$g-2$ of the muon: touchstone of the Standard Model, keyhole to New Physics?

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in collab. w. Kaoru Hagiwara, Ruofan Liao, Alan Martin and Daisuke Nomura

I. Testing Quantum Field Theory at the highest precision

II. The Standard Model prediction of $(g-2)_\mu$

III. SM vs. BNL: A sign for New Physics?

IV. The 'running coupling' $\alpha_{\rm QED}(q^2)$ and the indirect determination of the Higgs mass

V. The next round

I. Testing QFT at the highest precision

Definitions:

- g relates the *magnetic moment* $\vec{\mu}$ of a particle to its *spin* $\vec{s}$:

$$\vec{\mu} = g \frac{e}{2m} \vec{s} \qquad (m \text{ the particle's mass})$$

- Dirac equation $\rightsquigarrow$ $g = 2$, but *quantum corrections* lead to the anomalous magnetic moment: $a \equiv (g-2)/2$

I. Testing QFT at the highest precision

What happens in the $(g-2)_\mu$ experiment:

- Muon in an uniform magnetic field $\vec{B}$: circular motion with cyclotron frequency $\vec{\omega}_c = e\vec{B}/m$ and spin precession with $\vec{\omega}_s = ge\vec{B}/2m$.

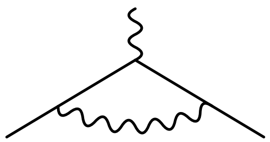
$\longrightarrow$ Direct measurement of the anomalous precession frequency

$$\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c = \frac{e}{m} a_\mu \vec{B}$$

via the time spectrum of the electrons from the muon decays $\mu^- \to \nu_\mu e^- \bar{\nu}_e$, and the $\vec{B}$ field via NMR magnetometers.

I. Testing QFT at the highest precision

- Theory prediction in QED at one loop (Schwinger '48):



$$a \equiv (g-2)/2 = \alpha/(2\pi) \simeq 11\,614\,097 \cdot 10^{-10}$$

- Experimentally: One of the most precisely measured quantities

$$a_\mu^{\rm exp} = 11\,659\,208\,(6) \cdot 10^{-10} \quad [0.5\,\text{ppm}] \quad \text{BNL 2004}$$
$$a_e^{\rm exp} = 11\,596\,521.8073\,(0.0028) \cdot 10^{-10} \quad [0.24\,\text{ppb}] \quad \text{Gabrielse 2008}$$

- $a_e \sim$ 2000 times more precise than a_μ (tests QED, measures α) $\longrightarrow$

  But: Sensitivity to *New Physics* at energy scale $\Lambda_{\rm NP}$ $\quad a^{\rm NP} \propto m^2/\Lambda_{\rm NP}^2$

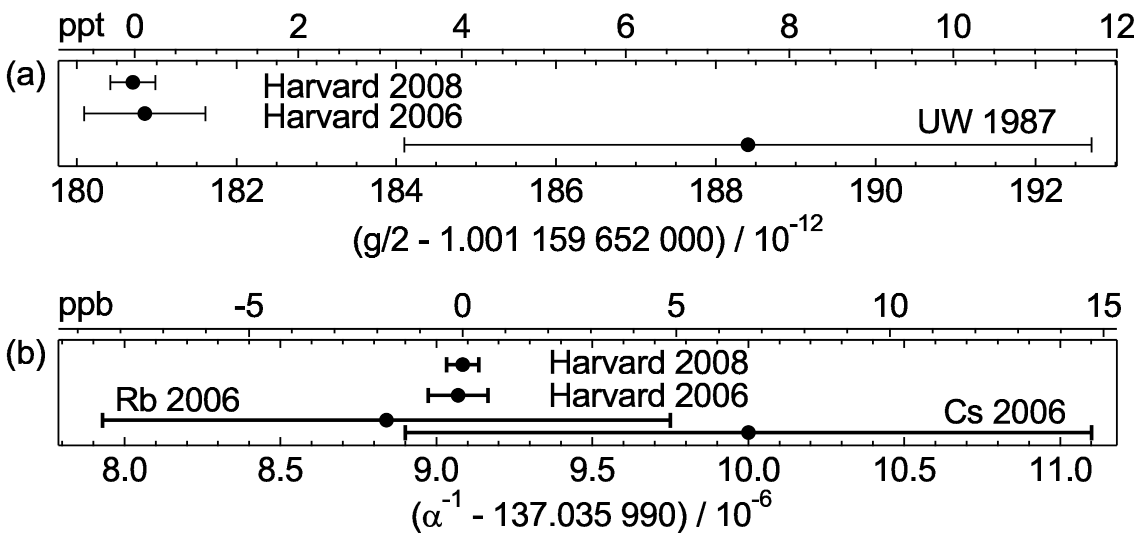
  $\longrightarrow$ muon wins by $m_\mu^2/m_e^2 \sim 43000$ (chirality flip)

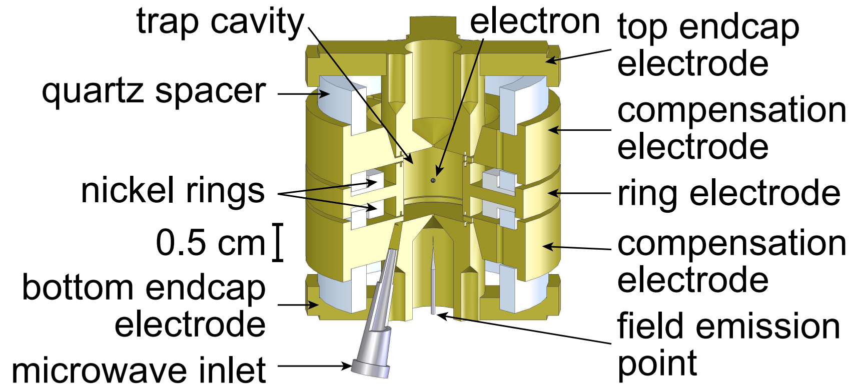
- IF $a_\mu^{\rm EXP} - a_\mu^{\rm SM} \sim 25 \cdot 10^{-10}$ $\rightsquigarrow$ test physics up to scale $\Lambda_{\rm NP} \sim 2$ TeV

  $\hookrightarrow$ establish or constrain *NP*

The electron's anomaly a_e

Hanneke, Fogwell, Gabrielse, PRL 100(2008)120801





- One electron quantum cyclotron, using quantum-jump spectroscopy and quantum non-demolition measurements of cyclotron and spin levels.
- Penning trap cavity inhibits spontaneous emission; one-particle self-excited oscillator.
- 15 times better than measurements from 1987, now 0.28 ppt for $g_e/2$.
- Resulting determination of α is 20 times better than other methods (measuring transition frequencies using atomic interferometers).
- a_τ: SM prediction $a_\tau = 117721(5) \times 10^{-8}$ Eidelman et al. (2007)

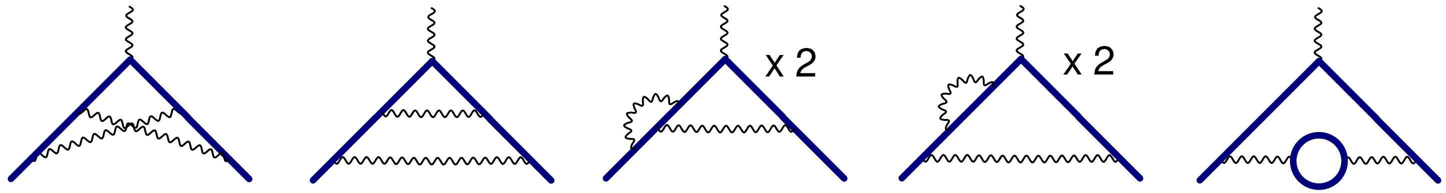
  but too short lived for precession exp.; DELPHI at LEP2: $(e^+e^- \to e^+e^-\tau^+\tau^-) \rightsquigarrow -0.052 < a_\tau < 0.013$

II. The Standard Model prediction of $(g-2)_\mu$

The different contributions in the SM: $a_\mu = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm had}$

✓ QED: Laporta+Remiddi, Kinoshita et al., Passera

* QED 2–loop Feynman graphs for illustration:



* Known to 4–loop (891 diagrams); 5–loop (12672 diagrams) estimates, ongoing effort
* Large compared to EW and hadronic, but very small error (from 4–, 5–loop, α)

→ Recent update from Passera, Kinoshita+Nio, details depend on input α, 5–loop est.

$$a_\mu^{\rm QED} = (116\ 584\ 718.08 \pm 0.15) \cdot 10^{-11}$$

* Further efforts ongoing; no big surprises so far for a_μ, but recent changes in a_e.

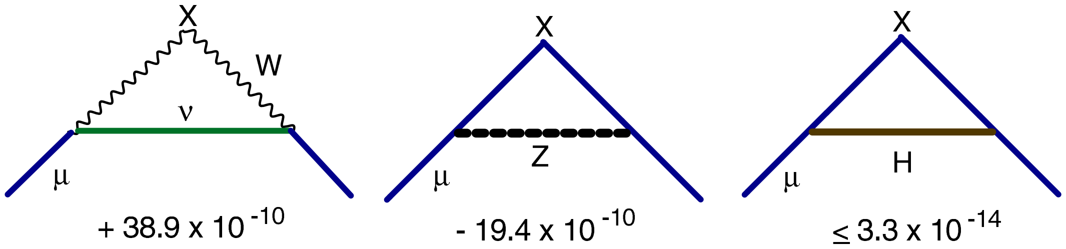
II. The Standard Model prediction of $(g-2)_\mu$

The different contributions in the SM: $a_\mu = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm had}$

✓ EW:

Czarnecki et al., Knecht et al.

* Electro-Weak 1–loop diagrams:



* Known to 2–loop (1650 diagrams; first EW 2–loop)

* Quite small and small error, but 2–loop relevant; results well checked

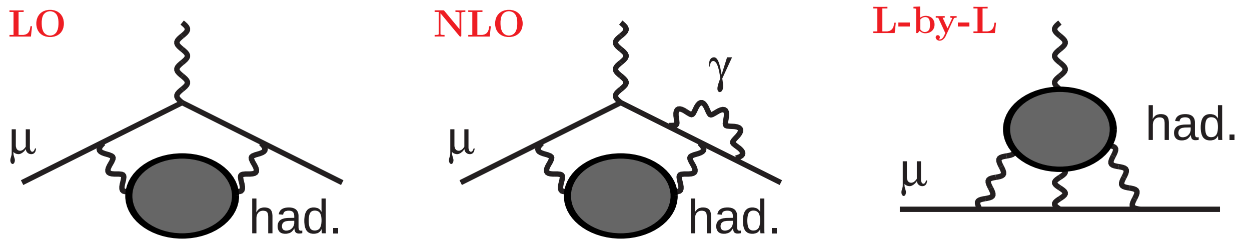
→ Czarnecki+Marciano+Vainshtein:

$$a_\mu^{\rm EW} = (154 \pm 2) \cdot 10^{-11}$$

$$\left[\, a_\mu^{\rm QED} = (116\ 584\ 718.08 \pm 0.15) \cdot 10^{-11} \,\right]$$

II. The Standard Model prediction of $(g-2)_\mu$

► Hadronic: $a_\mu^{\rm had} = a_\mu^{\rm had,LO} + a_\mu^{\rm had,NLO} + a_\mu^{\rm had,Light-by-Light}$



∗ Subleading but sizeable; uncertainty completely dominates the error of $a_\mu^{\rm SM}$

× 'Blob' cannot be calculated in perturbative QCD [pQCD applicable only for large virtualities]

• L-by-L: Model calculations with bumpy history:

$<$ Nov. 2001: $(-85 \pm 25) \cdot 10^{-11}$ with the 'famous' sign error of several groups

$>$ Nov. 2001: $(+80 \pm 40) \cdot 10^{-11}$ Nyffeler [$\rightsquigarrow$ much discussed $2.6\,\sigma \to 1.6\,\sigma$]

$>$ Dec. 2003: $\mathbf{(136 \pm 25) \cdot 10^{-11}}$ Melnikov+Vainshtein, or $\mathbf{(11 \pm 4) \cdot 10^{-10}}$ Bijnens+Prades

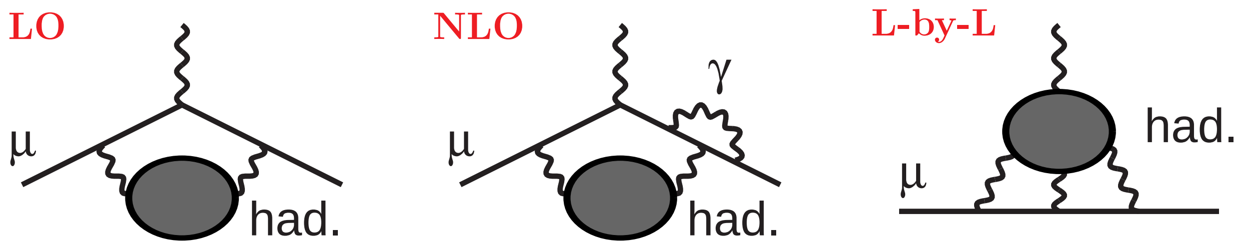
[Conservative estimate based on comparison of different works]

Recently: Convergence of different model calculations:

$\rightsquigarrow$ recent compilation from Prades, de Rafael, Vainshtein: $a_\mu^{\rm L-by-L} = (10.5 \pm 2.6) \cdot 10^{-10}$

II. The Standard Model prediction of $(g-2)_\mu$

▶ **Hadronic:** $a_\mu^{\text{had}} = a_\mu^{\text{had,LO}} + a_\mu^{\text{had,NLO}} + a_\mu^{\text{had,Light-by-Light}}$



- L-by-L: Model calculations with bumpy history:

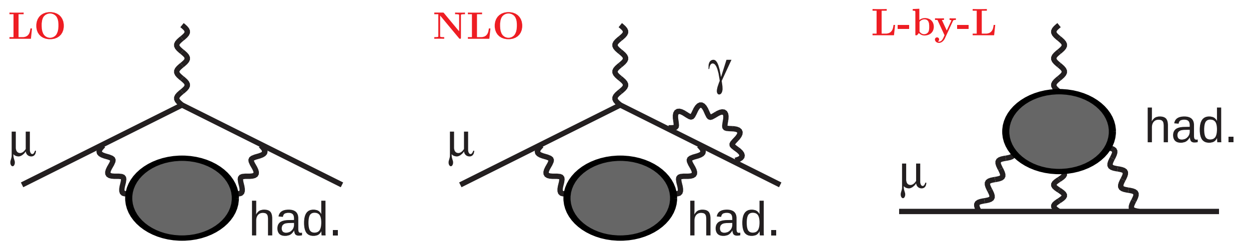
  ⇝ recent compilation from Prades, de Rafael, Vainshtein: $a_\mu^{\text{L-by-L}} = (10.5 \pm 2.6) \cdot 10^{-10}$

⟶ Help from Lattice QCD? ('first principles' numerical simulation, *i.e.* non-perturbative)
Two groups work with different approaches; promising but early stage, very challenging

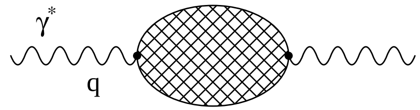
⟶ Situation already better than it seems:

* different groups have agreement in leading N_{colour} parts
* differences in (negative!) subleading contributions ⇝ modelling
* Upper bound can be defended; but more work needed, otherwise limiting factor!

▶ **Hadronic:** $a_\mu^{\rm had} = a_\mu^{\rm had,LO} + a_\mu^{\rm had,NLO} + a_\mu^{\rm had,Light-by-Light}$



- L-by-L: Stay for now with $\boldsymbol{a_\mu^{\rm L-by-L} = (10.5 \pm 2.6) \cdot 10^{-10}}$

▶ LO, NLO: all low energy hadronic resonances contribute through 

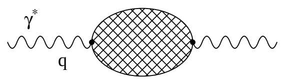
- pQCD not applicable; Lattice QCD results (still) not precise enough; but:
- Vacuum Polarisation contributions from exp. $\boldsymbol{\sigma(e^+e^- \to \gamma^* \to hadrons)}$ data

  (or from $\tau \to \nu_\tau + hadrons$ spectral functions, see below)

  via *dispersion integral* (based on analyticity and unitarity):

- $$a_\mu^{\rm had,LO} = \frac{1}{4\pi^3}\int_{m_\pi^2}^\infty {\rm d}s\, \sigma^0_{\rm had}(s) K(s)\,, \quad \text{with } K(s) = \frac{m_\mu^2}{3s}\cdot(0.63\ldots 1)$$

$\longrightarrow$ Weighting with kernel K towards smallest energies

$\longrightarrow$ σ^0 means without Vacuum Polarisation  , i.e. w/out running $\alpha_{\rm QED}(q^2)$

→ Data input for $\sigma^0_{\rm had}(s)$ from the experiment CMD-2 at Novosibirsk:

(Still the most precise $e^+e^- \to \pi^+\pi^-$ data with only 0.6% systematic error)

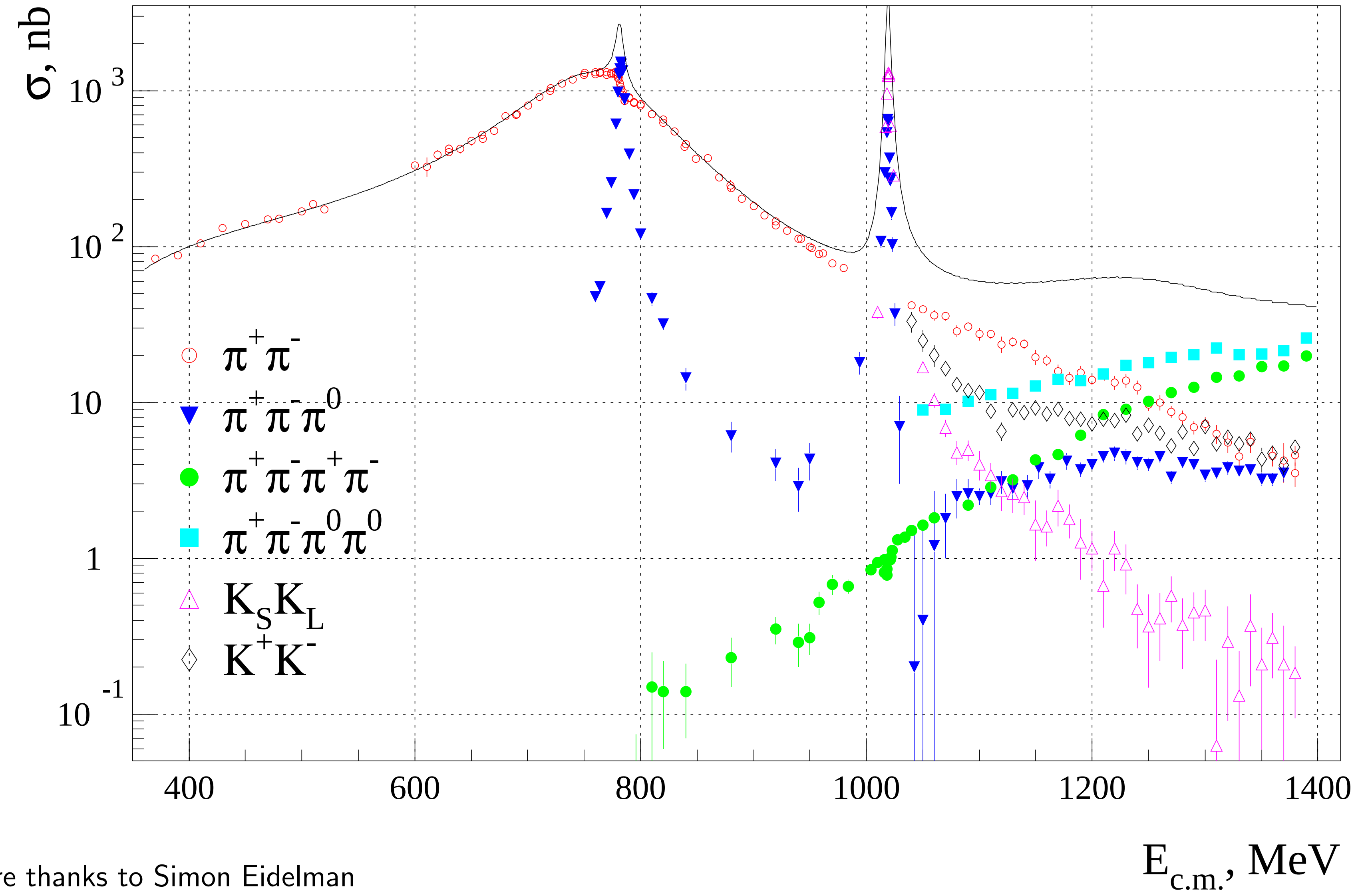


Figure thanks to Simon Eidelman

How to get the hadronic vac.-pol. contributions with precision: HLMNT

- Lowest energies most important, i.e. the hadronic channels 2π, 3π, KK, 4π, 5π, etc. Have to sum ~ 24 exclusive channels and inclusive data for $\sqrt{s}$ above $1.43 - 2$ GeV to get total $\sigma_{\rm had}$ with high precision.

  $\longrightarrow$ Use of *state-of-the-art* perturbative QCD only above ~ 11 GeV.

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- In each channel: Combine data from many different experiments:
  $\longrightarrow$ nontrivial w.r.t. error analysis / correlations / different energy ranges:
  use e.g. non-linear $\chi^2_{\rm min}$ fit (and do error analysis with covariance matrix)

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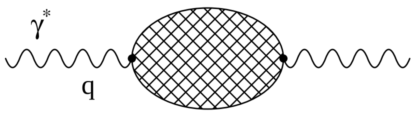
- In each channel: Combine data from many different experiments: use e.g. non-linear $\chi^2_{\rm min}$ fit (and do error analysis with covariance matrix)
- Before averaging and $\sum$: Check Radiative Corrections of each data set:

$\longrightarrow$ Additional final state photons must be fully *included/estimated*

$\longrightarrow$ For σ^0, running $\alpha(q^2)$ effects must be *subtracted*

(otherwise double-counting with $a_\mu^{\rm had, NLO}$)



$\longrightarrow$ but effects can cancel in $\sigma_{\rm had}/\sigma_{\rm norm}$, and corrections often done already partly... MANY COMPLICATIONS

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- Lowest energies most important, i.e. the hadronic channels 2π, 3π, KK, 4π, 5π, etc. Have to sum ~ 24 exclusive channels and inclusive data for $\sqrt{s}$ above $1.43 - 2$ GeV to get total σ_{had} with high precision.
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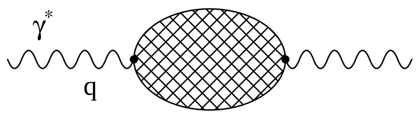


$\longrightarrow$ but effects can cancel in $\sigma_{\mathrm{had}}/\sigma_{\mathrm{norm}}$, and corrections often done already partly.

$\longrightarrow$ PRECISION ONLY FROM TH + MonteCarlo + EXP

$\longrightarrow$ Important detail: Use of running $\alpha(q^2)$ for time-like $q^2 = s$:

$$\alpha(s) \;=\; \alpha \,/\, \left(1 - \Delta\alpha_{\rm lep}(s) - \Delta\alpha^{(5)}_{\rm had}(s) - \Delta\alpha^{\rm top}(s)\right)$$

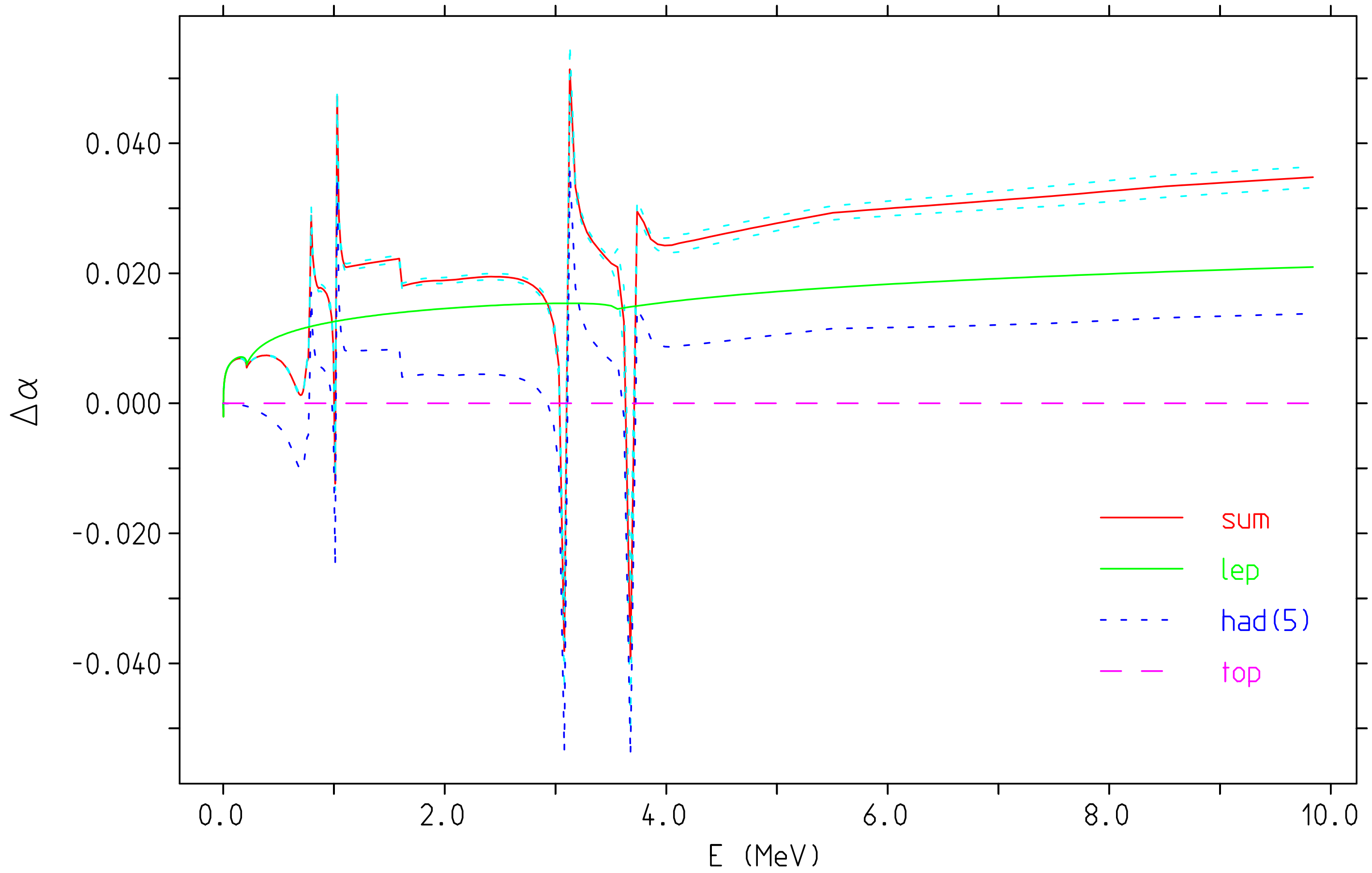
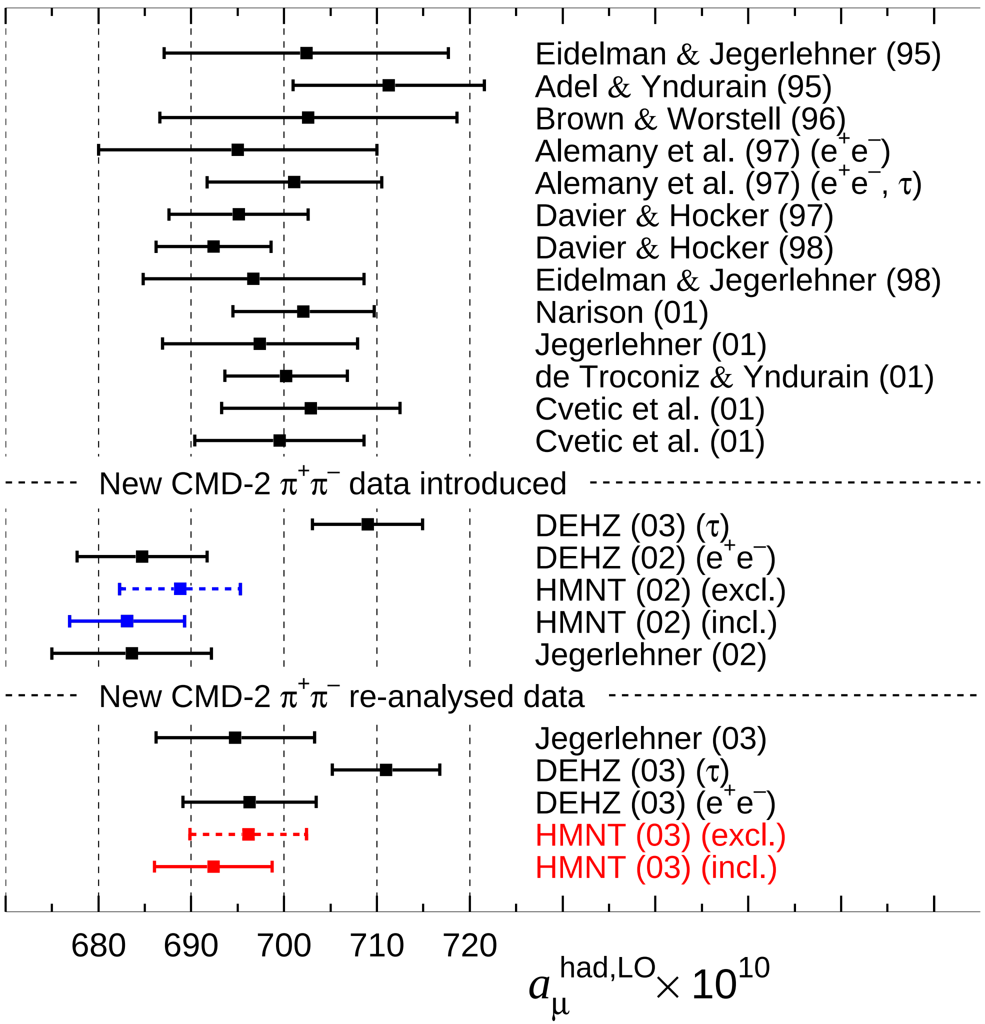


Figure from Fred Jegerlehner

$\longrightarrow$ In total these radiative corrections lead to an additional uncertainty of $\delta a_\mu^{\rm had, VP+FSR} \simeq 1.8 \times 10^{-10}$ $[\sim\; 10 \cdot \Delta a_\mu^{\rm EW}\;]$ (HLMNT analysis)

A brief history of different evaluations of $a_\mu^{\text{had, LO}}$:



★ since then the dominant CMD-2 data have been 'confirmed' by other measurements

★ the compilations of σ_{had} and hence a_μ^{had} have become even more precise since then

► Most important channels with changes in input data since $\sim$2006

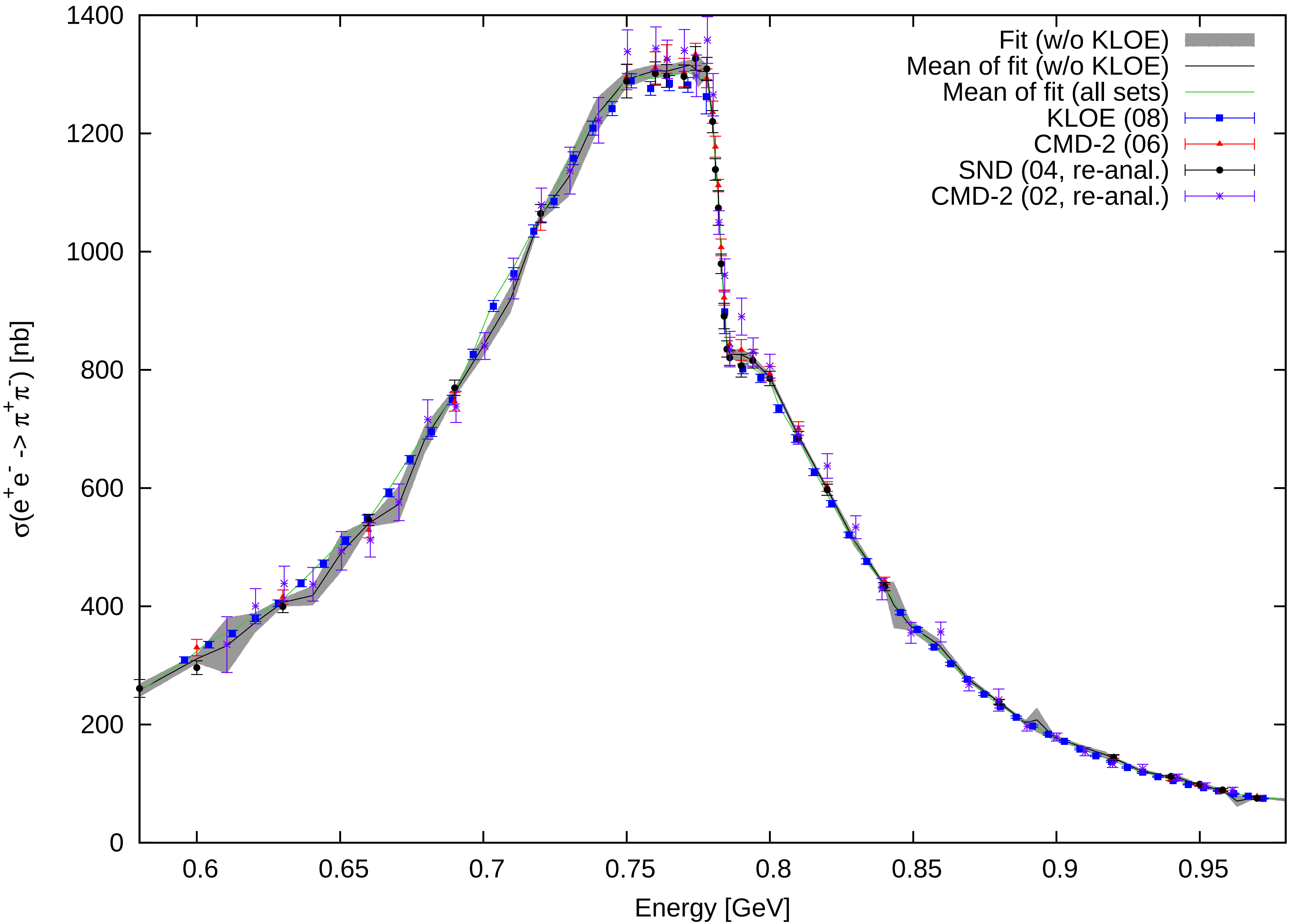
The main players for 'low' energy hadronic cross sections in e^+e^-:

- CMD-2, [VEPP-2M], Novosibirsk (K^+K^-, $2\pi^+2\pi^-\pi^0$, $2\pi^+2\pi^-2\pi^0$)
- SND, [VEPP-2M], Novosibirsk (K^+K^-, $K^0_SK^0_L$)
- KLOE, [DAΦNE], Frascati ($\pi^+\pi^-(\gamma)$, $\omega\pi^0$)
- BaBar, [PEP-II], SLAC, Stanford ($K^+K^-\pi^0$, $K^0_S\pi K$, $2\pi^+2\pi^-\pi^0$, $K^+K^-\pi^+\pi^-\pi^0$, $2\pi^+2\pi^-\eta$, $2\pi^+2\pi^-2\pi^0$)
- BELLE, [KEKB], KEK, Tsukuba
- BES, [BEPC], Beijing (inclusive $R = \sigma(e^+e^- \to hadrons)/\sigma(e^+e^- \to \mu^+\mu^-)$ data)
- CLEO, [CESR], Cornell (inclusive R)

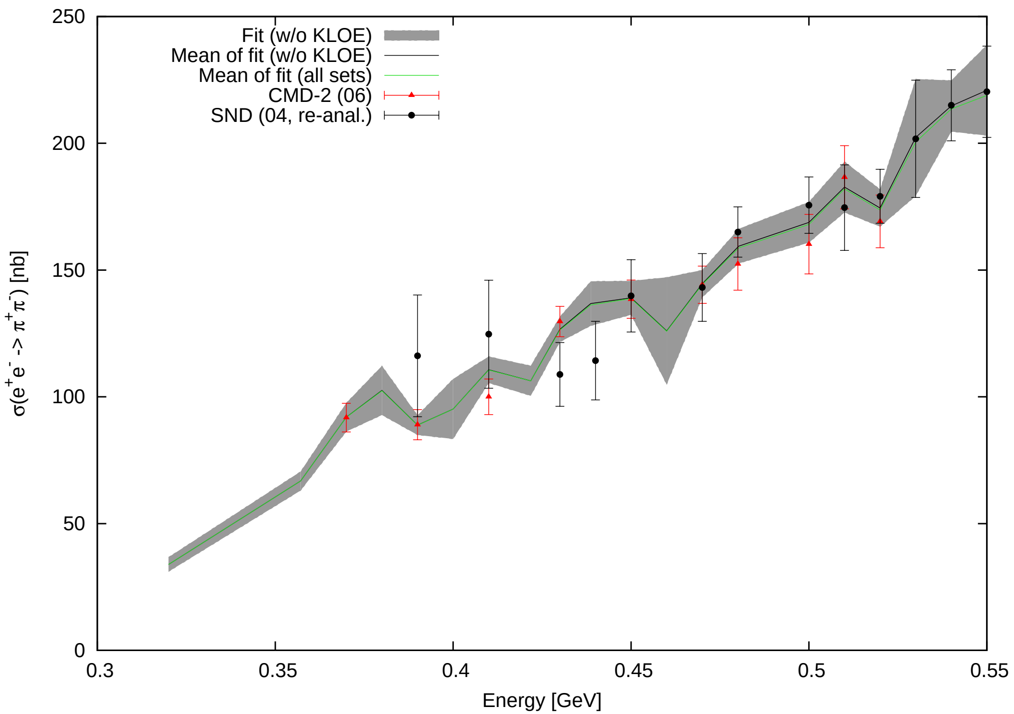
- In principle inclusion of new data in updated analysis straightforward..

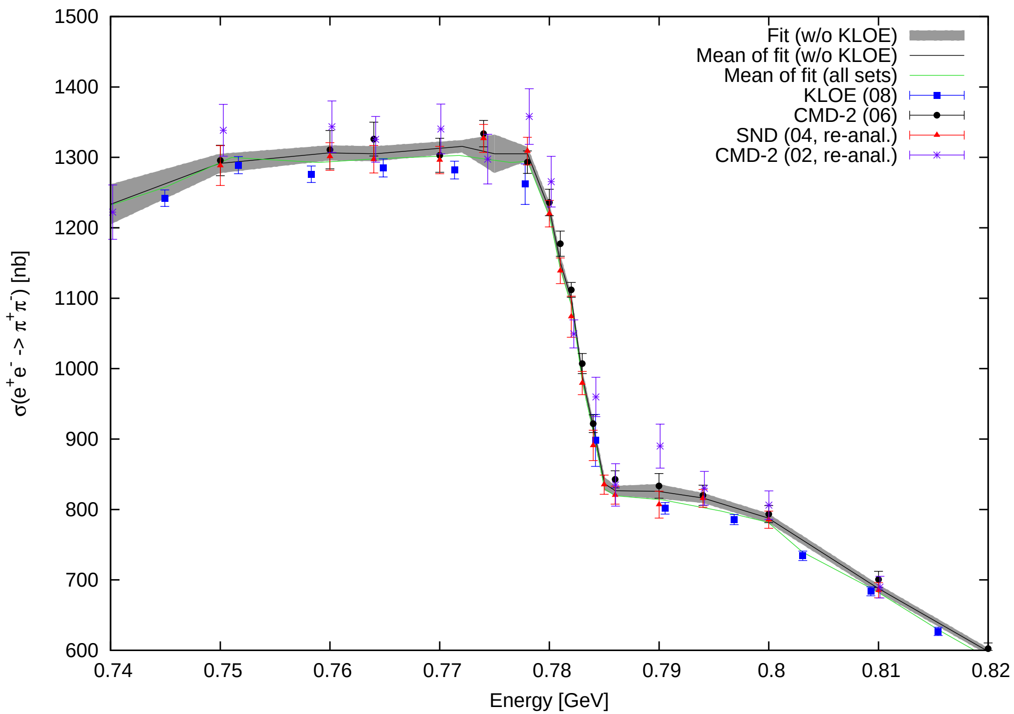
  Concentrate on two cases where not: most important $\mathbf{2\pi}$ and the $\mathbf{1.43 - 2}$ GeV region.

▶ The most important $e^+e^- \longrightarrow \rho \longrightarrow \pi^+\pi^-$ channel: Overall picture very good



Zoom in low energy and peak and $\rho - \omega$ interference region



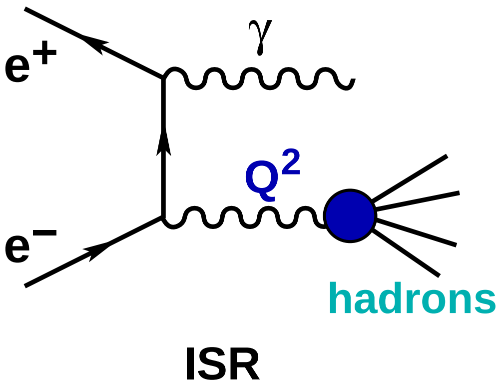


- Very good agreement between data from CMD-2 and SND, fully consistent with earlier data.
- Low energy points crucial for recent improvement of $a_\mu^{\pi\pi}$.
- $g-2$ integral over KLOE data agrees extremely well with the corresponding integral over all other sets:
  KLOE alone: $a_\mu^{\pi\pi} = (384.16 \pm 3.47) \cdot 10^{-10}$, all data without KLOE: $(384.12 \pm 2.51) \cdot 10^{-10}$.

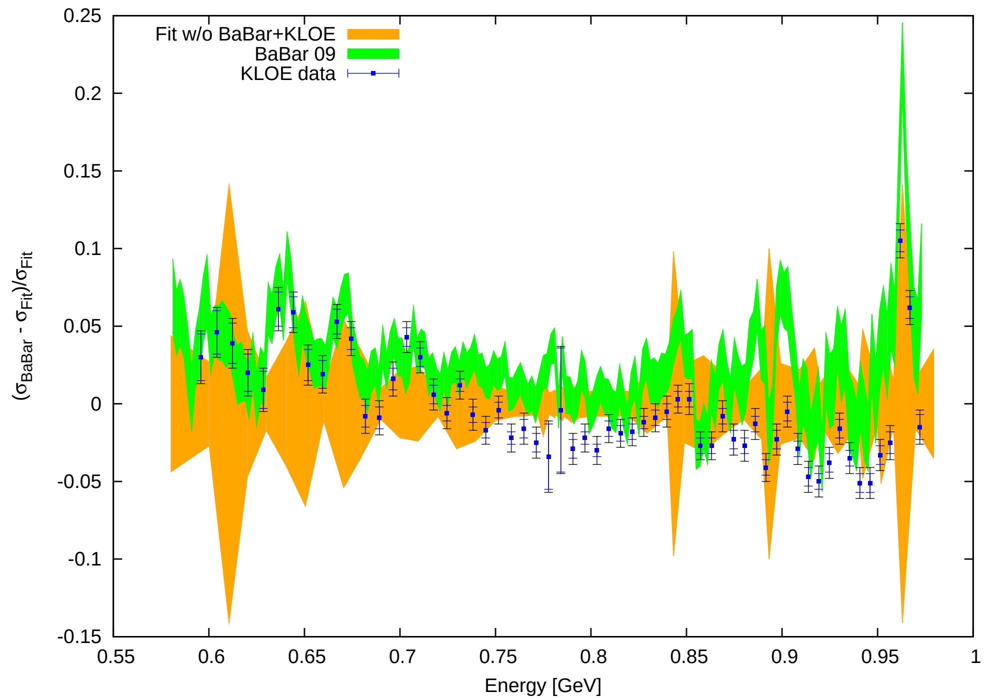
$\longrightarrow$ *However:* some differences in shape prevent good point-by-point combination:

KLOE 08 and BaBar 09 $\pi\pi(\gamma)$ Radiative Return result compared to comb. of all other:

Radiative Return (at fixed e^+e^- energy) has recently developed (TH + EXP) into a powerful method with great potential, complementary to direct energy scan!



Normalised difference of cross sections



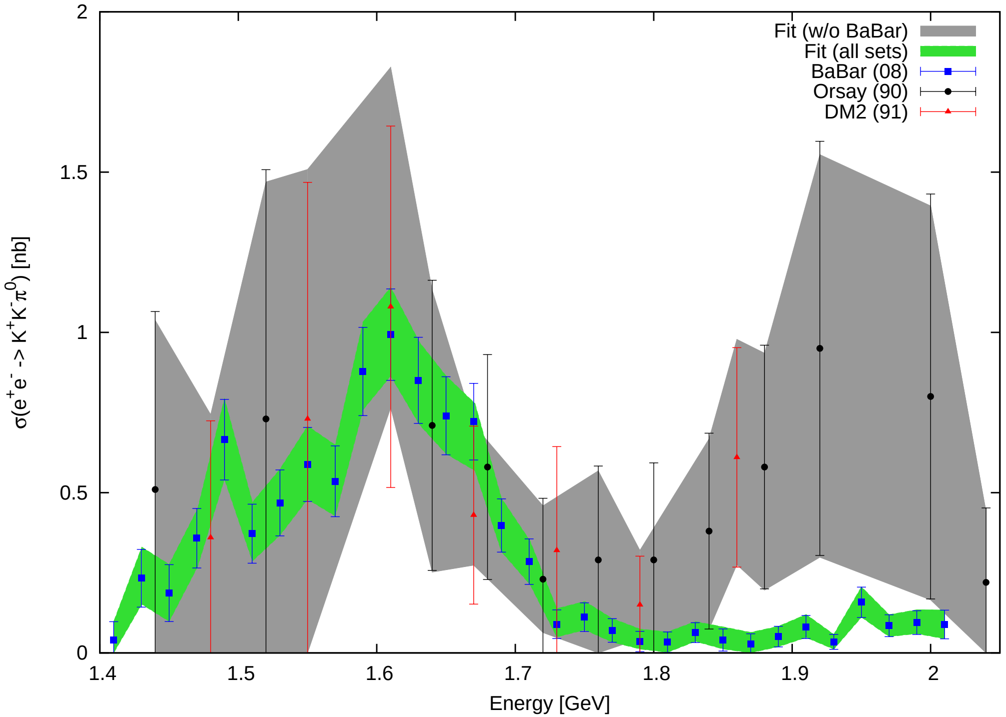
- New method used by 'meson factories', where high statistics compensates α/π suppression of γ radiation.
- Results for 2π channel slightly different in shape, but completely different method, Monte Carlos etc.

⇝ HLMNT 09: Combination with KLOE data after integration: (BaBar not yet in as only available recently)

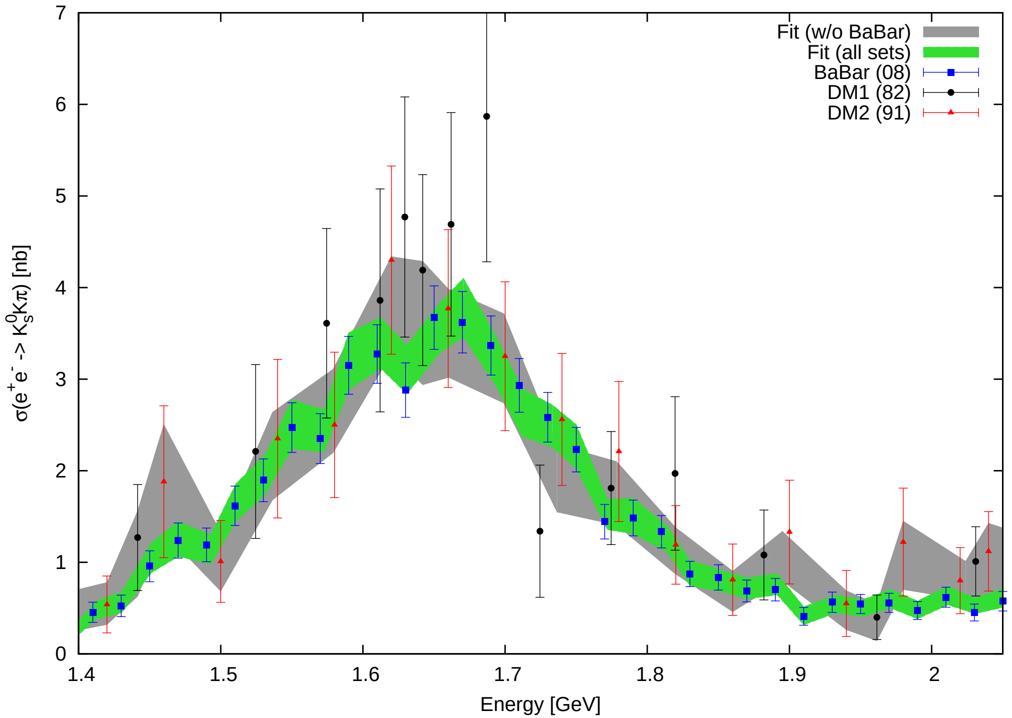
$$a_\mu^{\pi\pi}(0.596\,\mathrm{GeV} < \sqrt{s} < 0.972\,\mathrm{GeV}) = (384.13 \pm 2.03) \cdot 10^{-10}$$

▶ Region below 2 GeV: influence of recent BaBar Radiative Return analyses

$K^+K^-\pi^0$ channel



$K^0_S K\pi$ channel

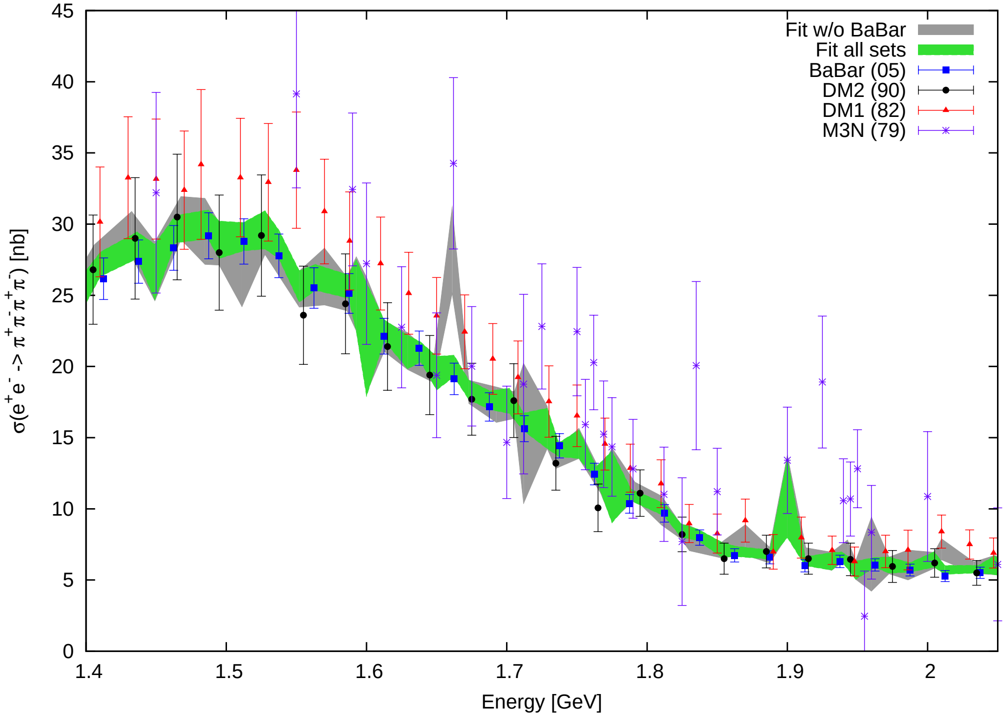


$\longrightarrow$ Important improvements over earlier data compilations.

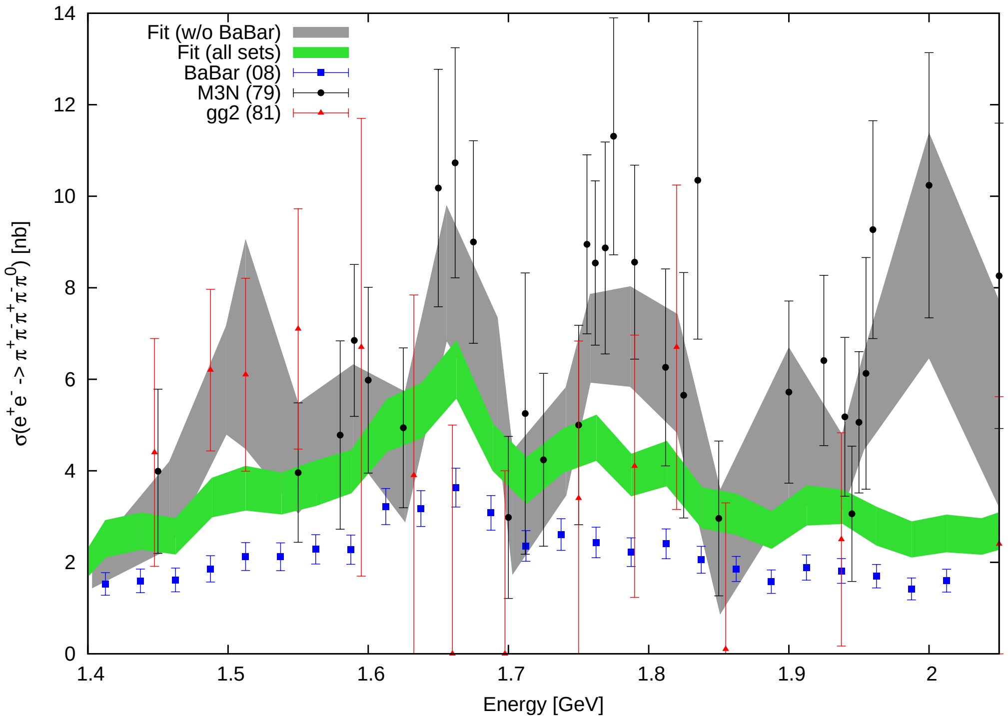
BaBar Radiative Return data lower than less precise older data in most channels.

Region below 2 GeV: influence of recent BaBar Radiative Return analyses (contd)

$2\pi^+2\pi^-$ channel



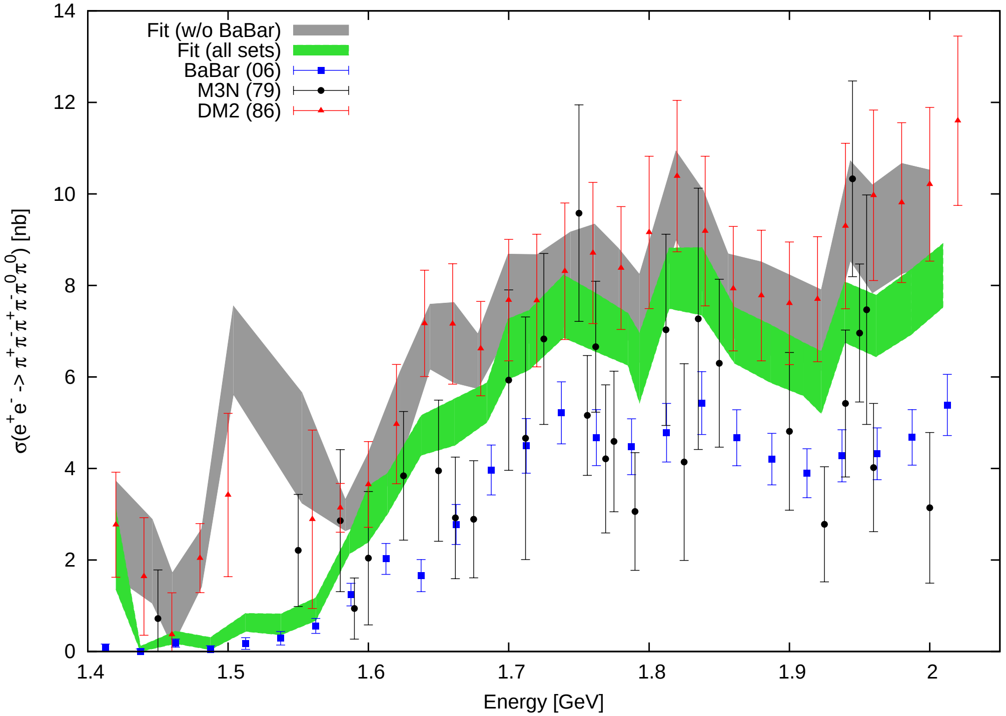
$2\pi^+2\pi^-\pi^0$ channel



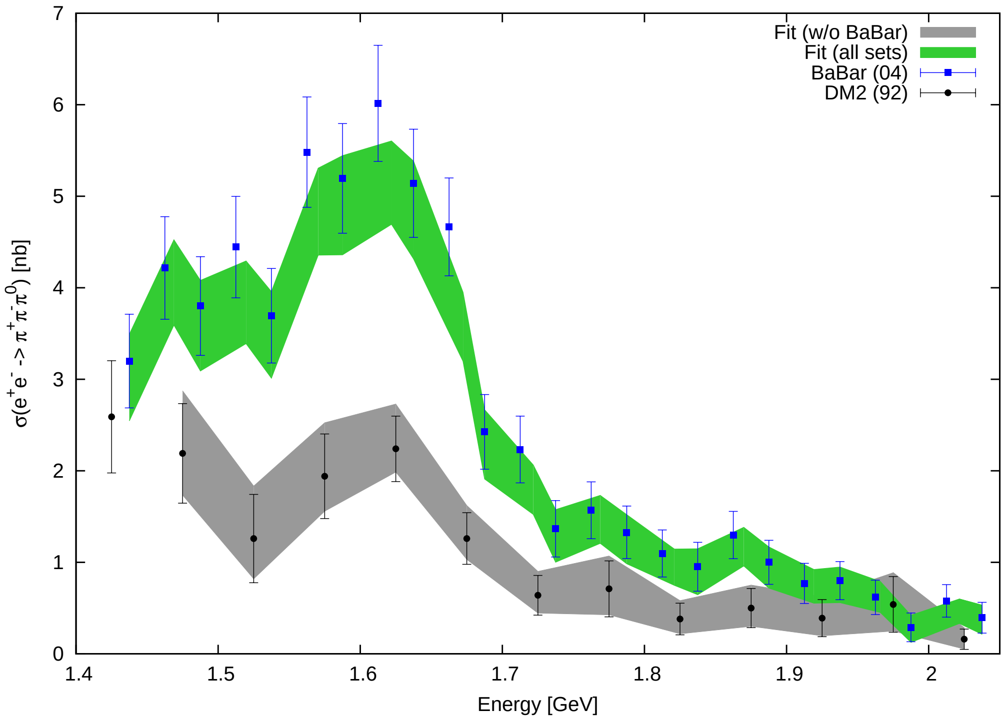
⟶ BaBar lower in $2\pi^+2\pi^-\pi^0$ channel $\rightsquigarrow$ errors for $g-2$ scaled up by $\sqrt{\chi^2_{\min}/\mathrm{dof}} = 1.29$.

▶ Region below 2 GeV: influence of recent BaBar Radiative Return analyses (contd 2)

$\boldsymbol{2\pi^+2\pi^-2\pi^0}$ channel

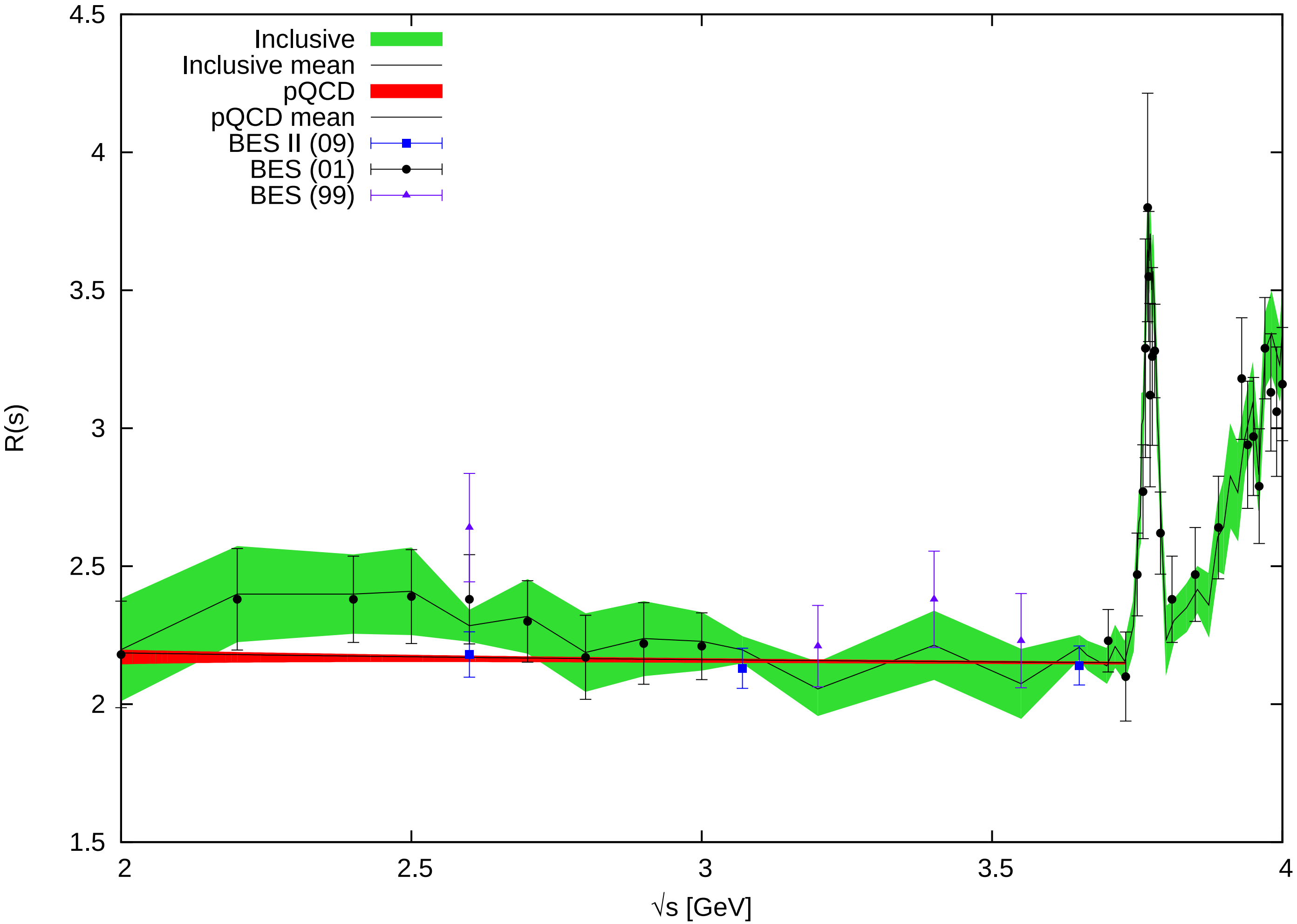


$\boldsymbol{\pi^+\pi^-\pi^0}$ channel



$\longrightarrow$ Again 'bad' $\chi^2_{\min}/\mathrm{dof}$ of 2.7 and 2.9. Data not really compatible, inflate error.

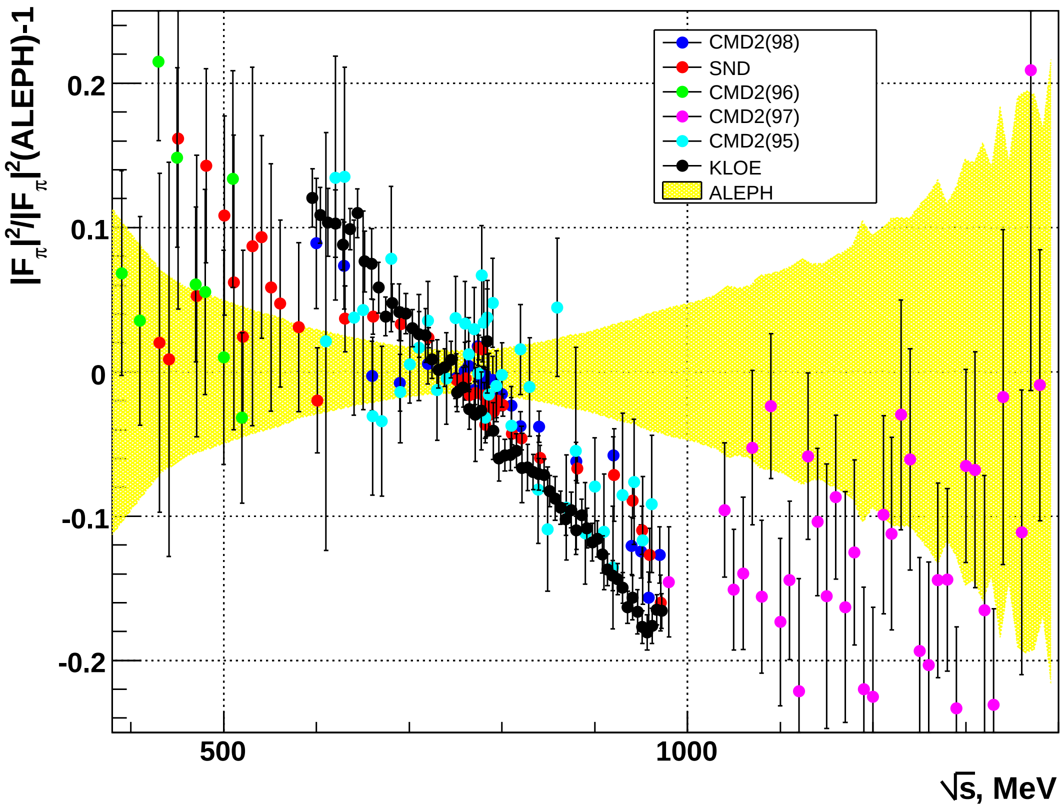
Perturbative QCD vs. inclusive data above 2 GeV (below charm threshold)



- R_{uds} from pQCD mostly below data in region above 2 GeV
- Latest BES data agree very well with pQCD
- shift downwards relevant for $g-2$ and $\Delta\alpha$

- # What about the τ data?
  - CVC hypothesis (Isospin-symm.) connects $\tau^- \to \pi^-\pi^0\nu_\tau$ to $e^+e^- \to \rho,\omega \to \pi^+\pi^-$
  - Sizeable Isospin-symmetry violations [from radiative corrections, mass differences ($m_{\pi^-} \neq m_{\pi^0}$), $\rho - \omega$ interf.]
    ($\to$ Cirigliano+Ecker+Neufeld)
  - Role of possible $\rho^0 - \rho^\pm$ mass difference?
  - Width difference $\Gamma_{\rho^0} \neq \Gamma_{\rho^\pm}$?
    Large effects possible!
    Are the model calculations reliable?

S. Eidelman (ICHEP06): τ compared to e^+e^- data



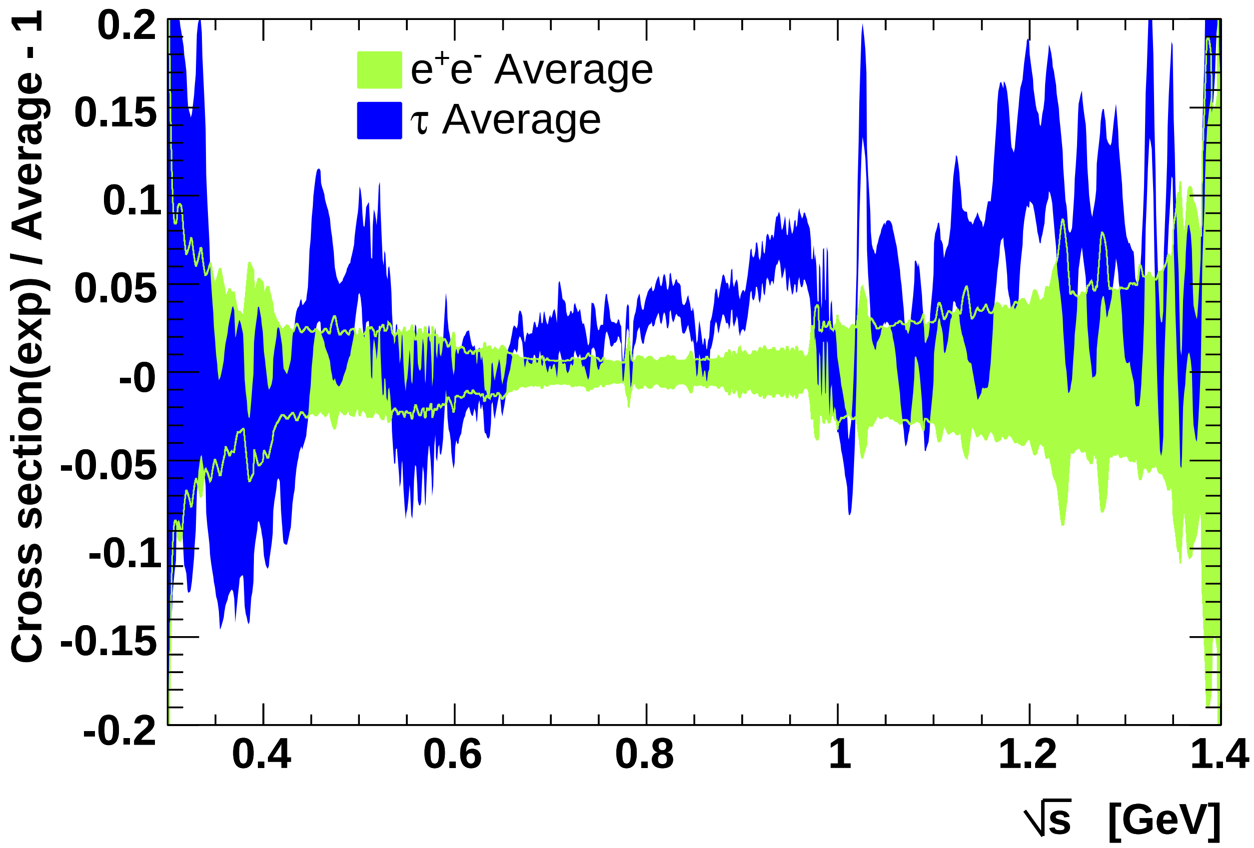
$\longrightarrow$ Disagreement between τ and e^+e^- data already for $[B_\tau - B_{CVC}]_{\pi\pi^0}$: up to 4.5 σ?!

$\longrightarrow$ Recent work of Davier et al. gives better agreement.. $\longrightarrow$

$\hookrightarrow$ Is everything under control *at the % level*? Is something wrong with data? H^-?

- KLOE Radiative Return agrees much better with e^+e^- scan experiments.
- Current 'consensus': Better NOT use τ data for $g-2$ predictions.

Recent compilation from Davier et al. [Fig. from 0908.4300]:



$\longrightarrow$ Disagreement between τ and e^+e^- data less severe than previously but still not solved.

$\longrightarrow$ Work from Benayoun et al. [EPJC55 (2008) 199; and recent 0907.4047,5603]:
mixing + isospin breaking effects in model based on *Hidden Local Symmetry*
$\rightsquigarrow$ τ compatible with and confirm e^+e^- ?!

• Results of HLMNT09 compilation

- Accidental cancellation of mean value shifts between different energy regions (compared to HMNT 2006 analysis, units of 10^{-10}):
  - low energy exclusive channels, $0.32 - 1.43$ GeV: -0.76
  - inclusive–exclusive region, $1.43 - 2$ GeV: $+$2.10
  - higher energy inclusive, $2 - 11.09$ GeV: -1.35

  $\longrightarrow$ ... the power of statistical fluctuations ...

▶ $a_\mu(\mathrm{LO, had}) = 689.41 \pm 3.61_{\mathrm{exp}} \pm 1.82_{\mathrm{rad}}$

Note: will probably change soon as BaBar's 2π Radiative Return data now available.

However combination non-trivial as in 'tension' with KLOE as discussed. $\longrightarrow$

Our first new fit results (ongoing work) suggest shift of $\sim 5 \times 10^{-10}$ upwards.

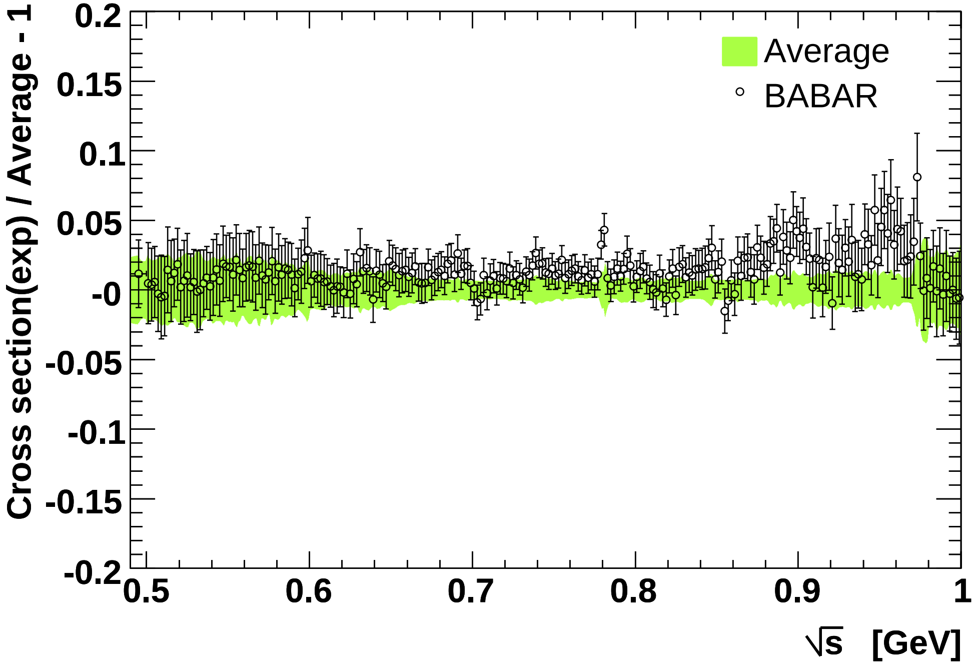
New KLOE 2009 data *(to be published soon)* will also have impact. $\longrightarrow$

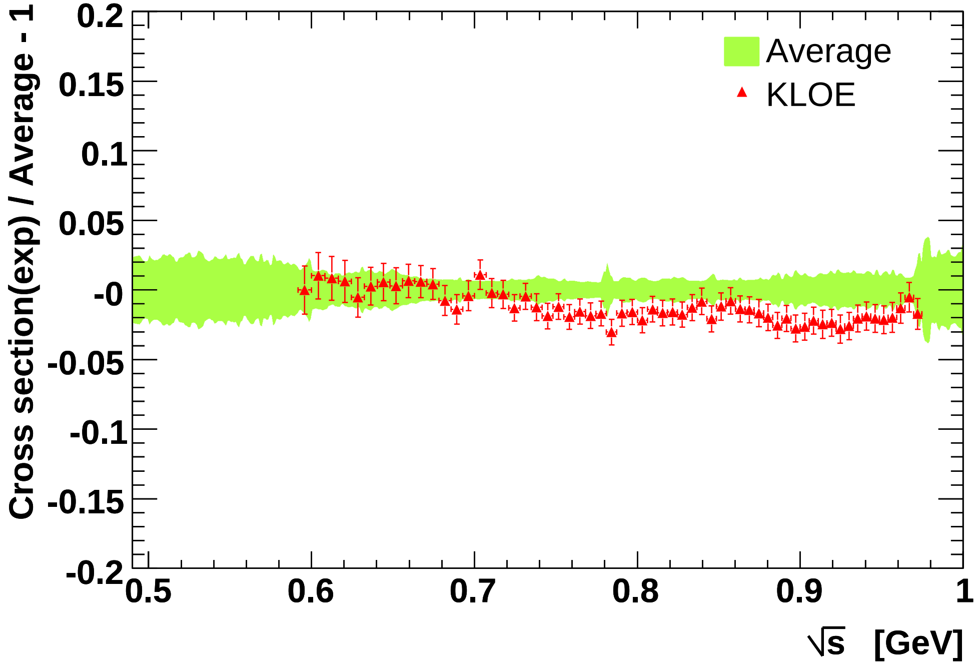
▶ $a_\mu(\mathrm{NLO, had}) = -9.79 \pm 0.06_{\mathrm{exp}} \pm 0.03_{\mathrm{rad}}$

[Was $a_\mu(\mathrm{NLO, had}) = -9.79 \pm 0.09_{\mathrm{exp}} \pm 0.03_{\mathrm{rad}}$.]

BaBar's new and KLOE's 2008 2π data from Radiative Return:

Difference plots as in Davier et al.'s analysis [Figs. from 0908.4300]

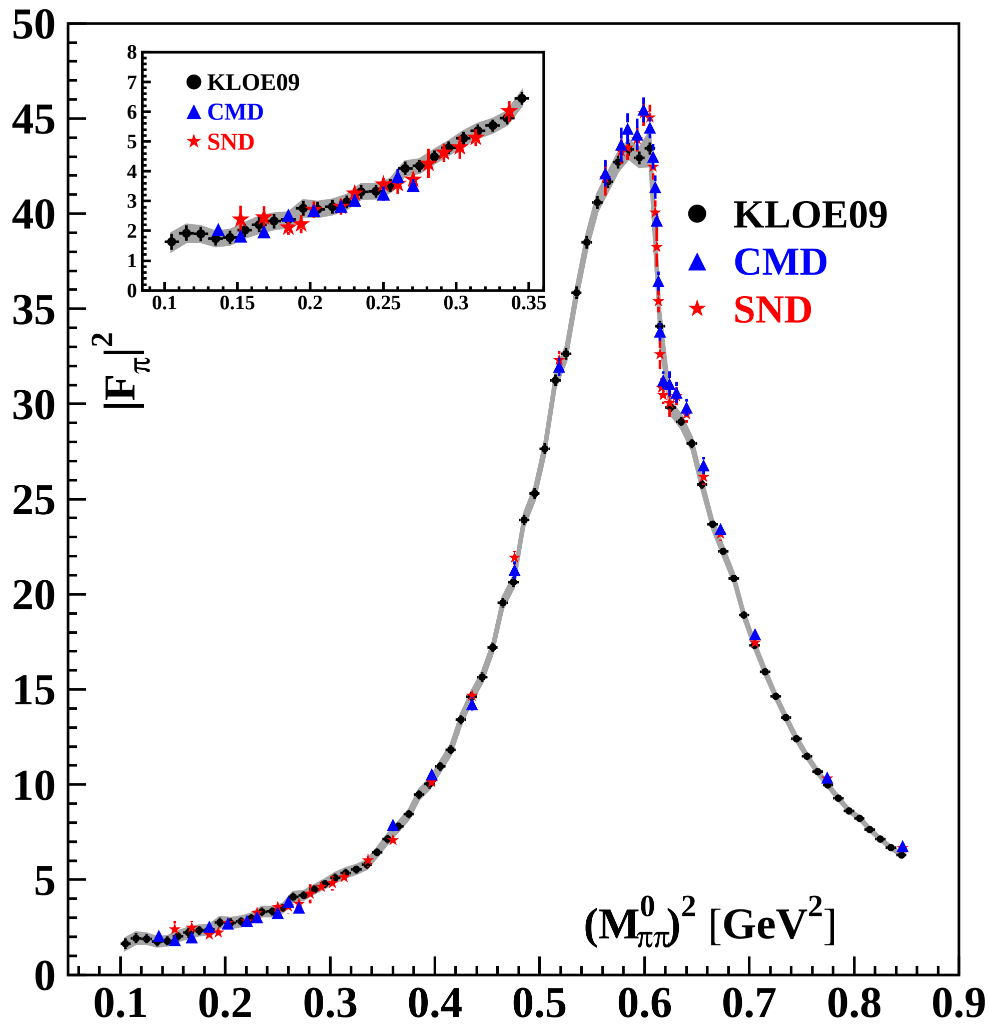


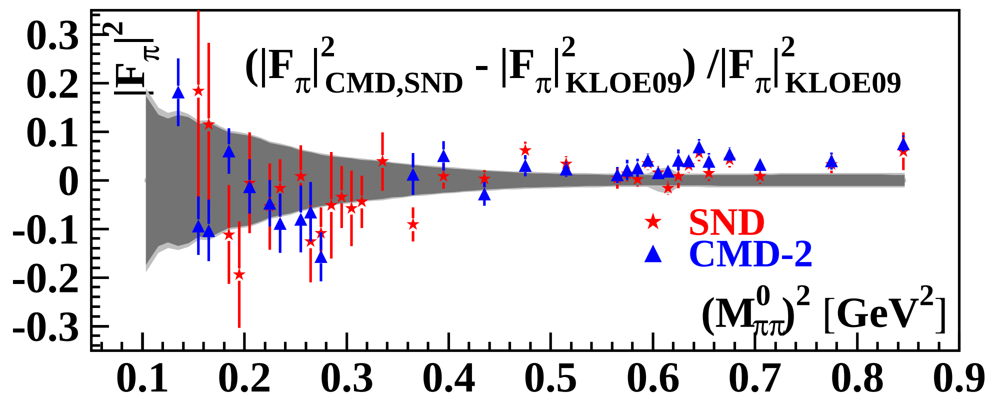


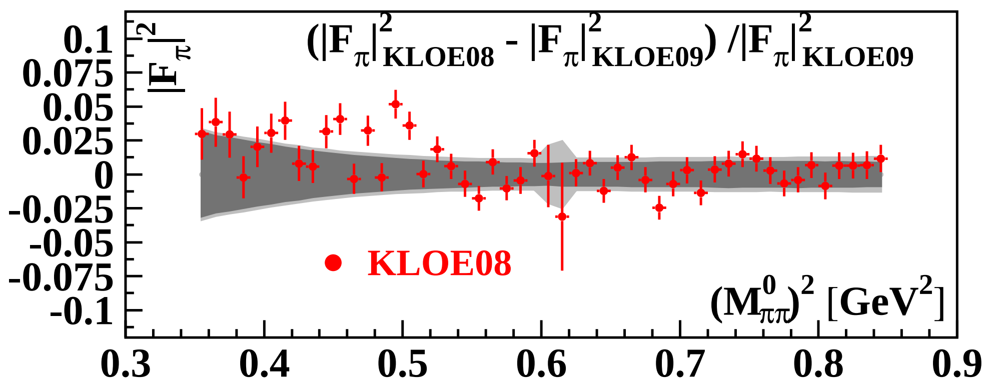
$\longrightarrow$ Disagreement between BaBar's and KLOE's 2π spectral functions (both from Rad. Ret.), especially at medium and larger energies.

KLOE's 2009 2π data from Radiative Return as presented at $\Phi\psi09$ last October:

Comparison with Novosibirsk's data and difference plots [Figs. from 0912.2205]







The different SM contributions numerically:

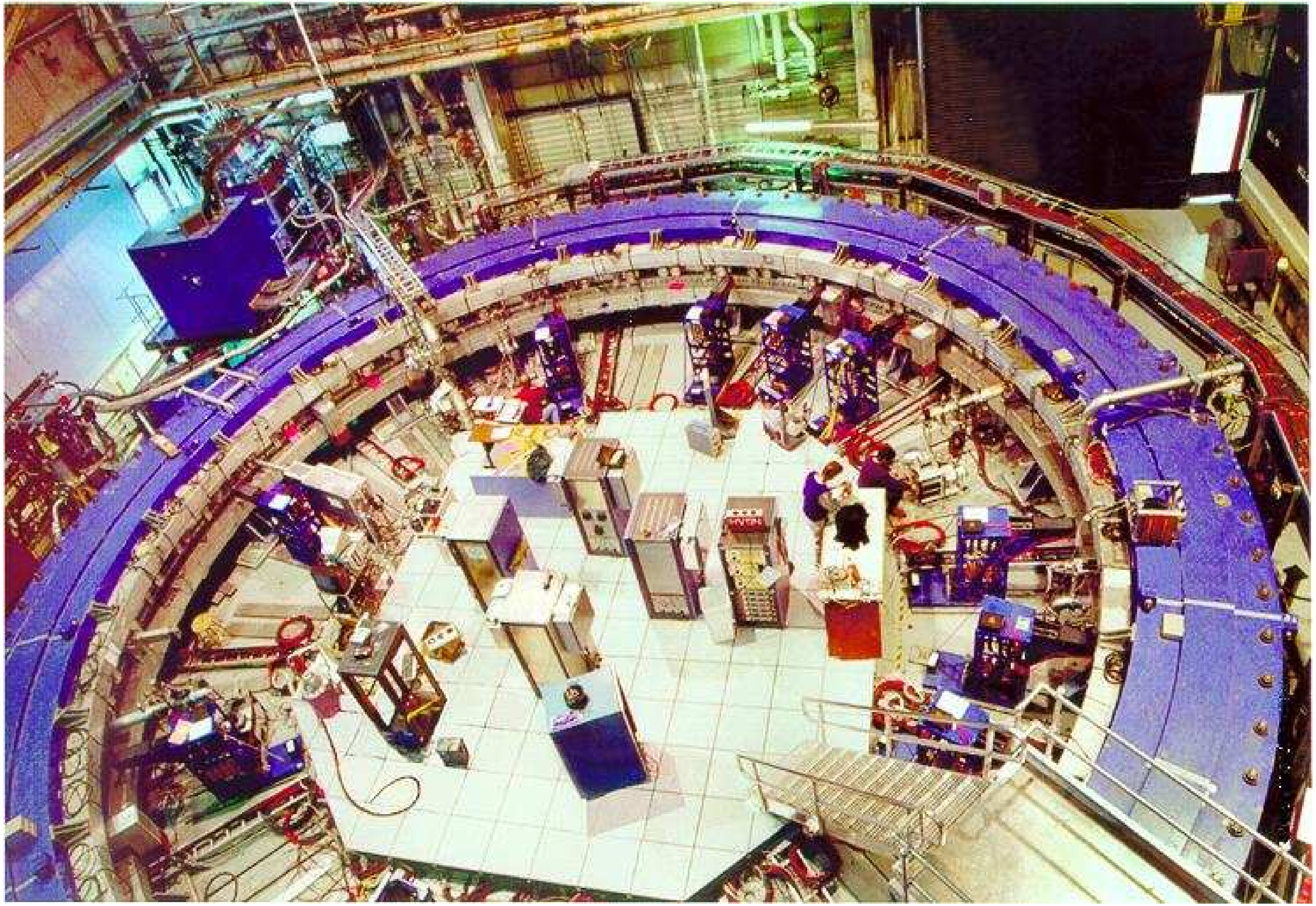
HLMNT09

Source	contr. to $a_\mu \times 10^{11}$	remarks
QED	$116\,584\,718.10 \pm 0.16$	up to 5-loop! (Kinoshita+Nio, Passera)
	(was $116\,584\,719.35 \pm 1.43$)	▶ incl. recent updates of α
EW	154 ± 2	2-loop, Czarnecki+Marciano+Vainshtein
		(agrees very well with Knecht+Peris+Perrottet+deRafael)
LO hadr.	$7110 \pm 50 \pm 8 \pm 28$	Davier+Eidelman+Hoecker+Zhang '03b (τ)
	$6963 \pm 62 \pm 36$	Davier+Eidelman+Hoecker+Zhang '03b (e^+e^-)
	$\mathbf{6924 \pm 59 \pm 24}$	Hagiwara+Martin+Nomura+T 03
new data:	$\mathbf{6894 \pm 36 \pm 18}$	HLMNT09, incl. recent CMD-2, SND, KLOE data
NLO hadr.	$\mathbf{-97.9 \pm 0.6 \pm 0.3}$	HLMNT, in agreem. with Krause '97, Alemany+D+H '98
L-by-L	105 ± 26	▶ Prades+deRafael+Vainshtein
$<$ Nov. 2001:	(-85 ± 25)	the 'famous' sign error, $2.6\,\sigma \to 1.6\,\sigma$
$\sum$	$\mathbf{116591773 \pm 48}$	with HLMNT09 (e^+e^-)

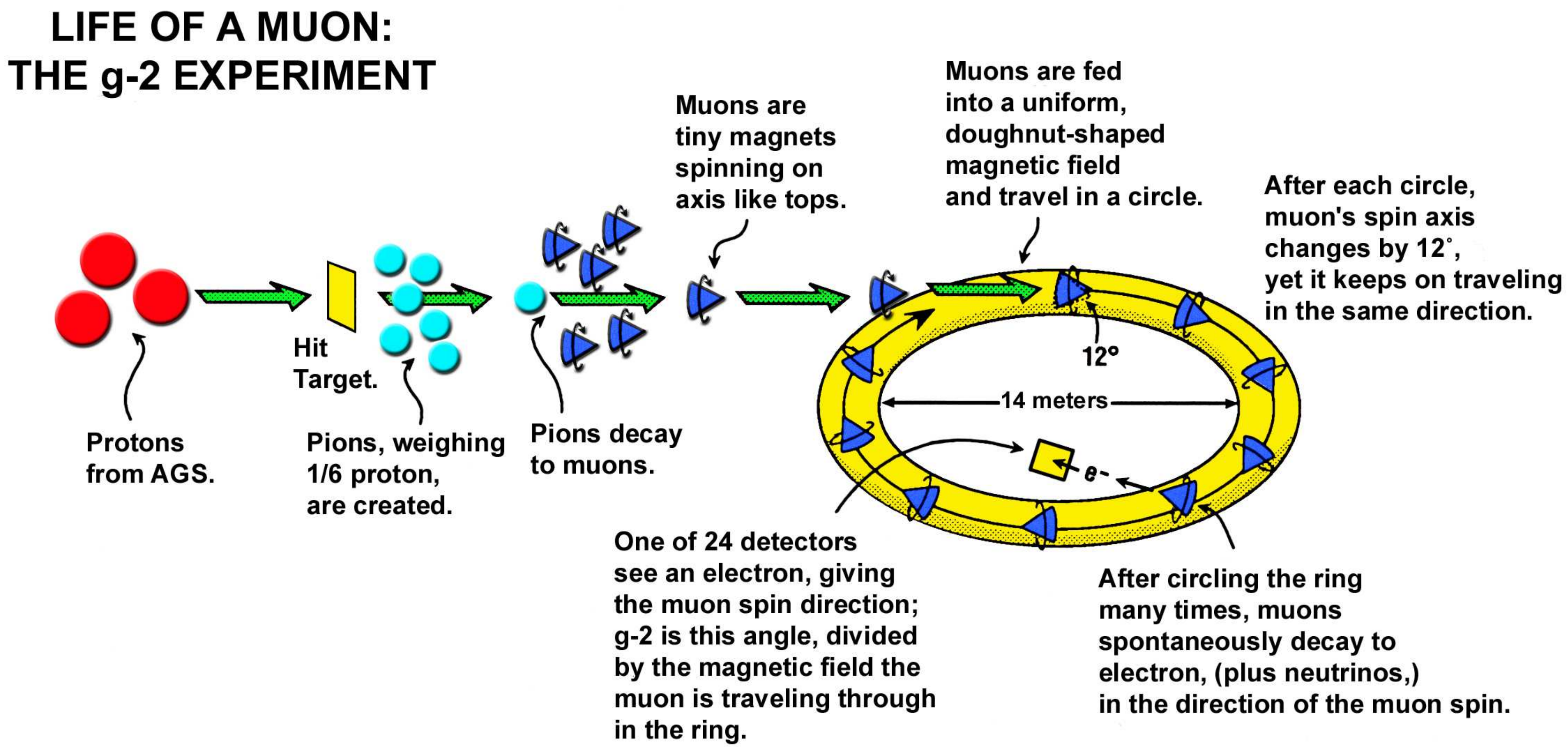
For the first time the theory prediction of $g-2$ is more precise than its measurement

III. SM vs. BNL: A sign of New Physics?

The experiment E821 at Brookhaven (Uncovered storage ring with three scientists)

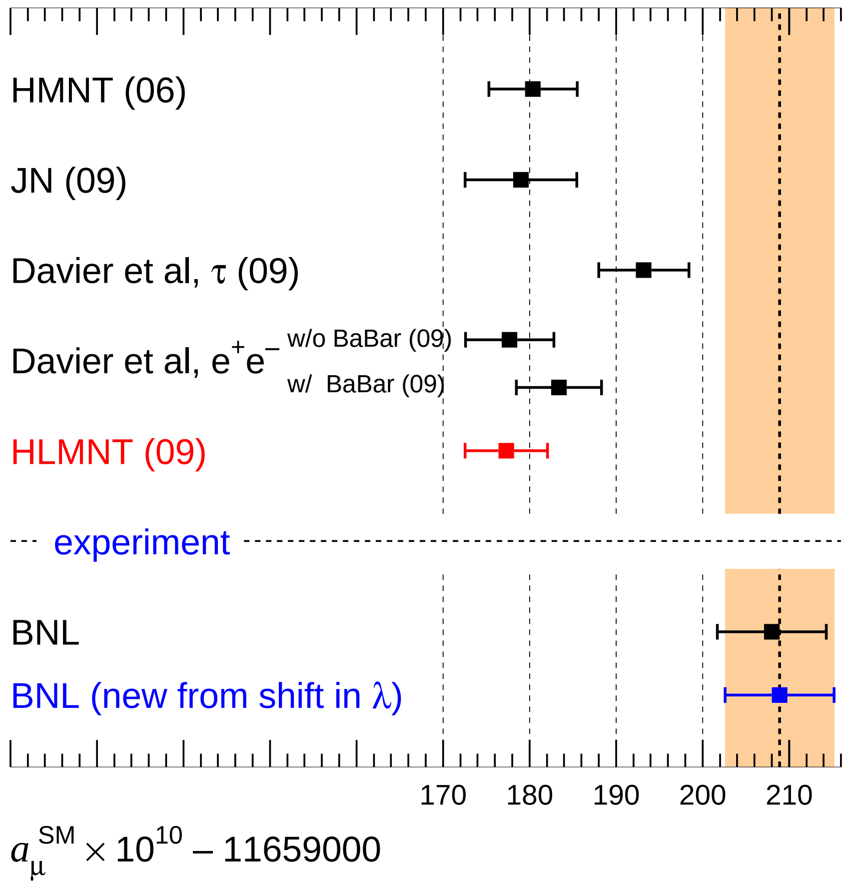


Sketch of how the measurement works (from the g–2 Collaboration webpage)



Covered muon storage ring; running (Pictures from the g–2 Collaboration)

a_μ^{SM} compared to BNL world av.



Davier et al.: $1.8/3.9/3.1\,\sigma$

JN 09: $3.2\,\sigma$ $[179.0 \pm 6.5]$

Recent changes

TH: Improved LO hadronic (from e^+e^-):

[New data from CMD-2, SND, KLOE, BaBar, CLEO, BES. Combination of excl. (BaBar RadRet) and incl. data below 2 GeV.]

$(6894 \pm 46) \cdot 10^{-11} \longrightarrow (6894 \pm 40) \cdot 10^{-11}$

TH: Use of recent L-by-L compilation [PdeRV]: $a_\mu^{\rm L-by-L} = (10.5 \pm 2.6) \cdot 10^{-10}$

EXP: Small shift of BNL's value due to CODATA's shift of muon to proton magn. moment ratio:

Was $a_\mu = 116\;592\;080(63) \cdot 10^{-11}$

$\rightarrow a_\mu = 116\;592\;089(63) \cdot 10^{-11}$ $(0.5\,{\rm ppm})$

▶ With this input HLMNT 09 get:

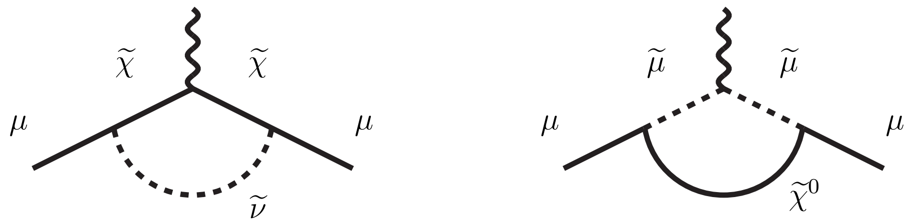
$a_\mu^{\rm EXP} - a_\mu^{\rm TH} = (31.6 \pm 7.9) \cdot 10^{-10}$, $\sim \mathbf{4.0\sigma}$

... $3-4\,\sigma$, not going away (but not 5) ... $a_\mu^{\rm SM}$ always stayed $< a_\mu^{\rm EXP}$ ... SUSY? ...

SUSY contributions in a_μ ?

$$a_\mu^{\mathrm{SUSY,1-loop}} \simeq \frac{\alpha}{8\pi \sin^2\theta_W} \tan\beta \; \mathrm{sign}(\mu) \frac{m_\mu^2}{M_{\mathrm{SUSY}}^2}$$

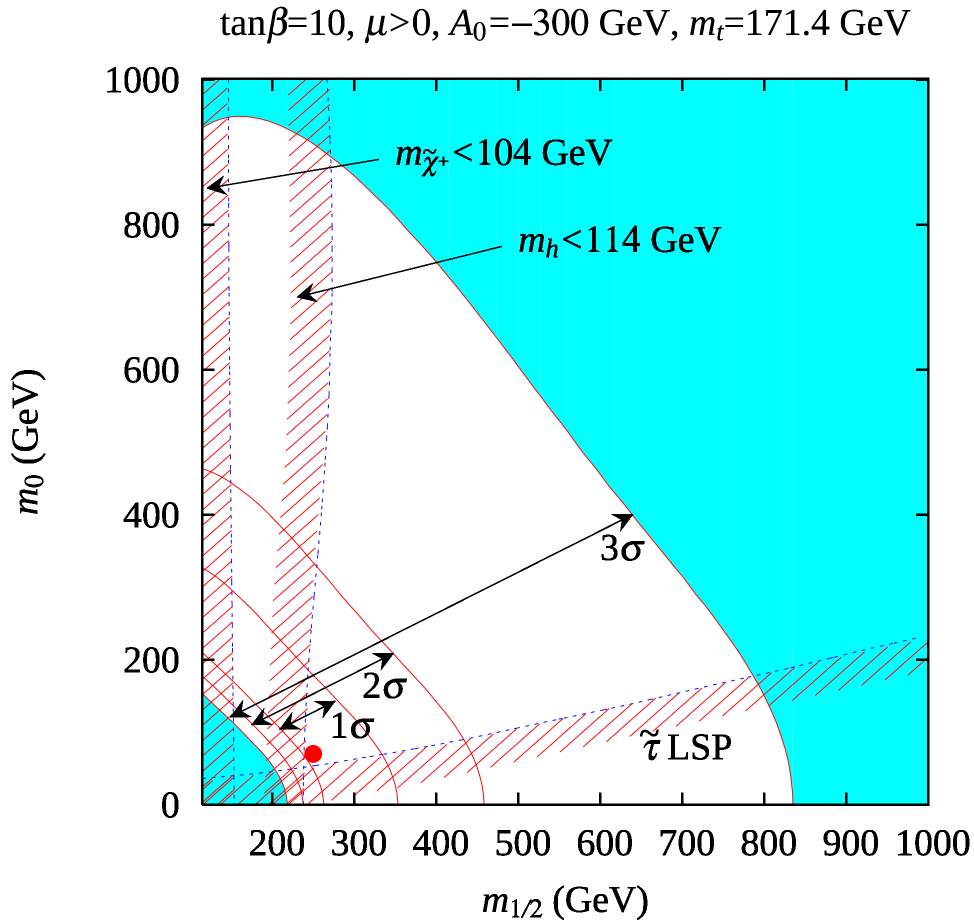
They mainly come from:



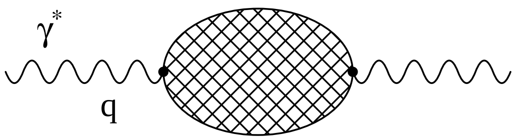
— SUSY is a good candidate to explain $\Delta a_\mu = a_\mu^{\mathrm{EXP}} - a_\mu^{\mathrm{SM}}$, but

- no chargino at LEP
- so far no light Higgs
- limits on lightest charged SUSY part.
- + limits from direct searches
- SPS 1a′ in 1σ band from $g-2$

— Many other BSM scenarios, like e.g. Universal Extra Dimensions, would not 'do the job'.



IV. The 'running coupling' $\alpha_{\rm QED}(q^2)$ and the Higgs mass



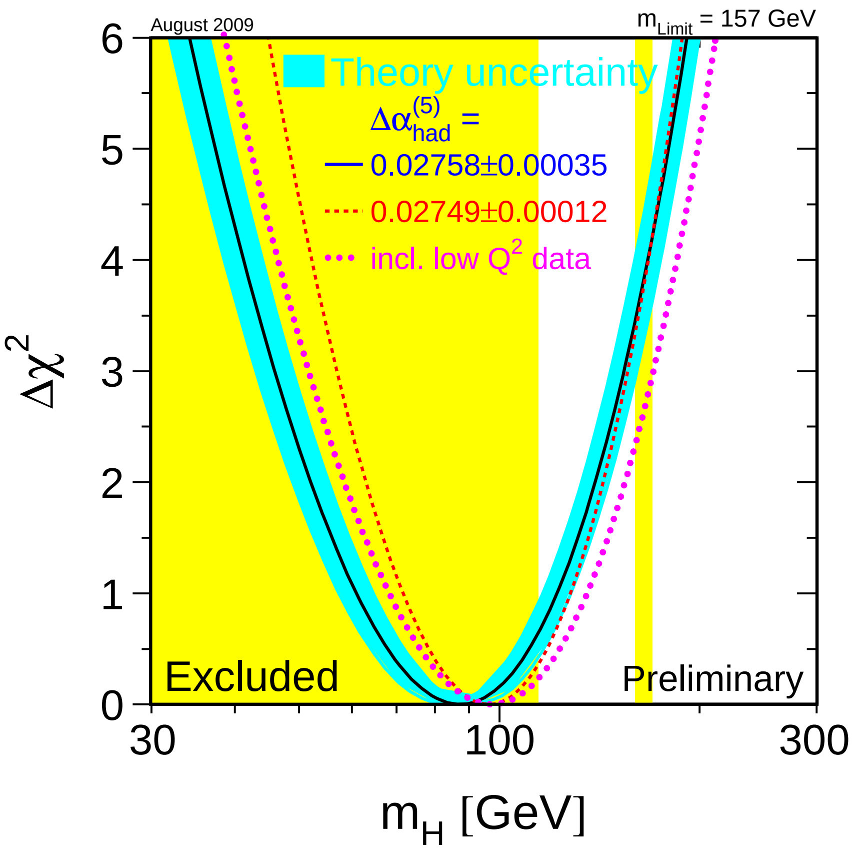
- Vacuum polarisation leads to the 'running' of α from $\alpha(q^2 = 0) = 1/137.035999084(51)$ to $\alpha(q^2 = M_Z^2) \sim 1/129$
- $\alpha(s) = \alpha / \left(1 - \Delta\alpha_{\rm lep}(s) - \Delta\alpha_{\rm had}(s)\right)$
- Again use of a dispersion relation:

$$\Delta\alpha_{\rm had}^{(5)} = -\frac{\alpha s}{3\pi} P \int_{s_{\rm th}}^{\infty} \frac{R_{\rm had}(s')\, ds'}{s'(s'-s)}$$

- Hadronic uncertainties $\rightsquigarrow$ α is the least well known Electro-Weak SM parameter of [G_μ, M_Z and $\alpha(M_Z^2)$] !
- We find: $\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = 0.02760 \pm 0.00015$

i.e. $\alpha(M_Z^2)^{-1} = 128.947 \pm 0.020$ (HLMNT '09)

Fit of the SM Higgs mass: LEP EWWG



- $m_H = 87\,^{+35}_{-26}$ GeV ($m_t = (173.1 \pm 1.3)$ GeV)

($m_H < 157$ GeV (95% CL), < 186 GeV incl. direct limit $m_H < 114$ GeV.)

- m_H would move further down with new $\Delta\alpha$.

V. The next round

Where is improvement needed most urgently? Hadronic VP still with the biggest error.

Pie diagrams of contributions to a_μ and $\alpha(M_Z)$ and their errors^2: enjoy!

Big changes and most critical regions:

$\rightarrow a_\mu$:

$1.4 - 2$ GeV already significantly improved;

low energies, i.e. ρ (central and low),

dominant source of contr. and error,

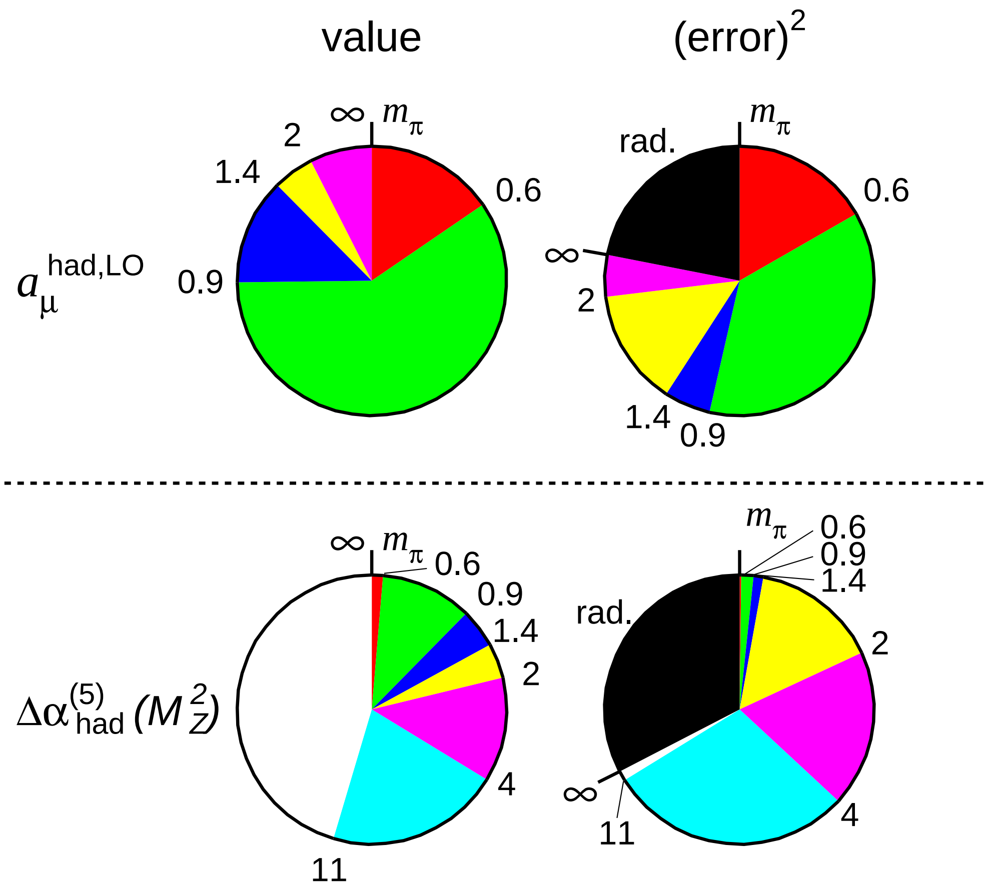
and again the region below 2 GeV.

$\rightarrow \alpha(M_Z)$:

inclusive data need improvement in wide

region up to where pQCD is used;

better control of radcors!



Summary:

- $g-2$ strongly tests *all sectors* of the SM and constrains possible physics beyond.
- Recently new data for the $\pi\pi$ channel from Novosibirsk (CMD-2 and SND) and Frascati (KLOE, using the new method of *Radiative Return*) have lead to a considerable error reduction (Δa_μ down by $\sim 20 \cdot 10^{-11}$).
- Further improvements and consolidation through many hadronic cross section measurements, with important input from BaBar, BES and CLEO.

$\longrightarrow$ Interaction of TH + EXP was and will be most important to achieve even higher precision!

▶ At present a $\mathbf{3-4\,\sigma}$ deviation persists.

- SUSY could, quite naturally, explain the discrepancy; the SUSY parameter space is already strongly constrained by $g-2$.
- With the same data compilation as for $g-2$ also the hadronic contributions to $\Delta\alpha(q^2)$ have been determined; in turn $\alpha(M_Z^2)$ has been improved considerably.

Outlook:

- Further Radiative Return analyses from Frascati (KLOE) are reported and in progress; they will check $\pi\pi$ down to the threshold and hopefully squeeze the error further.
- BaBar is very successful with Rad. Ret. for higher multiplicity final states and has also published very precise new 2π results which are now available.
- Even better prospects (factor 2–3) with VEPP2000 (CMD-3, SND), possibly DAΦNE-2.
- At higher energies, BES-III at BEPCII in Beijing just started; opportunities for BELLE.

$\longrightarrow$ Very good prospects to bring the error of $a_\mu^{\rm SM}$ (and $\Delta\alpha$) down significantly!

▶ The $g-2$ Collaboration is planning to move (partly by helicopter..) the ring from BNL to Fermilab, upgrading the experiment E821 to P989, designed to achieve a 0.14ppm measurement of $g-2$. Hopefully the US politicians will value this great opportunity!

▶ J-PARC, a new high intensity proton accelerator near KEK plans to host a radically new designed $g-2$ experiment. Improvement by a factor 5-10 might be possible!

- For $g-2$, Light-by-Light may well become the limiting factor.

▶ The coming years will be most exciting, and not only for the LHC ... `Stay tuned!`