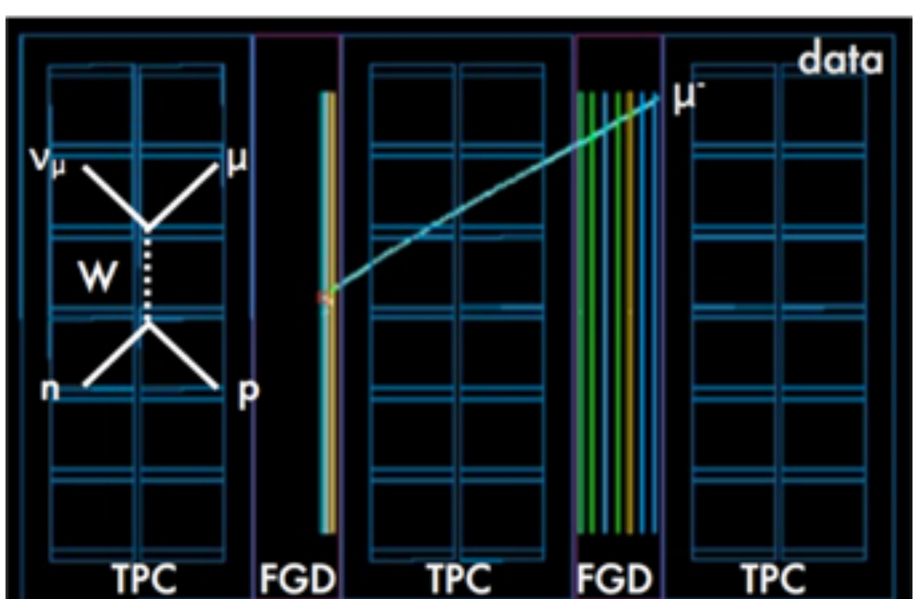
Carbon and Oxygen target materials

Acceptance differs from far detector

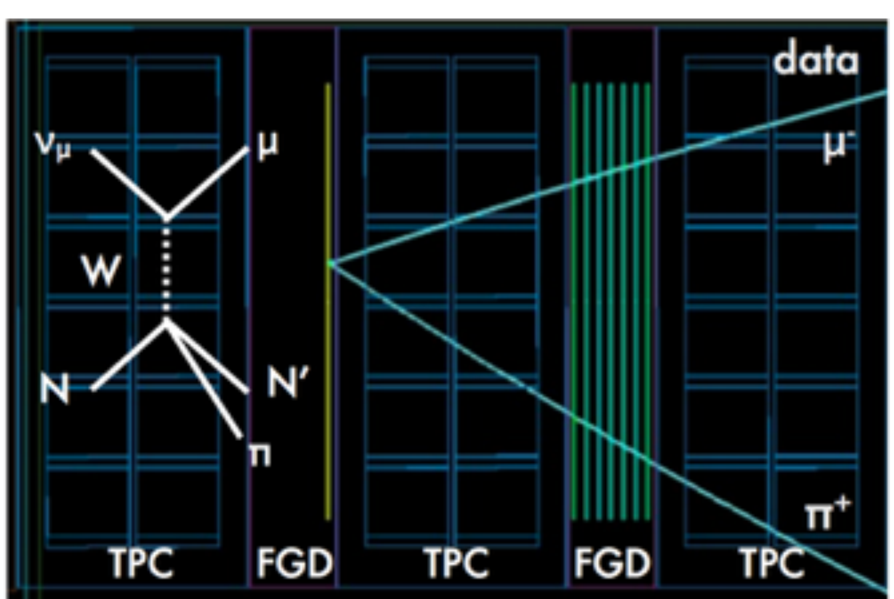
Magnetic field for sign selection



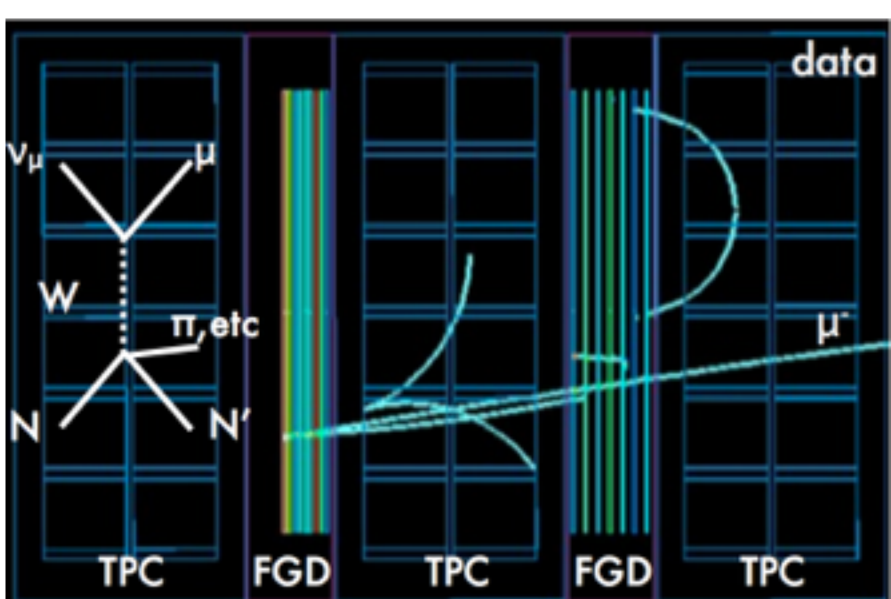
Near Detector (ND280)



CC 1μ + 0π + X



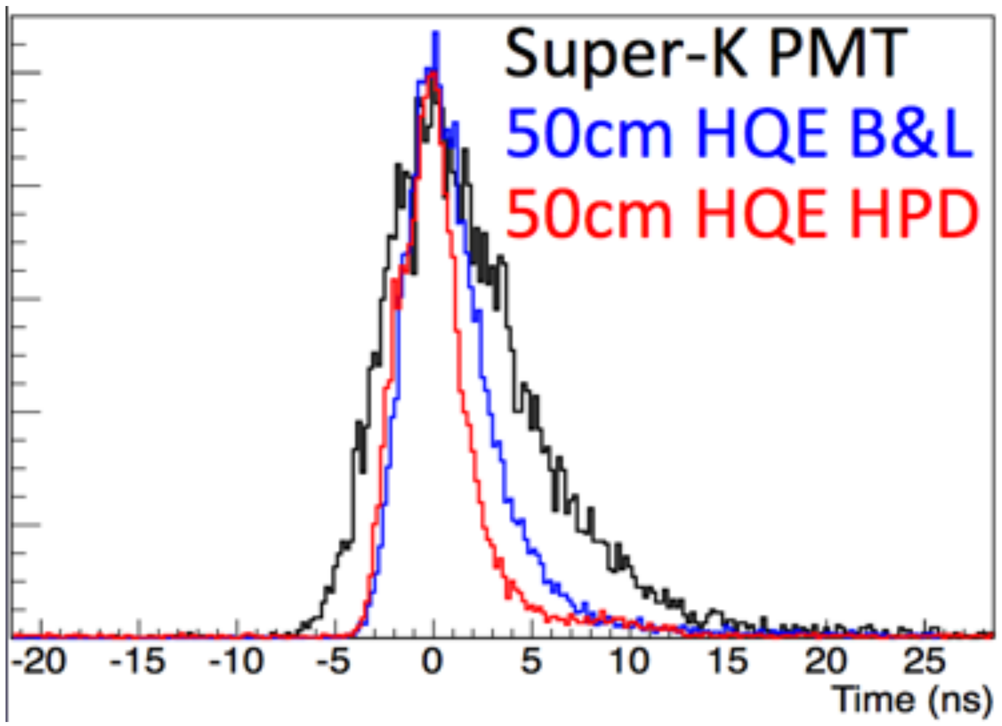
CC 1μ + 1π⁺ + X



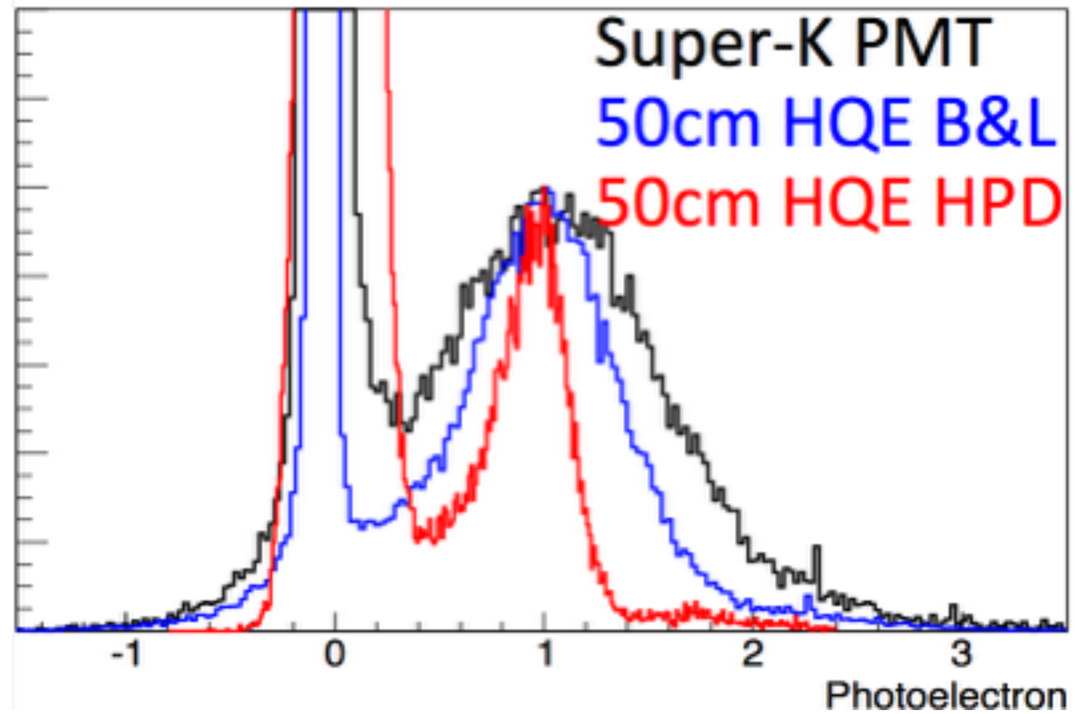
CC other

Photo Sensors

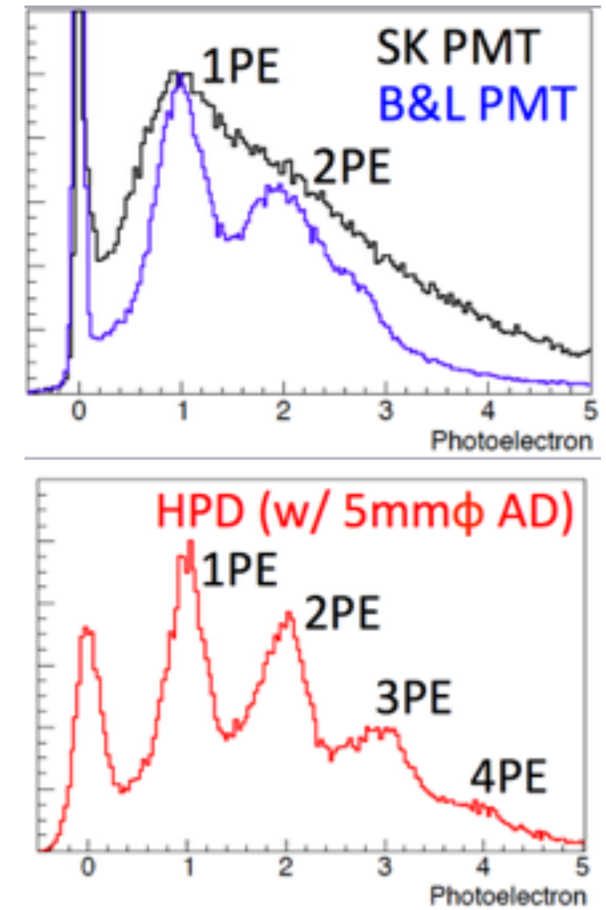
Time Resolution



1p.e. charge distribution

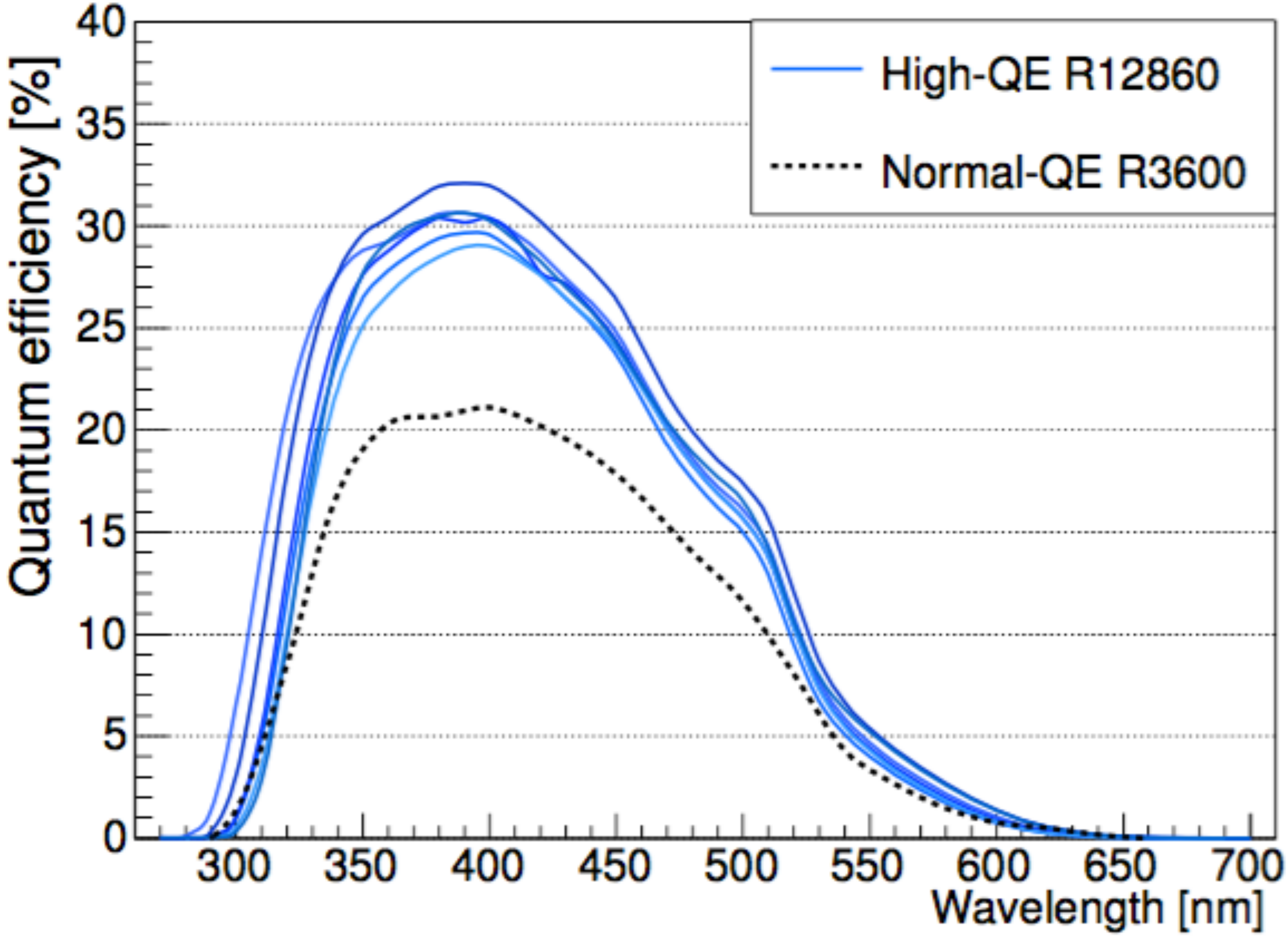


Multi-p.e. charge distribution



	SK PMT	B&L PMT	50cm HPD (20cm)
1PE T resolution σ (ns)	2.1	1.1	1.4 (1.1)
FWHM (ns)	7.3	4.1	3.4 (3.3)
1PE Q resolution σ/mean	53%	35%	16% (12%)
Peak-to-Valley ratio	2.2	4.3	3.9 (5.2)

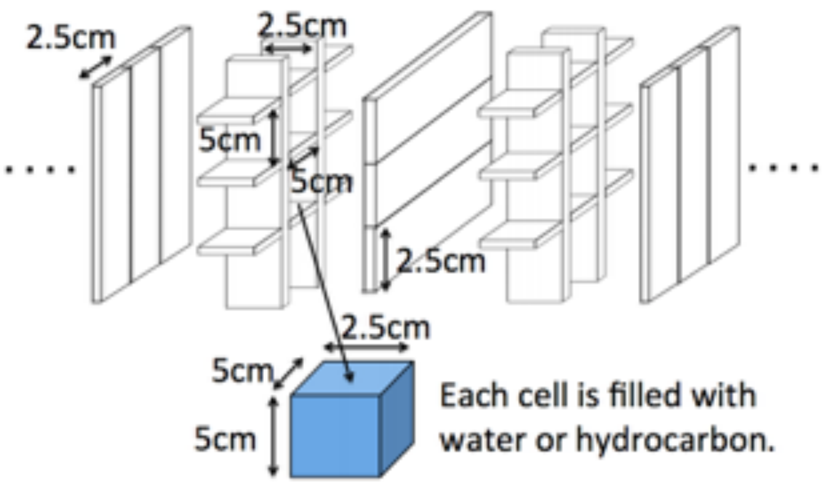
Photo Sensors

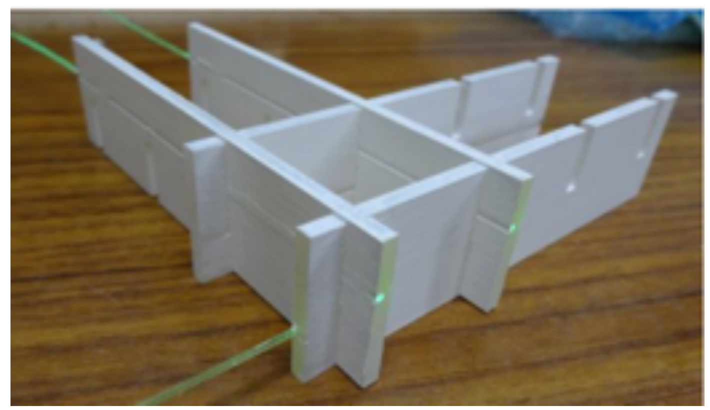


Near Detector Development

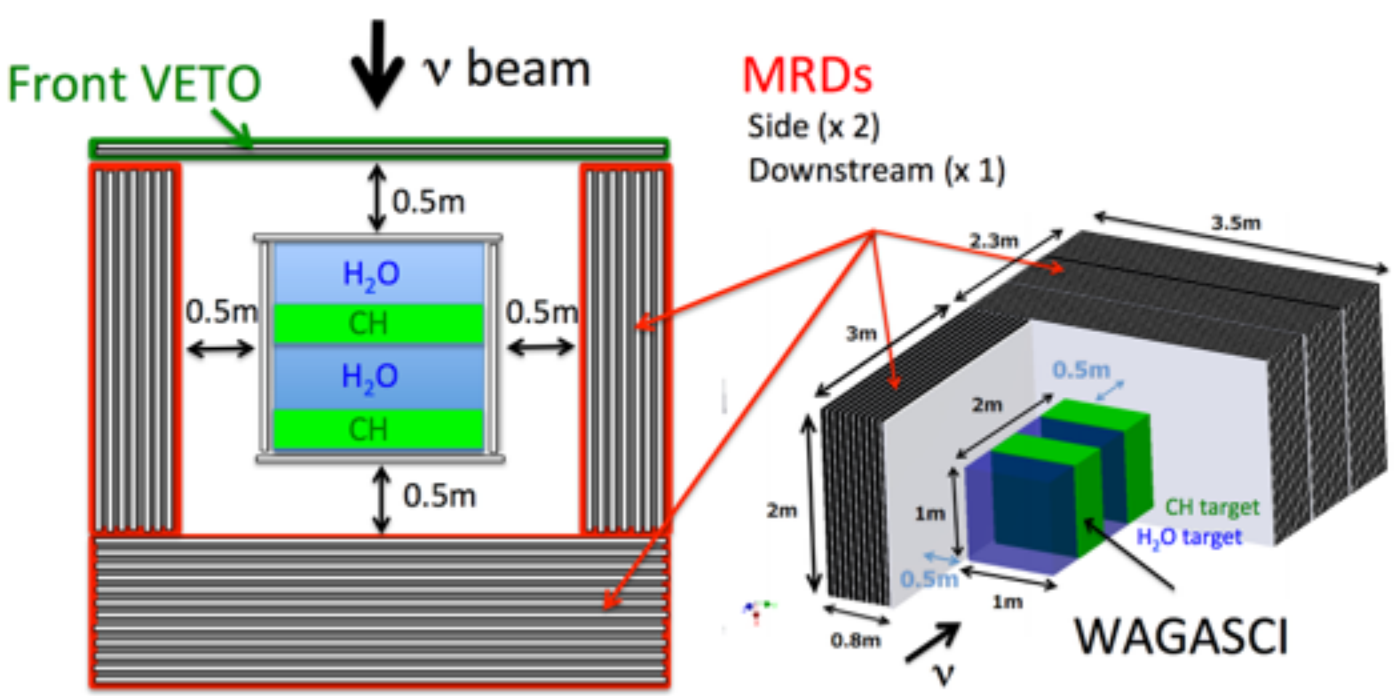
New/Upgraded Detectors in the Existing ND280 Complex

WAGASHI

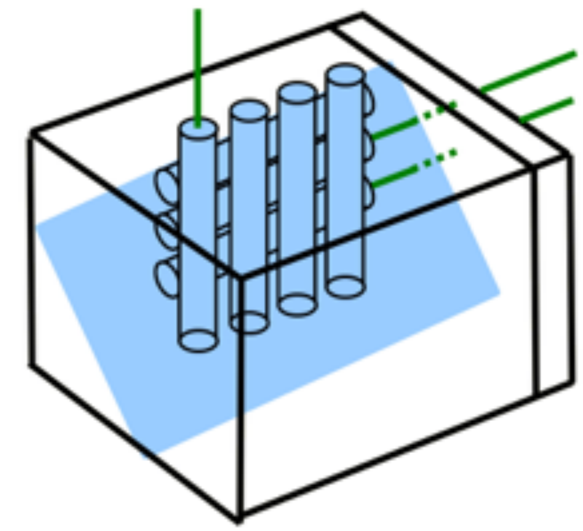




Water dominated target
4π acceptance



Water based liquid scintillator



An alternative approach is to improve knowledge of neutrino-nucleus interactions

e.g. High Pressure Gas TPC

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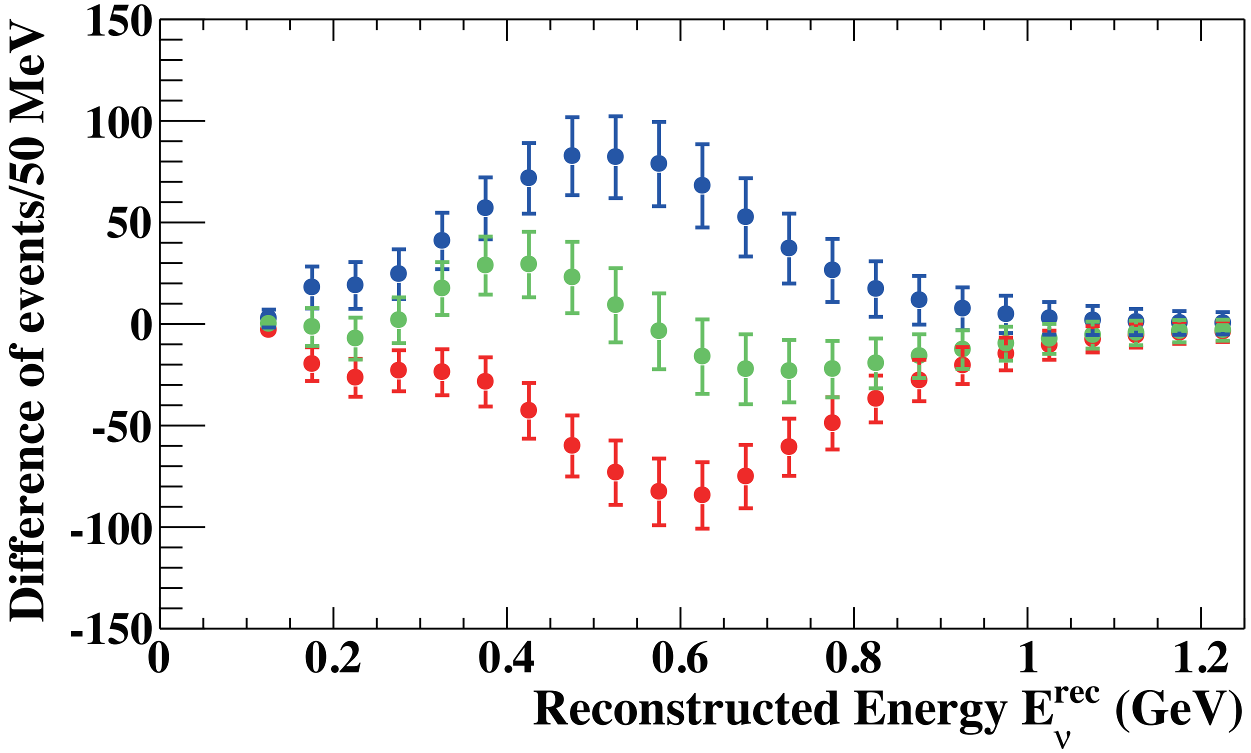
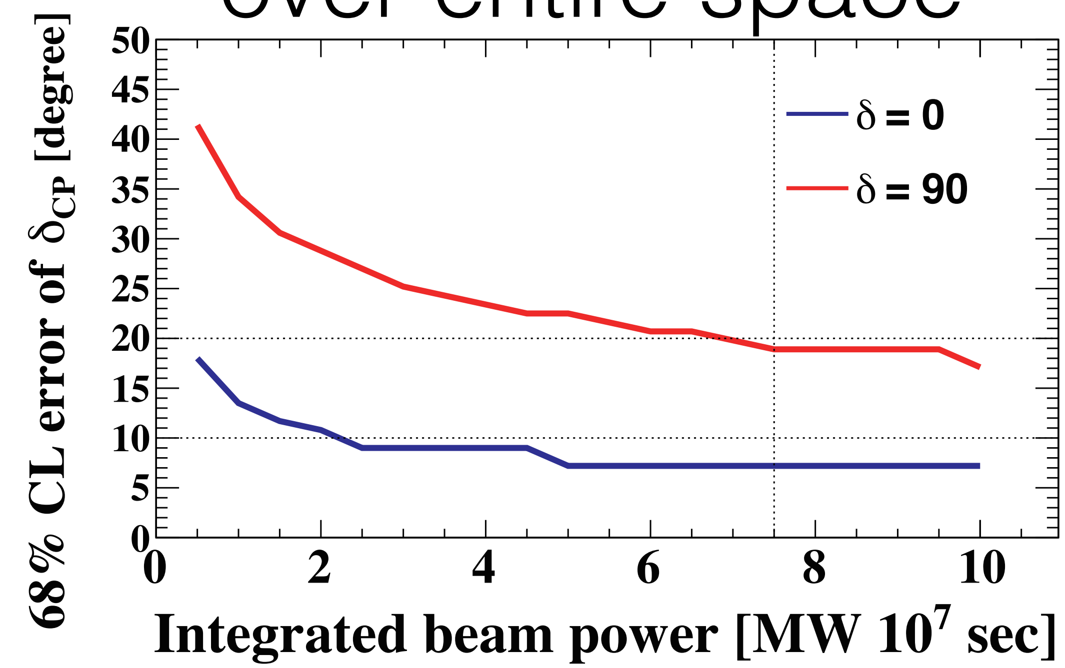
Leptonic CP Violation

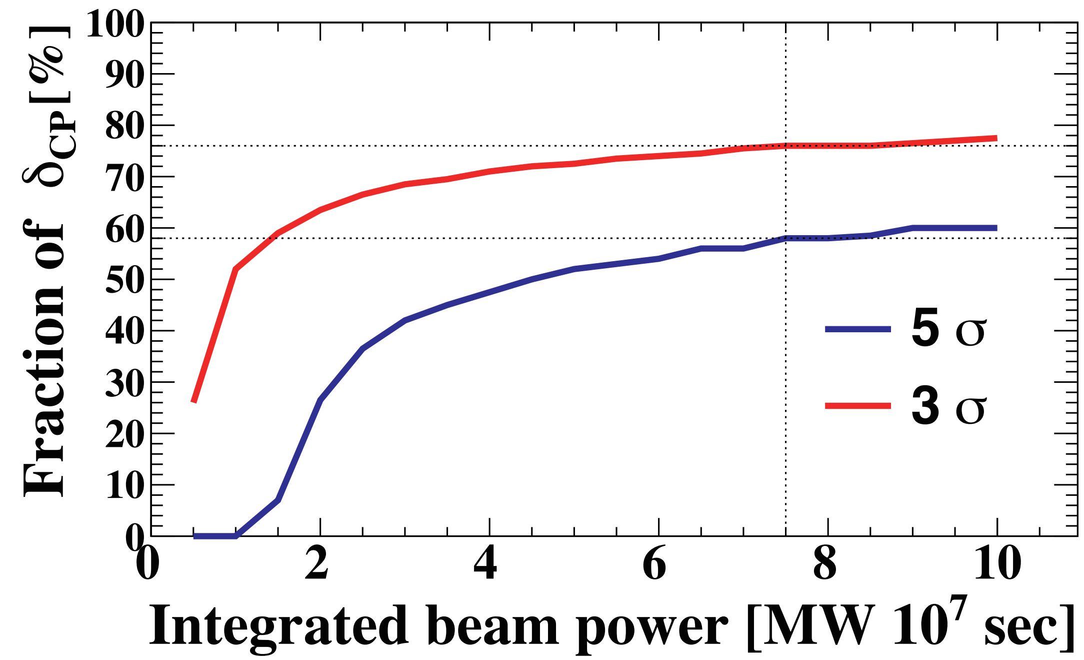
Measure δ_{CP} by comparing data with beam in ν-mode with anti-ν mode



CP violation can be established at 3σ (5σ) for 76% (58%) of δ_{CP} space.

δ_{CP} measured to < 20° over entire space





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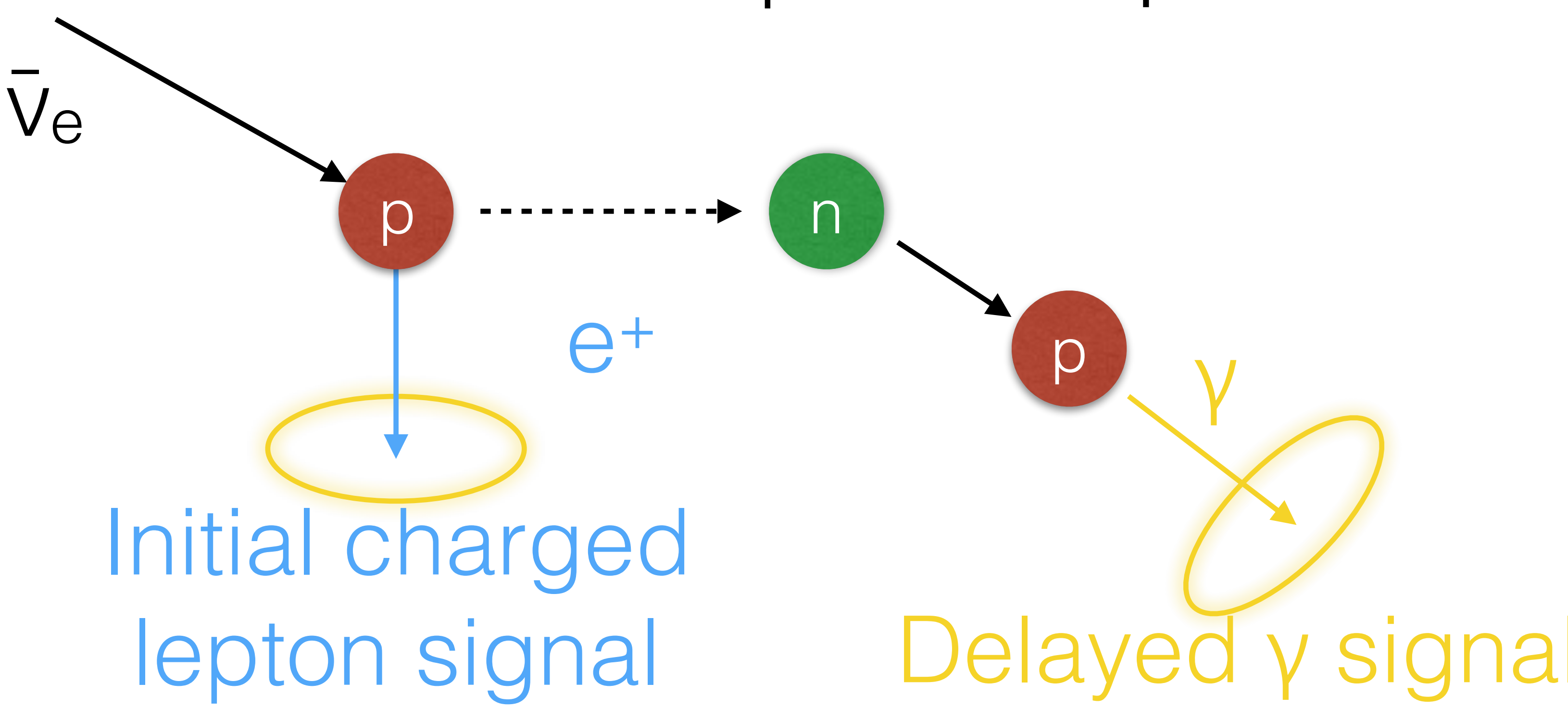
Neutron Capture on Hydrogen

$\bar{\nu}_e + p \rightarrow e^+ + n$

$n + p \rightarrow d + \gamma$



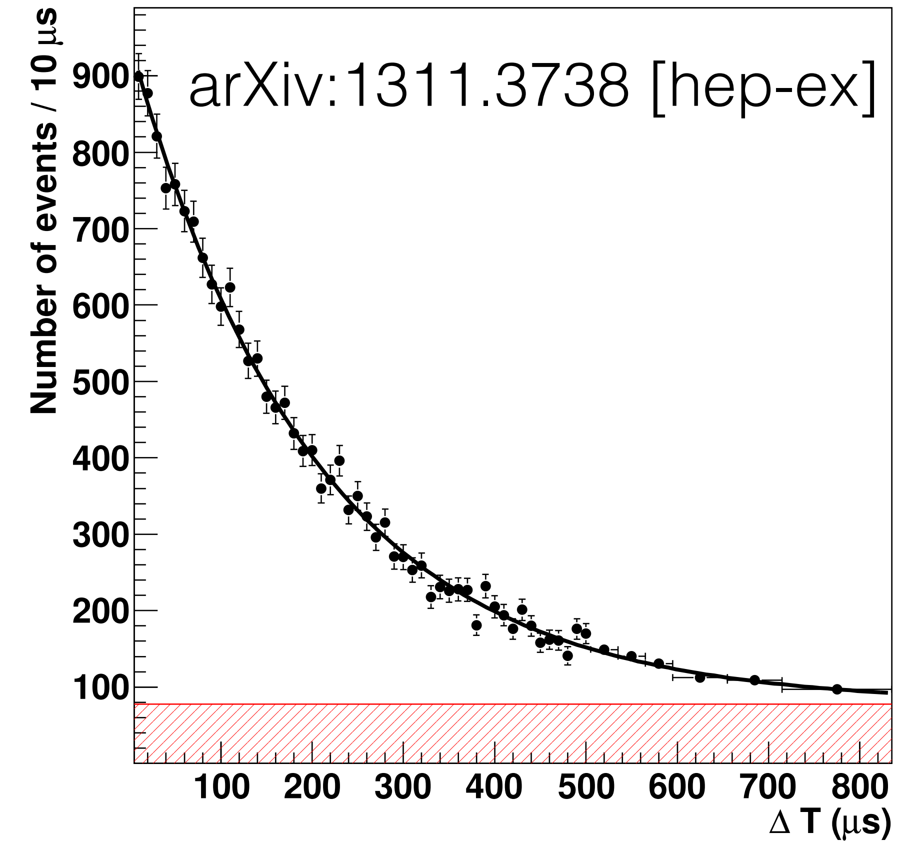
200 μs capture time

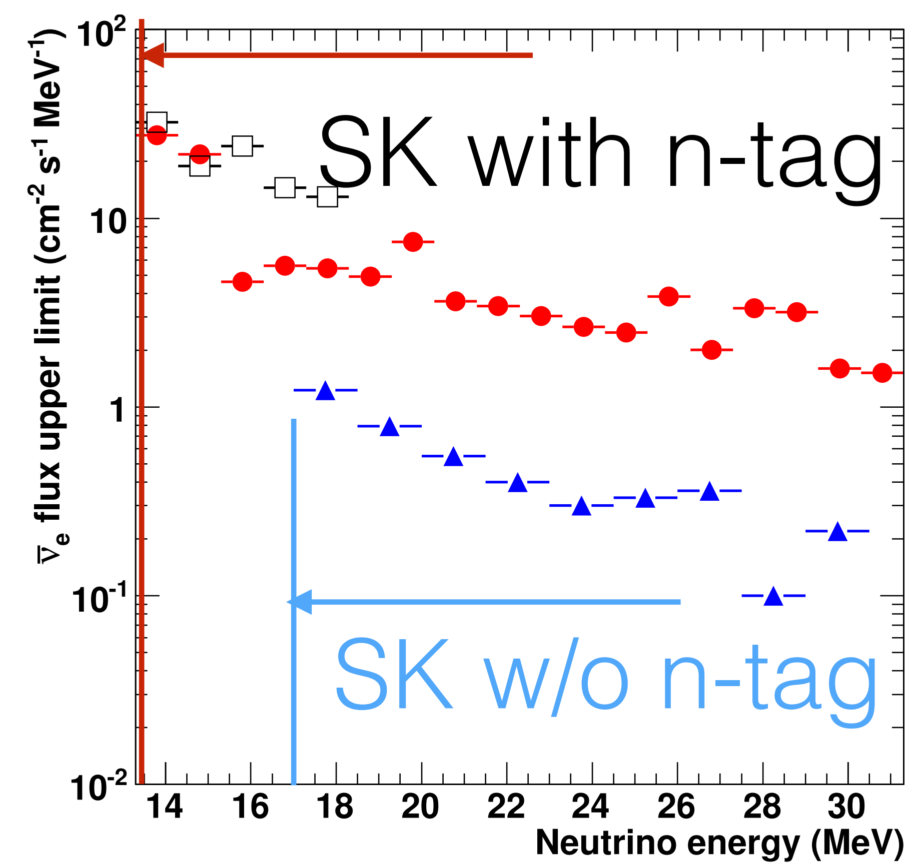
E_γ = 2.2 MeV

Low light yield

Close to or below trigger threshold

Low detection efficiency (~18%)

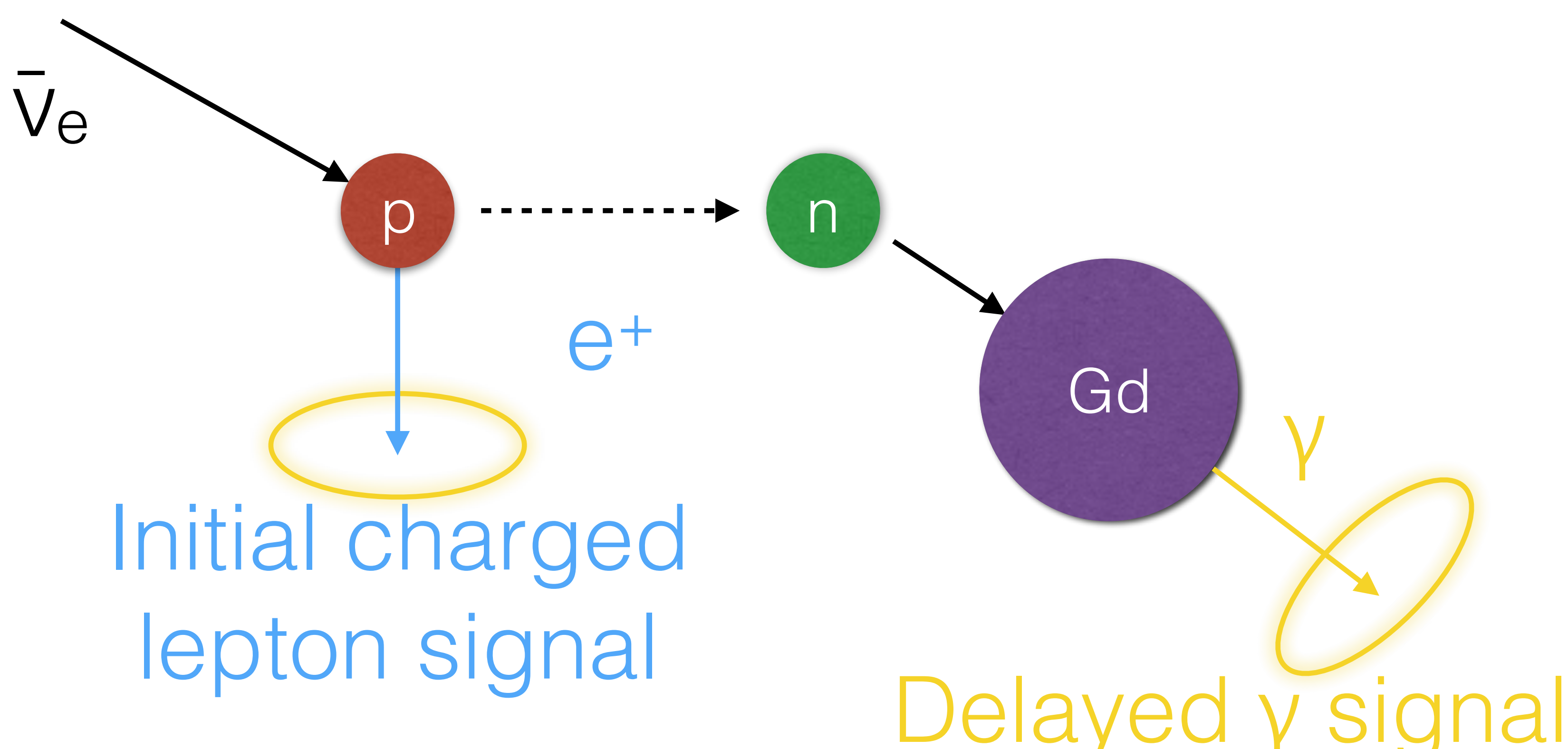


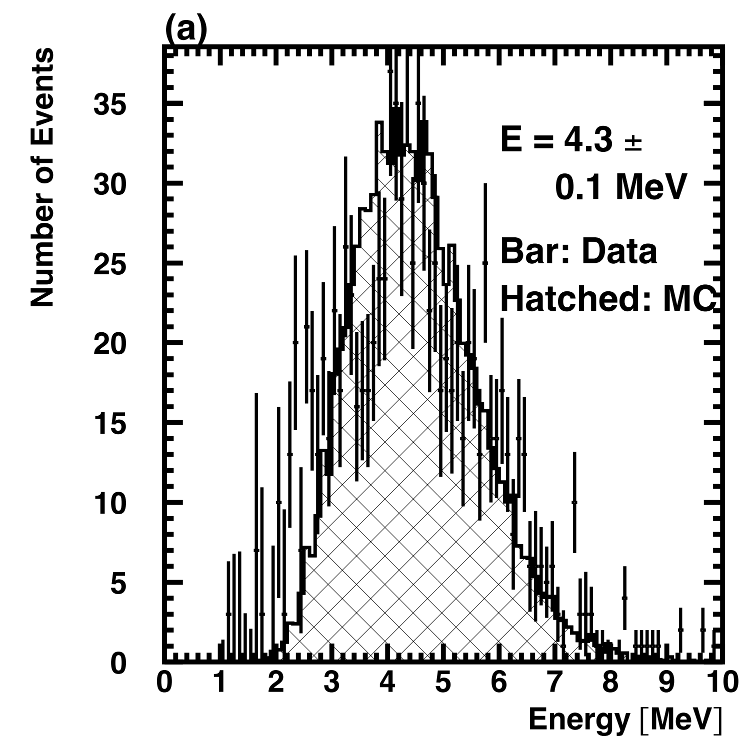


Neutron Capture on Gadolinium

arXiv:0811.0735 [hep-ex]

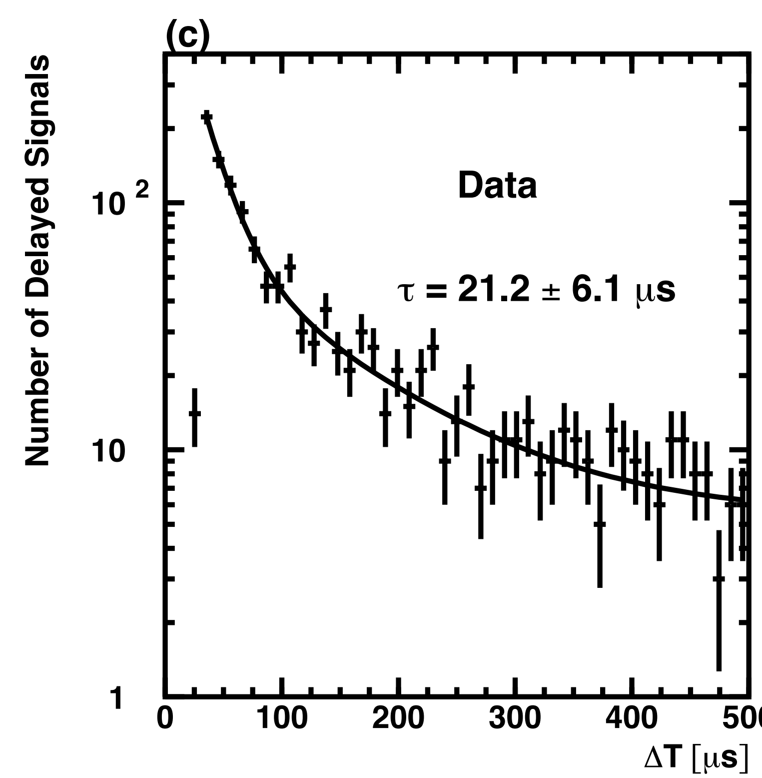
$\bar{\nu}_e + p \rightarrow e^+ + n$





20 μs capture time

E_γ ~ 8 MeV cascade (~4 MeV visible)



Fast capture time (small ΔT window)

Higher energy γ signal

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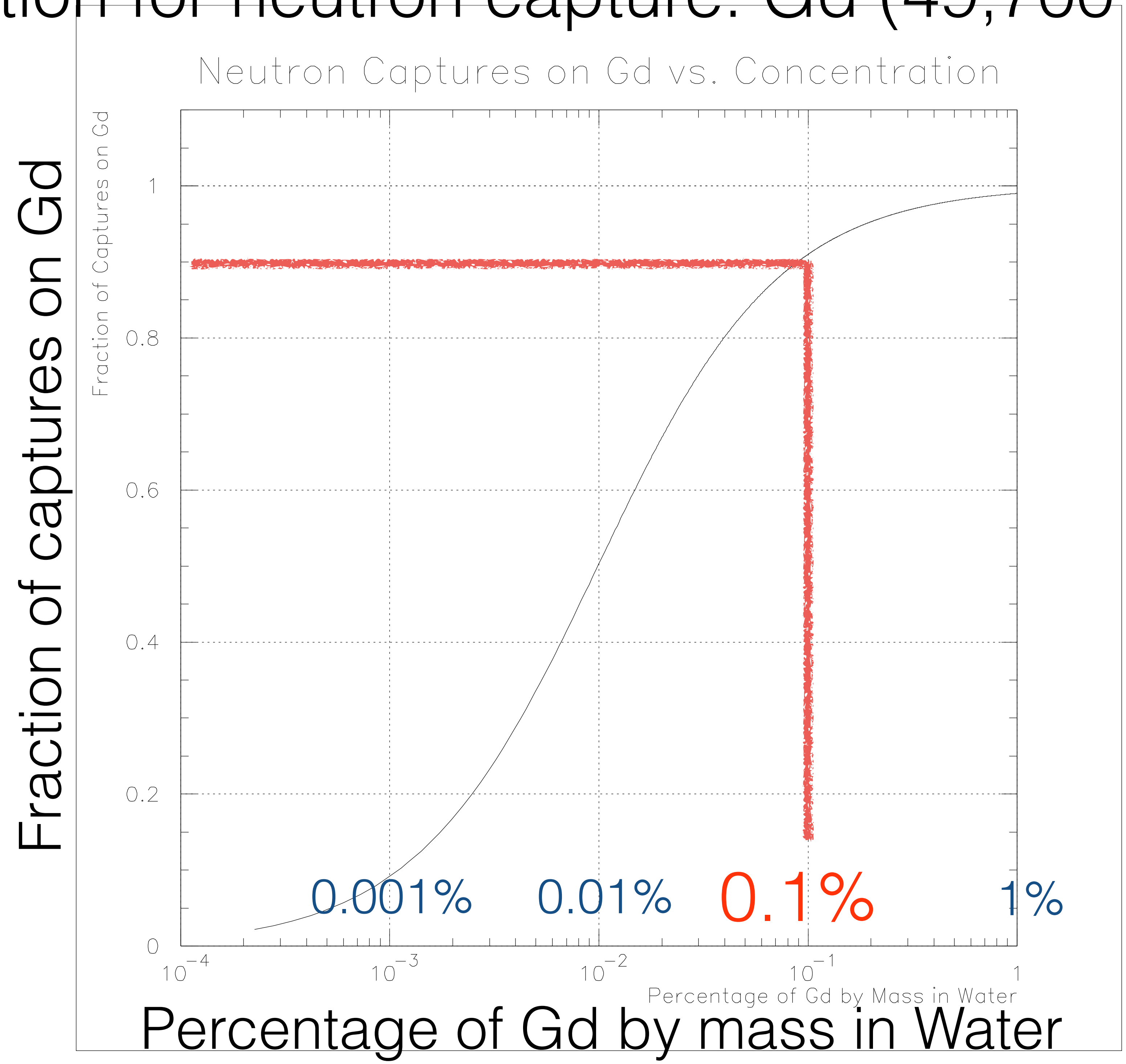
Neutron Capture on Gadolinium

Cross section for neutron capture: Gd (49,700 b), H (0.3 b)



0.1% Gd fraction gives 90% neutrons captured on Gd.

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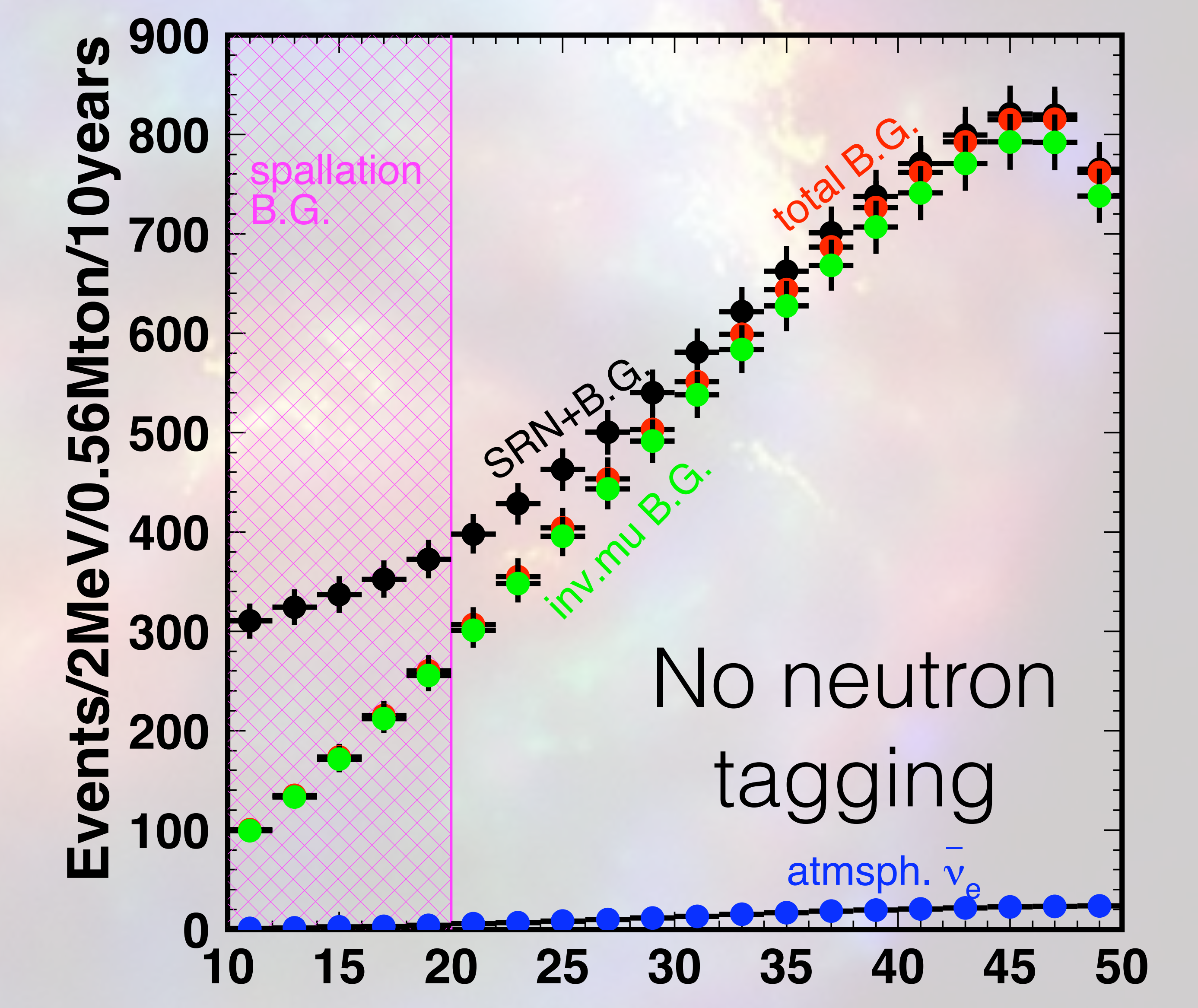
Applications: Supernova Relic Neutrinos

A low energy example

Directly observable local supernova are all too rare

Alternative is to measure diffuse supernova background DSNB/SRN

Very low rate
Large backgrounds



arXiv:1109.3262 [hep-ex]

Applications: Supernova Relic Neutrinos

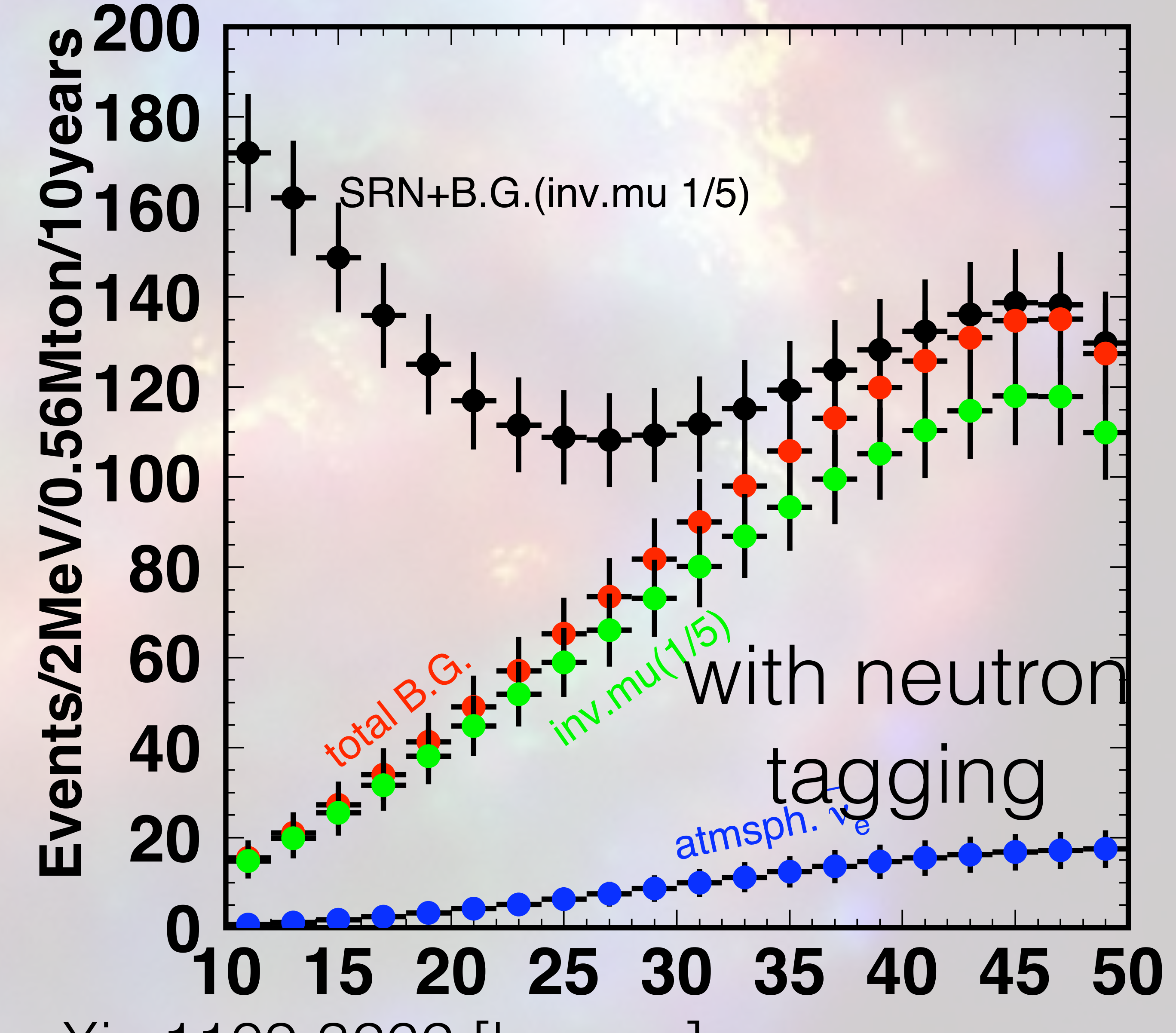
A low energy example

Directly observable local supernova are all too rare

Alternative is to measure diffuse supernova background DSNB/SRN

Very low rate
Large backgrounds

Removed by requiring coincidence with neutron



arXiv:1109.3262 [hep-ex]

A few clean events per year in SK
~100s per year in HK

Tank Parameters

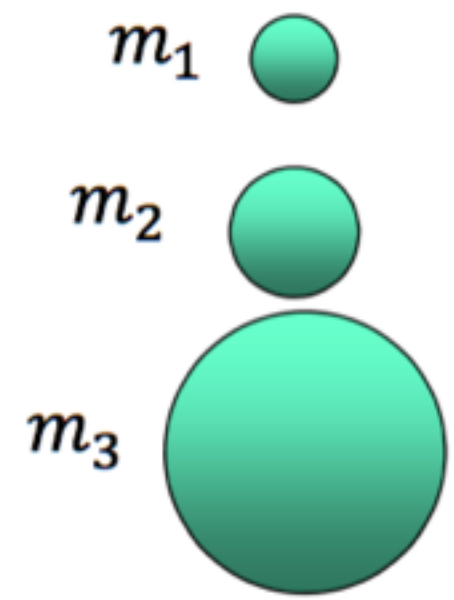
	KAM	SK	HK-1TankHD
Depth	1,000 m	1,000 m	650 m
Dimensions of water tank			
diameter	15.6 m ϕ	39 m ϕ	74 m ϕ
height	16 m	42 m	60 m
Total volume	4.5 kton	50 kton	258 kton
Fiducial volume	0.68 kton	22.5 kton	187 kton
Outer detector thickness	$\sim$ 1.5 m	$\sim$ 2 m	1 $\sim$ 2 m
Number of PMTs			
inner detector (ID)	948 (50 cm ϕ)	11,129 (50 cm ϕ)	40,000 (50 cm ϕ)
outer detector (OD)	123 (50 cm ϕ)	1,885 (20 cm ϕ)	6,700 (20 cm ϕ)
Photo-sensitive coverage	20%	40%	40%
Single-photon detection efficiency of ID PMT	unknown	12%	24%
Single-photon timing resolution of ID PMT	$\sim$ 4 nsec	2-3 nsec	1 nsec

Three Flavor Mixing in Lepton Sector

Weak eigenstates

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \mathrm{U}_{\mathrm{MNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

mass eigenstates



$$\mathrm{U}_{\mathrm{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & +c_{23} & +s_{23} \\ 0 & -s_{23} & +c_{23} \end{pmatrix} \begin{pmatrix} +c_{13} & 0 & +s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & +c_{13} \end{pmatrix} \begin{pmatrix} +c_{12} & +s_{12} & 0 \\ -s_{12} & +c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(c_{ij} = \cos\theta_{ij}, s_{ij} = \sin\theta_{ij})$$

$\boldsymbol{\theta_{12}, \theta_{23}, \theta_{13}, \delta,}$
$\Delta m^2_{21}, \Delta m^2_{32}, \Delta m^2_{31}$

$^*\Delta m^2_{ij} = m^2_i - m^2_j$
Out of three Δm^2's, number of free parameters is two. ($\Delta m^2_{31} = \Delta m^2_{21} + \Delta \mathrm{m}^2_{32}$)

ν_μ disappearance probability

θ_{13}=0 case

$$P_{\mu\to x} \approx 1 - \sin^2 2\theta_{23} \cdot \sin^2\left(\frac{\Delta m_{32}^2 L}{4E_\nu}\right)$$

For non-zero θ_{13}

$$P_{\mu\to x} \approx 1 - \left(\cos^4\theta_{13}\cdot\sin^2 2\theta_{23} + \sin^2\theta_{23}\cdot\sin^2 2\theta_{13}\right)\sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right)$$

$$\Delta m^2 \approx \Delta m_{32}^2 \approx \Delta m_{31}^2$$

Maximal disappearance occurs at $\sin^2\theta_{23} = \frac{1}{2\cos^2\theta_{13}} = 0.513$

more on ν_μ disappearance

- ν_μ disappearance probability in vacuum

$$
\begin{aligned}
P(\nu_\mu \to \nu_\mu) &= 1 - (c_{13}^4 \ \sin^2 2\theta_{23} + s_{23}^2 \ \sin^2 2\theta_{13}) \sin^2 \Delta_{atm} \\
&+ \left\{ c_{13}^2 (c_{12}^2 - s_{13}^2 \ s_{23}^2) \sin^2 2\theta_{23} + s_{12}^2 \ s_{23}^2 \ \sin^2 2\theta_{13} - c_{13} \ \sin 2\theta_{13} \ \sin 2\theta_{23} \ \sin 2\theta_{12} \ \cos\delta \right\} \\
&\times \left\{ \frac{1}{2} \sin 2\Delta_{solar} \ \sin 2\Delta_{atm} + 2\sin^2 \Delta_{solar} \ \sin^2 \Delta_{atm} \right\} \\
&- \left\{ \sin^2 2\theta_{12} \left(c_{23}^2 - s_{13}^2 \ s_{23}^2\right)^2 + s_{13}^2 \ \sin^2 2\theta_{23} \left(1 - c_\delta^2 \ \sin^2 2\theta_{12}\right) \right. \\
&\quad + 2 s_{13} \ \sin 2\theta_{12} \ \cos 2\theta_{12} \ \sin\theta_{23} \ \cos 2\theta_{23} \ c_\delta \\
&\quad - \frac{1}{2} c_{13} \ \sin 2\theta_{13} \sin 2\theta_{23} \sin 2\theta_{12} \ \cos\delta \ s_{23}^2 \ s_{12}^2 \\
&\quad \left. + \sin^2 2\theta_{23} \ c_{13}^2 \ \left(c_{12}^2 - s_{13}^2 \ s_{12}^2\right) + s_{13}^2 \ s_{23}^2 \ \sin^2 2\theta_{13} \right\} \times \sin^2 \Delta_{solar} \qquad (26)
\end{aligned}
$$

$$
\begin{aligned}
s_{ij} &= \sin\theta_{ij} \\
c_{ij} &= \cos\theta_{ij} \\
c_\delta &= \cos\delta \\
\Delta_{atm} &= \frac{\Delta m_{13}^2 \ L}{4 \ E_\nu} \\
\Delta_{solar} &= \frac{\Delta m_{21}^2 \ L}{4 \ E_{/nu}}
\end{aligned}
$$

T2K: L = 295 km, E_ν peaks at ~ 0.6 GeV -> $\sin^2\Delta_{solar}$ ~ 0, $\sin 2\Delta_{atm}$ ~ 0

$$
P(\nu_\mu \to \nu_\mu) \sim 1 - \left(\underbrace{\cos^4\theta_{13} \cdot \sin^2 2\theta_{23}}_{\text{Leading-term}} + \underbrace{\sin^2 2\theta_{13} \cdot \sin^2\theta_{23}}_{\text{Next-to-leading}}\right) \cdot \sin^2 \frac{\Delta m_{31}^2 \cdot L}{4E}
$$

Leading-term Next-to-leading

ν_μ disapp. probability depends on $\sin^2 2\theta_{13} \cdot \sin^2\theta_{23}$ to second order

-> Can be used in combination with known $\sin^2 2\theta_{13}$ to resolve the θ_{23} octant

ν_e appearance probability
Leading term only

$$P_{\mu \to e} \approx \sin^2 \theta_{23} \cdot \sin^2 2\theta_{13} \cdot \sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right)$$

$$\Delta m^2 \approx \Delta m_{32}^2 \approx \Delta m_{31}^2$$

ν_e appearance probability (exact formula in vacuum)

$$P(\nu_\mu \to \nu_e) = 4c_{13}^2 s_{13}^2 s_{23}^2 \sin^2\Delta_{31}$$

Leading term — θ_{13}

$$+8c_{13}^2 s_{12}s_{13}s_{23}(c_{12}c_{23}\cos\delta - s_{12}s_{13}s_{23})\cos\Delta_{32}\sin\Delta_{31}\sin\Delta_{21}$$

CPC

$$-8c_{13}^2 c_{12}c_{23}s_{12}s_{13}s_{23}\sin\delta\sin\Delta_{32}\sin\Delta_{31}\sin\Delta_{21}$$

CPV

$$+4s_{12}^2 c_{13}^2 (c_{12}^2 c_{23}^2 + s_{12}^2 s_{23}^2 s_{13}^2 - 2c_{12}c_{23}s_{12}s_{23}s_{13}\cos\delta)\sin^2\Delta_{21}$$

Solar

$c_{ij} = \cos\theta_{ij}, s_{ij} = \sin\theta_{ij}$

$\Delta_{ij} = \Delta m_{ij}^2 \frac{L}{4E_\nu}$

replace δ by –δ for $P(\overline{\nu_\mu} \to \overline{\nu_e})$

CP violating term introduced by interference among three-flavor mixing

ν_e appearance probability with 1st order matter effect

$$P(\nu_\mu \to \nu_e) \approx 4c_{13}^2 s_{13}^2 s_{23}^2 \sin^2\Delta_{31}\left(1+\frac{2a}{\Delta m_{31}^2}\left(1-2s_{13}^2\right)\right)$$

Leading including matter effect

$$+8c_{13}^2 s_{12}s_{13}s_{23}(c_{12}c_{23}\cos\delta - s_{12}s_{13}s_{23})\cos\Delta_{32}\sin\Delta_{31}\sin\Delta_{21}$$

CP conserving

$$-8c_{13}^2 c_{12}c_{23}s_{12}s_{13}s_{23}\sin\delta\sin\Delta_{32}\sin\Delta_{31}\sin\Delta_{21}$$

CP violating

$$+4s_{12}^2 c_{13}^2(c_{12}^2 c_{23}^2 + s_{12}^2 s_{23}^2 s_{13}^2 - 2c_{12}c_{23}s_{12}s_{23}s_{13}\cos\delta)\sin^2\Delta_{21}$$

Solar

$$-8c_{13}^2 s_{13}^2 s_{23}^2(1-2s_{13}^2)\frac{aL}{4E}\cos\Delta_{32}\sin\Delta_{31}$$

Matter effect (small)

$$c_{ij} = \cos\theta_{ij}, s_{ij} = \sin\theta_{ij}$$

$$\Delta_{ij} = \Delta m_{ij}^2 \frac{L}{4E_\nu}$$

$$a \equiv 2\sqrt{2}G_F n_e E = 7.56\times10^{-5}\,\mathrm{eV}^2 \frac{\rho}{\mathrm{gcm}^{-3}}\frac{E}{GeV}$$

replace δ by $-\delta$ and a by $-a$ for $P(\overline{\nu_\mu} \to \overline{\nu_e})$

ν_e appearance probability

approximation at around oscillation maximum

$$P(\nu_\mu \to \nu_e) \approx \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2\left(\frac{\Delta m_{32}^2 L}{4E_\nu}\right)\left(1+\frac{2a}{\Delta m_{31}^2}\left(1-2\sin^2\theta_{13}\right)\right)$$

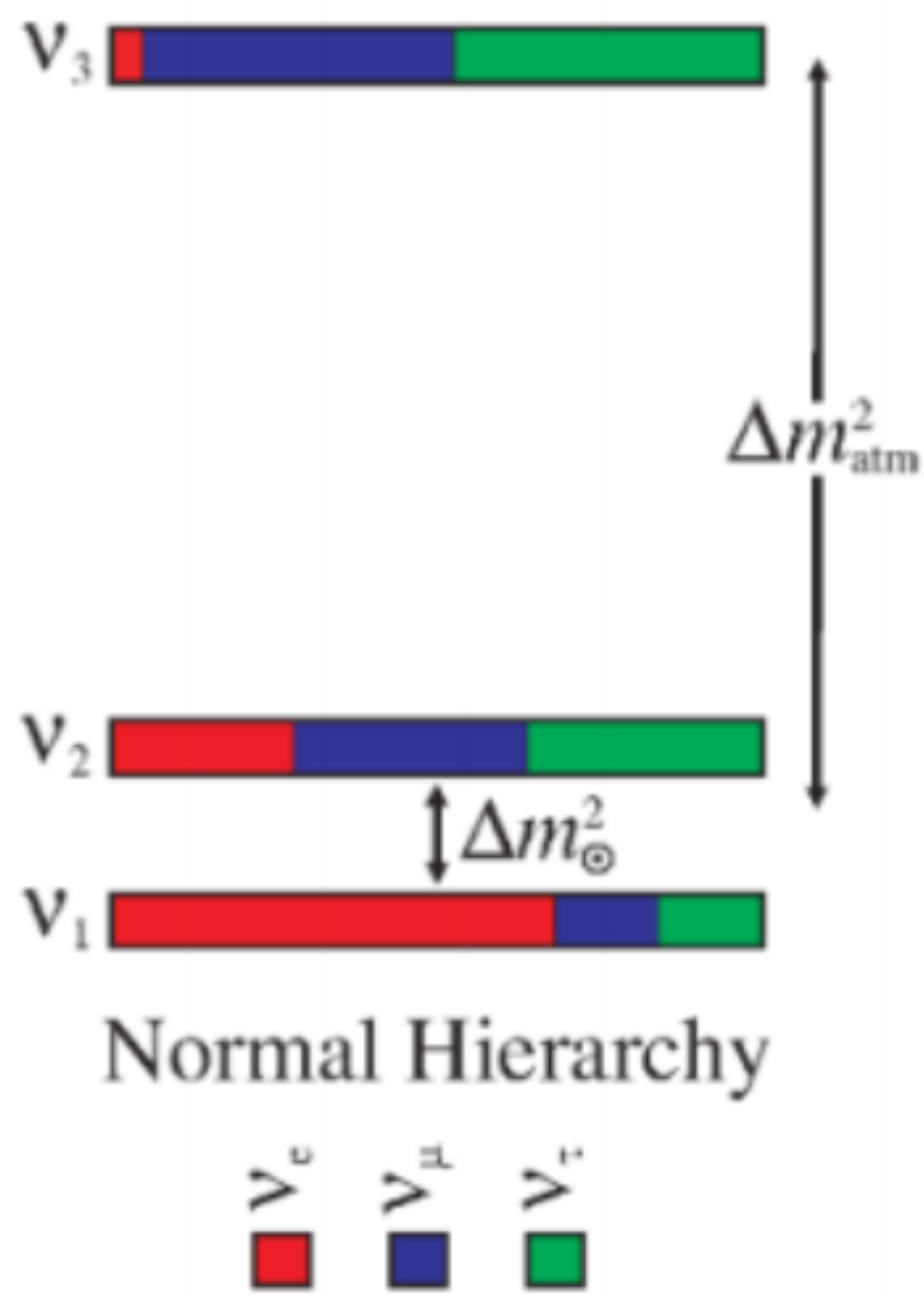
Leading including matter effect

$$-\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}\sin\delta\sin^2\left(\frac{\Delta m_{32}^2 L}{4E_\nu}\right)\sin\left(\frac{\Delta m_{21}^2 L}{4E_\nu}\right)$$

CP violating

replace δ by $-\delta$ and a by $-a$ for $P(\overline{\nu_\mu} \to \overline{\nu_e})$

$$a \equiv 2\sqrt{2}G_F n_e E = 7.56\times 10^{-5}\,\mathrm{eV}^2\,\frac{\rho}{\mathrm{gcm}^{-3}}\,\frac{E}{GeV}$$


ν3
ν2
ν1
Δm²atm
Δm²⊙
Normal Hierarchy
νe
νμ
ντ