Non-Zero Reactor Mixing Angle and its Implications for Neutrino Physics

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PPD seminar
Rutherford Appleton Laboratory, 27 March 2013

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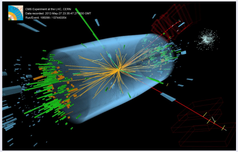
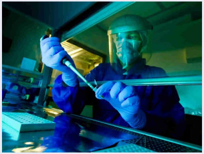
Science Magazine's Breakthrough of 2012 and Ten Runners-up

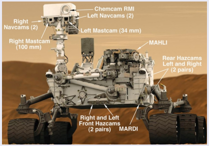
The Higgs Discovery



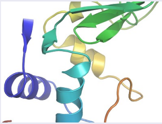
Denisovan Genome



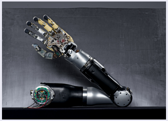
Curiosity Landing



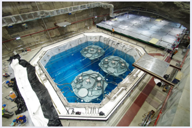
X-ray Laser Advances



Controlling Bionics



Neutrino Mixing angle



Reactor Neutrino Oscillation

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- Electron antineutrinos from reactor core
- A few kilometres

$$\bar{\nu}_e \rightarrow \bar{\nu}_\mu, \bar{\nu}_\tau$$

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- Daya Bay, RENO, Double Chooz, and long baseline beam experiments T2K, MINOS.
- Reactor oscillation - Mixing angle θ_{13} and neutrino mass-squared difference Δm^2_{13}
- Global fit

$$\sin^2 \theta_{13} = 0.0181 - 0.0327$$

Neutrino Oscillation

Neutrino Oscillation

- Flavour Eigenstate $\neq$ Mass Eigenstate
- Production - Flavour Eigenstates
  Propagation - Mass Eigenstates

Neutrino Oscillation

- Flavour Eigenstate $\neq$ Mass Eigenstate
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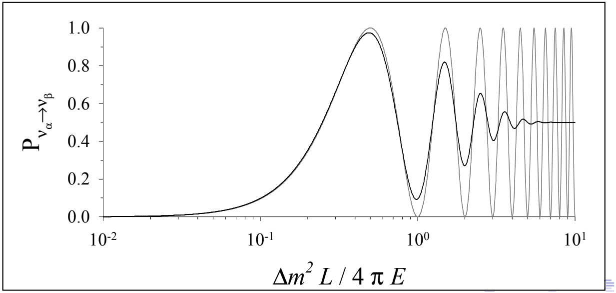
  Propagation - Mass Eigenstates
- Transition Probability

$$P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\theta \cos\left(\frac{\Delta m^2 L}{4E}\right) \quad (\alpha \neq \beta)$$



A little history

- Solar Neutrino Problem

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    - Standard Solar Model - neutrino flux
    - Neutrino deficit - MSW effect

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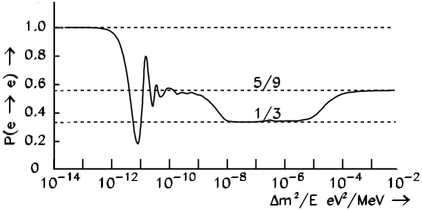
  - $\nu_1 = \nu_e \cos\theta + \nu_\mu \sin\theta, \quad \nu_2 = -\nu_e \sin\theta + \nu_\mu \cos\theta$



  - $\sin\theta = \frac{1}{\sqrt{3}} \qquad \Delta m^2 = 75 \mathrm{meV}^2$

A little history

- Atmospheric Neutrino Anomaly

A little history

- Atmospheric Neutrino Anomaly
  - Cosmic rays and muon flux
  - The double ratio

$$R = \frac{R^{data}_{\nu_\mu/\nu_e}}{R^{MC}_{\nu_\mu/\nu_e}} < 1$$

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  - $\sin\theta = \frac{1}{\sqrt{2}} \qquad \Delta m^2 = 2500 \text{meV}^2$
  - $\nu_\mu \to \nu_\tau$
- Reactor neutrinos and Beam neutrinos
  - No evidence for $\nu_e \to \nu_\mu$ compatible with Δm^2_{atm}.
  - New results - $\overline{\nu}_e$ disappearance - θ_{13} oscillations

The Standard Model Lagrangian - mass terms

- Charged leptons
  - $y_e \overline{L_e} H e_R + y_\mu \overline{L_\mu} H \mu_R + y_\tau \overline{L_\tau} H \tau_R$

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- How to add mass? Add a right-handed field ν_R.
- Dirac term
    - $y_w \overline{L} \tilde{H} \nu_R$
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- Gauge invariance - no $\overline{\nu_L^c} \nu_L$ term

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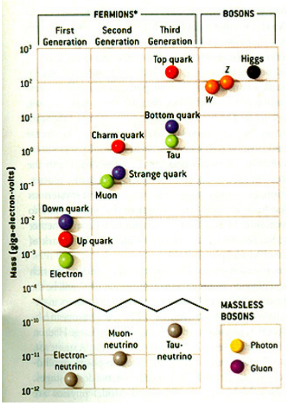

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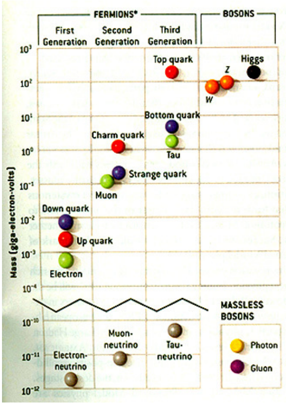

- The Seesaw mechanism
  - Dirac and Majorana mass terms

$$\begin{pmatrix}\overline{\nu_L} \\ \overline{\nu_R^c}\end{pmatrix}^T \begin{pmatrix}0 & m_w \\ m_w & m_{\mathrm{Maj}}\end{pmatrix}\begin{pmatrix}\nu_L^c \\ \nu_R\end{pmatrix}$$

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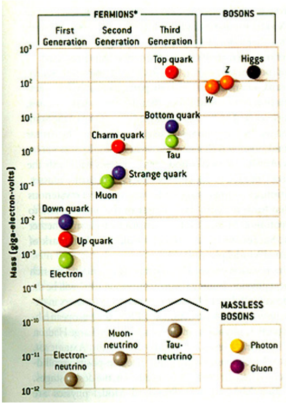

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  - Diagonalise
    Effective Seesaw mass matrix

$$\begin{pmatrix}\overline{\nu}_{\mathrm{light}}\\ \overline{\nu}_{\mathrm{heavy}}\end{pmatrix}^T \begin{pmatrix}\frac{m_W^2}{m_{\mathrm{Maj}}} & 0\\ 0 & m_{\mathrm{Maj}}\end{pmatrix}\begin{pmatrix}\nu_{\mathrm{light}}\\ \nu_{\mathrm{heavy}}\end{pmatrix}$$

The three families

- Effective Seesaw light neutrino mass

$$\frac{m_W^2}{m_{\mathrm{Maj}}}\overline{\nu}_{\mathrm{light}}\nu_{\mathrm{light}} = m_{\mathrm{ss}}\overline{\nu}_{\mathrm{light}}\nu_{\mathrm{light}}$$

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$$\begin{pmatrix} \overline{\nu_L} \\ \overline{\nu_R^c} \end{pmatrix}^T \begin{pmatrix} 0 & M_w \\ M_w & M_{\mathrm{Maj}} \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}$$

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$$\begin{pmatrix} \overline{\nu_L} \\ \overline{\nu_R^c} \end{pmatrix}^T \begin{pmatrix} 0 & M_w \\ M_w & M_{\mathrm{Maj}} \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}$$

- Complex symmetric effective seesaw mass matrix

$$M_w(M_{Maj})^{-1}(M_w)^T\overline{\nu}_{\mathrm{light}}\nu_{\mathrm{light}} = M_{\mathrm{ss}}\overline{\nu}_{\mathrm{light}}\nu_{\mathrm{light}}$$

The three families

- M_{ss} diagonalised using unitary matrix U
  - $U^\dagger M_{ss} U^* = diag(m_1, m_2, m_3)$

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  - $\nu_\alpha = U_{\alpha i}\nu_i$
- Parameterisation of the PMNS mixing matrix, U.

$$U_{MNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

  - δ - CP violation in oscillations
  - $\alpha_1,\ \alpha_2$ - Majorana phases - Neutrinoless Double β Decay

Observed values

- $\sin^2\theta_{12} = 0.273 - 0.354$
  $\sin^2\theta_{23} = 0.341 - 0.670$
  $\sin^2\theta_{13} = 0.0181 - 0.0327$
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- A first approximation - Tribimaximal mixing

$$|U_{\mathrm{TBM}}| \equiv \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

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- Mixing angles

$$\sin^2\theta_{12} = \frac{1}{3}, \quad \sin^2\theta_{23} = \frac{1}{2}, \quad \theta_{13} = 0$$

Discrete symmetries

Defined by 3 Symmetries:

$$U_{\text{TBM}} = \begin{array}{c} e \\ \mu \\ \tau \end{array} \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

CP Symmetry

$\mu - \tau$ *Symmetry*

Democracy

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CP Symmetry

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Democracy

Symmetries reflected in neutrino mass matrix in the flavour basis:

$$M_{ss} = \begin{array}{c|ccc} & \nu_e & \nu_\mu & \nu_\tau \\ \hline \nu_e & c+a+b & b & b \\ \nu_\mu & b & c+b & a+b \\ \nu_\tau & b & a+b & c+b \end{array}$$

CP Symm. ⇒ parameters Real; $\mu - \tau$ Symm. is Manifest;

Democracy ⇒ rows and cols sum to same value - Magic matrix.

Triχmaximal Mixing: Relax *CP* Symmetry

Retains two symmetries:

$$U_{MNS} = \begin{pmatrix} \sqrt{\frac{2}{3}}\cos\chi & \frac{1}{\sqrt{3}} & i\sqrt{\frac{2}{3}}\sin\chi \\ -\frac{\cos\chi}{\sqrt{6}} + i\frac{\sin\chi}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{\cos\chi}{\sqrt{2}} - i\frac{\sin\chi}{\sqrt{6}} \\ -\frac{\cos\chi}{\sqrt{6}} - i\frac{\sin\chi}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{\cos\chi}{\sqrt{2}} - i\frac{\sin\chi}{\sqrt{6}} \end{pmatrix}$$

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CP Violated

$\mu - \tau$ *Symmetry*

Democracy

Allows b param. to become complex $(b + id) \Rightarrow$ 3 masses and $\theta_{13} \neq 0$ arbitrary.

$$M_{ss} = \begin{array}{c|ccc} & \nu_e & \nu_\mu & \nu_\tau \\ \hline \nu_e & c + a + b & b + id & b - id \\ \nu_\mu & b + id & a + b - id & c + b \\ \nu_\tau & b - id & c + b & a + b + id \end{array}$$

$\mu - \tau$ Symm. still Manifest (need to take c. conjugate);

Democracy $\Rightarrow$ rows and cols still sum to same value.

Triχmaximal Mixing: Relax *CP* Symmetry

- TχM

$$U_{MNS} = \begin{pmatrix} \sqrt{\frac{2}{3}}\cos\chi & \frac{1}{\sqrt{3}} & i\sqrt{\frac{2}{3}}\sin\chi \\ -\frac{\cos\chi}{\sqrt{6}} + i\frac{\sin\chi}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{\cos\chi}{\sqrt{2}} - i\frac{\sin\chi}{\sqrt{6}} \\ -\frac{\cos\chi}{\sqrt{6}} - i\frac{\sin\chi}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{\cos\chi}{\sqrt{2}} - i\frac{\sin\chi}{\sqrt{6}} \end{pmatrix}$$

- Mixing angles
  - $|U^{\dagger}_{e3}|^2 = \sin^2\theta_{13} = \frac{2}{3}\sin^2\chi$
  - $|U^{\dagger}_{e2}|^2 = \sin^2\theta_{12}\cos^2\theta_{13} = \frac{1}{3}$
  - $|U^{\dagger}_{\mu 3}|^2 = \sin^2\theta_{23}\cos^2\theta_{13} = \frac{\sin^2\chi}{6} + \frac{\cos^2\chi}{2} \implies \sin^2\theta_{23} = \frac{1}{2}$
  - $\delta_{CP} = \pm\frac{\pi}{2}$
- Non-zero θ_{13} - opened the door for leptonic CP violation
  - TχM - $\delta_{CP} = \pm\frac{\pi}{2}$ - Maximal CP violation

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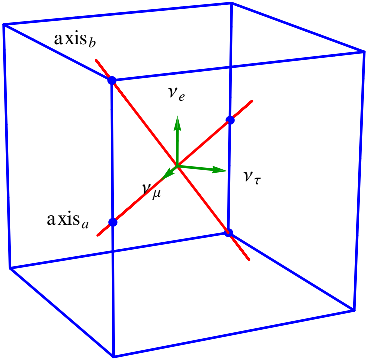
  - The pattern of VEVs - the structure of mass matrix

An example: S_4 discrete group

- Rotational symmetries of a cube



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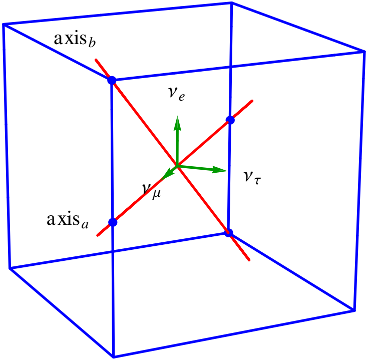
- 3D space - ν_e, ν_μ, ν_τ - **3′** a representation of S_4

Note the of alignment of ν_e, ν_μ, ν_τ



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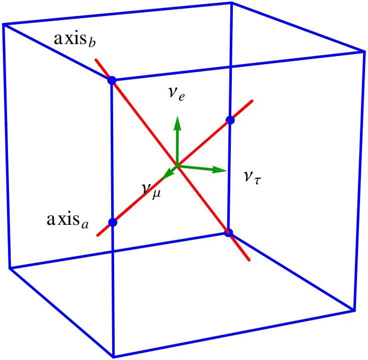
- Irreps of S_4 - **1**, **2**, **3**, **3′**



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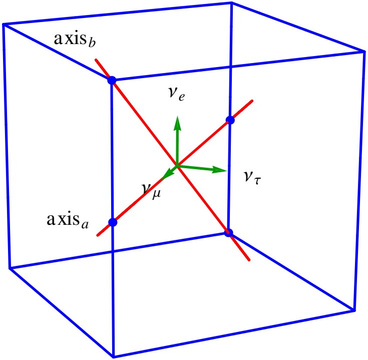
- Mass term - $\overline{\nu_R^c} M_{Maj} \nu_R$ - couplings among two triplets, ν_R



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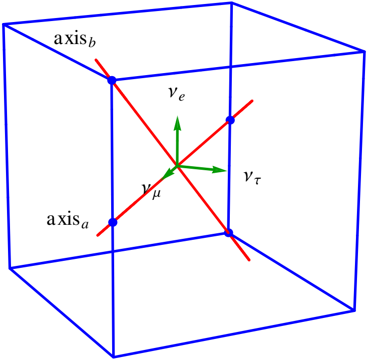
- Tensor product of two triplets



$$\mathbf{3'} \otimes \mathbf{3'} = \mathbf{1} + \mathbf{2} + \mathbf{3} + \mathbf{3'}$$

An example: S_4 discrete group

- Using $\mathbf{3'} = (\nu_e, \nu_\mu, \nu_\tau)$:

$$\psi_1 = \frac{1}{\sqrt{3}}(\nu_e.\nu_e + \nu_\mu.\nu_\mu + \nu_\tau.\nu_\tau),$$

$$\psi_2 = \begin{pmatrix} -\sqrt{\frac{2}{3}}\nu_e.\nu_e + \frac{1}{\sqrt{6}}\nu_\mu.\nu_\mu + \frac{1}{\sqrt{6}}\nu_\tau.\nu_\tau \\ \frac{1}{\sqrt{2}}(\nu_\mu.\nu_\tau + \nu_\tau.\nu_\mu) \end{pmatrix},$$

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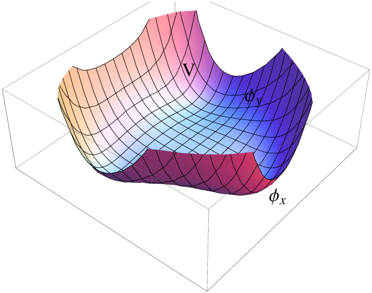
The flavon VEVs

- Flavon potential - like the Higgs potential
- The doublet flavon ϕ_2

$$V(\phi_2) \propto -\phi_x^3 + 3\phi_y^2\phi_x + \left(\phi_x^2 + \phi_y^2\right)^3$$



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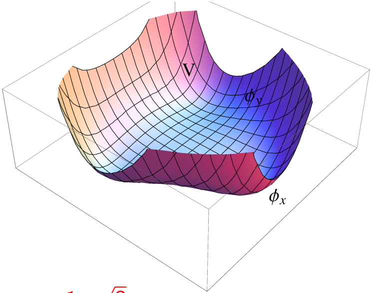

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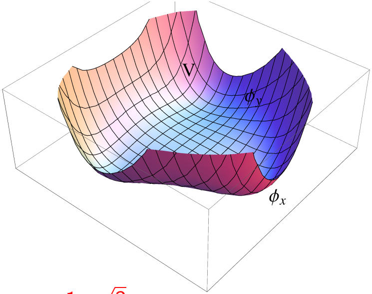

- VEV $\Rightarrow$ $\langle\Phi_2\rangle = (-\frac{1}{2}, \frac{\sqrt{3}}{2})$
- Similarly we obtain $\langle\Phi_1\rangle = 1$, $\langle\Phi_3\rangle = (1, 1, -1)$

The “Simplest” ansatz

- Using the above VEVs

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with $a, c, d = f(c_1, c_2, c_3)$ and $b = 0$

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- $M_{ss} \propto M_{\text{Maj}}^{-1}$ - neutrino masses ∝ inverse of eigenvalues

Neutrino mass prediction

- Neutrino mass squared differences and the non-zero θ_{13} - three parameters of the model
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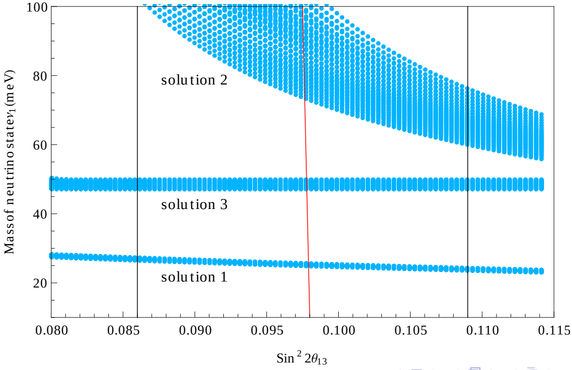
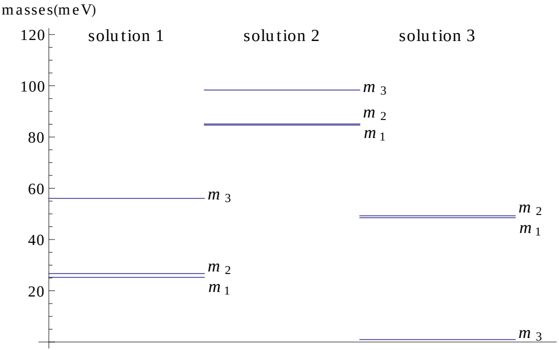
- Predict neutrino mass offset - All the neutrino masses
- The lightest neutrino mass



Neutrino mass prediction

- The neutrino masses - best fit



Summary

- A brief overview of neutrino oscillations
- See-saw mechanism and Majorana neutrinos
- Triχmaximal mixing - non-zero θ_{13}
- Discrete symmetries, eg: S_4 group
- Flavons and Model building
- “Simplest” ansatz and predictions