# Determination of $\gamma$ from $\boldsymbol{B}^{ \pm} \rightarrow$ DK $^{ \pm}$: LHCb and CLEOc Jim Libby (University of Oxford) 

## Outline

- Motivation for the precise determination of $\gamma$
- LHCb
- Overview
- Status
- Measuring $\gamma$ with $B^{ \pm} \rightarrow D K^{ \pm}$at LHCb
- Complementary measurements of $D$ decay at CLEO-c


## $C P$ violation in weak decays of quarks

- $C P$ violation implies differences between matter and antimatter
- In the Standard Model the weak and flavour eigenstates of the three generations of quarks
$\mathbf{V}_{\mathrm{CKM}}=\left(\begin{array}{ccc}V_{u d} & V_{u s} & V_{u b} \propto e^{i \gamma} \\ V_{c d} & V_{c s} & V_{c b} \\ V_{t d} \propto e^{i \beta} & V_{t s} & V_{t b}\end{array}\right)$ are related by a unitary matrix
- A complex phase introduces CP violating effects
- Represented in terms of the Unitarity Triangle

All sides and angles can be measured in $b$-hadron decay

$$
\mathbf{V}_{\mathbf{C K M}} \mathbf{V}_{\mathbf{C K M}}^{\dagger}=\mathbf{I} \Rightarrow V_{u d} V_{u b}^{*}+V_{c d} V_{c b}^{*}+V_{t d} V_{t b}^{*}=0
$$



## Searching for new physics



- Non Standard Model particles contribute within the virtual loops
- Differences between tree-level and loop-level triangles
- Signature of new physics
- Complements direct searches



## LHC Status

Last dipole lowered April $26^{\text {th }}$ this year! (First was in March 2005)

Last interconnect - Nov 2007
Quad triplet remediation - Sep 2007
Latest official schedule (August '07) had beam commissioning beginning in May '08, with then 2 months estimated before first 14 TeV collisions
Since then, there have been problems, eg. with shielding bellows in cold interconnects


## The LHCb Experiment

- Dedicated experiment for precision measurement of CP violation and rare decays of b-hadrons (and charm) at the LHC
a Collider-mode operation at same time as the general-purpose detectors, with less-focused beams $\rightarrow$ most events have a single pp interaction

Dipole magnet

Interaction point

## LHCb in a slide

- pp collisions at a centre of mass energy of 14 TeV - $10^{12} \mathrm{~b} /$ year
- Ring Imaging Cherenkov detectors
- hadron ID for momentum from 2 to $100 \mathrm{GeV} / \mathrm{c}$

- First level hardware trigger rate from $10 \rightarrow 1 \mathrm{MHz}$
- 10 MHz the rate of bunch crossings with 1 or more interaction
- Bunch crossing rate 30 MHz (offset interaction point)
- Software Higher Level Trigger (HLT):
- inclusive and exclusive selections to reduce storage rate to 2 kHz


## Si Vertex Locator (VELO)



- 21 stations of Si wafer pairs with $r$ and $\phi$ strip readout
- Split in two halves to allow retraction from beam line
- When closed 8 mm from beam
- Both detector halves now completed, installed in the pit




## RICH Detectors

3 radiators: RICH1 Aerogel (2-10 GeV), $\mathrm{C}_{4} \mathrm{~F}_{10}$ ( $10-60 \mathrm{GeV}$ ) RICH2 $\mathrm{CF}_{4}(16-100 \mathrm{GeV})$




Status: RICH2 ready: full DAQ exercised

## Trigger

## Full bandwidth for flavour unlike GPDs

Hardware trigger (LO)
$>$ Fully synchronized ( 40 MHz ), $4 \mu \mathrm{~s}$ fixed latency
$>$ High $\mathrm{p}_{\mathrm{T}}$ particles: $\mu, \mu \mu, \mathrm{e}, \gamma$ and hadron
$>$ (typically $\mathrm{p}_{\mathrm{T}} \sim 1-4 \mathrm{GeV} / \mathrm{c}$ )

## 1 MHz (readout of all detector components)

## Software trigger (HLT)

> Full detector info available, only limit is CPU time
$>$ Use more tracking info to re-confirm L0+high IP
$>$ Full event reconstruction: exclusive and inclusive streams tuned for specific final states


L0 HLT and
L0×HLT efficiency

## View of the cavern



## Introduction $\boldsymbol{B}^{ \pm} \rightarrow \boldsymbol{D K} \mathbf{K}^{ \pm}$

- $B \rightarrow D K$ decays involve $\mathrm{b} \rightarrow \mathrm{c}$ and $\mathrm{b} \rightarrow \mathrm{u}$ transitions

Strong phase difference
$A\left(B^{-} \rightarrow D^{0} K^{-}\right)=A_{B}$



- Access $\gamma$ via interference if $\mathrm{D}^{0}$ and ${\overline{D^{0}}}^{0}$ decay to the same final state
- These measurements are theoretically clean
- No penguin $\Rightarrow$ CKM standard candle
- largest correction is sub-degree from D-mixing
- LHCb looking at a number of strategies to study such decays
- B+: Atwood-Dunietz-Soni ('ADS'), 3 and 4 body Dalitz Plot Anal.


## $B^{ \pm} \longrightarrow D\left(K_{S}^{0} \pi^{+} \pi^{-}\right) K^{ \pm}$

- For $B^{+} \rightarrow D\left(K^{0} \pi^{+} \pi^{-}\right) K^{+}$

$$
\begin{aligned}
& A^{-}=f\left(m_{-}^{2}, m_{+}^{2}\right)+r_{B} e^{i(-\gamma+\delta)} f\left(m_{+}^{2}, m_{-}^{2}\right) \\
& A^{+}=f\left(m_{+}^{2}, m_{-}^{2}\right)+r_{B} e^{i(\gamma+\delta)} f\left(m_{-}^{2}, m_{+}^{2}\right)
\end{aligned}
$$

$m_{ \pm}=K_{S}^{0} \pi^{ \pm}$invariant mass and $f\left(m_{ \pm}^{2}, m_{\mp}^{2}\right)$ Dalitz amplitudes

- Assume isobar model (sum of Breit-Wigners) Number of resonances

$$
f\left(m_{+}^{2}, m_{-}^{2}\right)=\left[\sum_{j=1}^{N} a_{j} e^{i \alpha_{j}} A_{j}\left(m_{+}^{2}, m_{-}^{2}\right)\right]+b e^{i \beta}
$$

Amplitude and phase extracted from $D^{*}+\rightarrow D^{0} \pi^{+}$sample at B-factories

Non-resonant

- Fit $D$-Dalitz plots from $B$-decay to extract $\gamma, r_{B}$ and $\delta_{B}$


## Current $\mathbf{e}^{+} \mathbf{e}^{-}$results

- Current best direct constraints on $\gamma$ :

PRD 73, 112009 (2006) hep-ex/0607104

$$
\begin{aligned}
& \phi_{3}=\left(53_{-18}^{+15}(\text { stat }) \pm 3(\text { syst }) \pm 9(\text { model })\right)^{\circ}[\text { Belle }] \\
& \gamma=(92 \pm 41(\text { stat }) \pm 11(\text { syst }) \pm 12(\text { model }))^{\circ}[\text { BABAR }]
\end{aligned}
$$

- Based on ~300 events each (1/3 of final data set)
- However, large error from isobar model assumptions
- BABAR and Belle use large samples of flavour tagged $D^{*+} \rightarrow D^{0} \pi^{+}$ events to find parameters of the isobar model
- Excellent knowledge of $|f|^{2}$ but phases less well known
- Model uncertainties from assumptions about the resonance structures in the model


## Isobar model uncertainty

- Most challenging aspects

BABAR (PRL 95 121802,2005)

| Resonance | Amplitude | Phase $(\mathrm{deg})$ | Fit fraction |
| :--- | :---: | :---: | :---: |
| $K^{*}(892)^{-}$ | $1.781 \pm 0.018$ | $131.0 \pm 0.8$ | 0.586 |
| $K_{0}^{*}(1430)^{-}$ | $2.45 \pm 0.08$ | $-8.3 \pm 2.5$ | 0.083 |
| $K_{2}^{*}(1430)^{-}$ | $1.05 \pm 0.06$ | $-54.3 \pm 2.6$ | 0.027 |
| $K^{*}(1410)^{-}$ | $0.52 \pm 0.09$ | $154 \pm 20$ | 0.004 |
| $K^{*}(1680)^{-}$ | $0.89 \pm 0.30$ | $-139 \pm 14$ | 0.003 |
| $K^{*}(892)^{+}$ | $0.180 \pm 0.008$ | $-44.1 \pm 2.5$ | 0.006 |
| $K_{0}^{*}(1430)^{+}$ | $0.37 \pm 0.07$ | $18 \pm 9$ | 0.002 |
| $K_{2}^{*}(1430)^{+}$ | $0.075 \pm 0.038$ | $-104 \pm 23$ | 0.000 |
| $\rho(770)$ | $1($ fixed $)$ | $0($ fixed $)$ | 0.224 |
| $\omega(782)$ | $0.0391 \pm 0.0016$ | $115.3 \pm 2.5$ | 0.006 |
| $f_{0}(980)$ | $0.482 \pm 0.012$ | $-141.8 \pm 2.2$ | 0.061 |
| $f_{0}(1370)$ | $2.25 \pm 0.30$ | $113.2 \pm 3.7$ | 0.032 |
| $f_{2}(1270)$ | $0.922 \pm 0.041$ | $-21.3 \pm 3.1$ | 0.030 |
| $\rho(1450)$ | $0.52 \pm 0.09$ | $38 \pm 13$ | 0.002 |
| $\sigma$ | $1.36 \pm 0.05$ | $-177.9 \pm 2.7$ | 0.093 |
| $\sigma^{\prime}$ | $0.340 \pm 0.026$ | $153.0 \pm 3.8$ | 0.013 |
| Non Resonant | $3.53 \pm 0.44$ | $128 \pm 6$ | 0.073 | of the model uncertainty come from $\mathrm{K} \pi$ and $\pi \pi \mathrm{S}$ wave

Fit to flavour tag sample


## $B^{ \pm} \rightarrow D\left(K^{0}{ }_{S} \pi^{+} \pi^{-}\right) K^{ \pm}$at LHCb

- Simulation studies performed to determine the expected yields and backgrounds at LHCb
- One 'nominal' year of data-taking $2 \mathrm{fb}^{-1}$
- Total luminosity goal $10 \mathrm{fb}^{-1}$
- Selection studies performed on PYTHIA/EVTGEN/GEANT4 simulated samples of signal and background events
- Limited statistics available for background estimates
- 34 million $b$-inclusive events corresponds to $\sim 15$ minutes of data taking at nominal luminosity
- Trigger simulation is applied for Level-0 and large impact parameter with $p_{t}$ HLT


## $B^{ \pm} \longrightarrow D\left(K_{S}^{0} \pi^{+} \pi^{-}\right) K^{ \pm}$at LHCb

- Selection based on large impact parameter, RICH particle ID and good $p$ resolution
- Efficiency $=0.7 \times 10^{-3}$
- Backgrounds:

$$
\begin{aligned}
- & B^{ \pm} \rightarrow D\left(K^{0}{ }_{S} \pi^{+} \pi^{-}\right) \pi^{ \pm} \\
& B / S=0.24
\end{aligned}
$$

- Combinatoric B/S<0.7 at 90\% c.l.


5000 events/ $\mathbf{2 f b}^{-1}$

## Model uncertainty impact on LHCb

- The model-dependent likelihood fit yields an uncertainty on $\gamma$ between $7-12^{\circ}$ for an $\mathrm{r}_{\mathrm{B}}=0.1$
- Range represents differing assumptions about the background
- However, the current model uncertainty is 10$15^{\circ}$ with an $r_{B}=0.1$
- Uncertainties $\propto 1 / r_{B}$
- Without improvements LHCb sensitivity will be dominated by model assumptions within 1 year of data taking
- Motivates a model-independent method that relies on a binned analysis of the Dalitz plot
- Disadvantage is that information is lost via binning


## Binned method

- Proposed in the original paper by Giri, Grossman, Soffer and Zupan and since been extended significantly by Bondar and Poluektov
- GGSZ, PRD 68, 054018 (2003)
- BP, most recently arXiv:0711.1509v1 [hep-ph]
- Bin the Dalitz plot symmetrically about $\mathrm{m}_{-}{ }^{2}=\mathrm{m}_{+}{ }^{2}$ then number of entries in $\mathrm{B}^{-}$ decay given by:
$\propto$ \# events in bin of flavour tagged $\mathrm{D}^{0}$ decays
$\begin{aligned} N_{i}^{-} & \propto \int_{D_{i}}\left|f\left(m_{-}^{2}, m_{+}^{2}\right)\right|^{2} d D+r_{B}^{2} \int_{D_{i}}\left|f\left(m_{+}^{2}, m_{-}^{2}\right)\right|^{2} d D \\ & +2 \sqrt{\int_{D_{i}}\left|f\left(m_{-}^{2}, m_{+}^{2}\right)\right|^{2} d D \int_{D_{i}}\left|f\left(m_{+}^{2}, m_{-}^{2}\right)\right|^{2} d D}\left(x_{-} \stackrel{\rightharpoonup}{c_{i}+y_{-}} s_{i}\right)\end{aligned}$
$x_{ \pm}=r_{B} \cos \left(\delta_{B} \pm \gamma\right) \quad y_{ \pm}=r_{B} \widehat{\sin \left(\delta_{B} \pm \gamma\right)}$
'Cartesian coordinates'


Average cosine and sine of strong phase difference between $D^{0}$ and
$\overline{\mathrm{D}^{0}}$ decay amplitudes $\left(\Delta \delta_{\mathrm{D}}\right)$ in this bin

## Binned method continued

- Can determine $s_{i}$ and $c_{i}$ at the same time as extracting $\gamma, r_{B}$ and $\delta_{B}$ from $B$ data
$-3+N_{\text {bins }}$ free parameters $\left(\mathrm{C}_{\mathrm{i}}=\mathrm{C}_{-\mathrm{i}}\right.$ and $\mathrm{S}_{\mathrm{i}}=-\mathrm{S}_{-\mathrm{i}}$ )
- Huge loss in $\gamma$ sensitivity not practical until you have $O\left(10^{6}\right)$ events (2500/fb-1 @ LHCb)
- However, CP-correlated $e^{+} e^{-} \rightarrow \psi^{\prime \prime} \rightarrow D^{0} \overline{D^{0}}$ data where one decay is to $K_{S} \pi \tau$ and the other decays to a CP eigenstate and $K_{s} \tau \pi$ allows $\mathrm{c}_{\mathrm{i}}$ and $\mathrm{s}_{\mathrm{i}}$ to be determined, respectively



## Enter CLEO-c

CLEO is the grand-daddy of flavour physics, with history of achievement dating back over 25 years

CLEO-c is latest incarnation. Dedicated programme of data-taking at and above the $c \bar{c}$ threshold Important studies for LQCD and B physics

Oxford LHCb physicists (with Bristol) have joined CLEO-c in order to measure quantities essential for the $\gamma$ studies

Cornell University, Ithaca NY, USA


## CLEO-c detector



## CLEO-c data samples

## CLEO-c: Oct. 2003 - April Fool's Day 2008

368 .MeV, $54 \mathrm{pb}^{-1}, \quad \mathrm{~N}(\Psi(2 \mathrm{~S})) \approx 27 \mathrm{M} \quad \mathbf{e}^{+} \mathbf{e}^{-} \rightarrow \Psi(2 \mathrm{~S}) \rightarrow \pi \pi \mathrm{J} / \Psi, \gamma \chi_{\mathrm{c}}$ etc.
377 BMeV, $800 \mathrm{pb}^{-1}$ delivered, $\sim 3$ milion $\psi(3770) \rightarrow D^{0} \bar{D}^{0}$
$4170 \mathrm{MeV}, 195 \mathrm{pb}^{-1} \rightarrow \sim 300 \mathrm{pb}^{-1} \rightarrow$ more $\rightarrow \sim 720 \mathrm{pb}^{-1}, \quad \mathrm{D}_{(\mathrm{s})}(*) \overline{\mathrm{D}}_{(\mathrm{s})}(*)$ 3970-4260MeV energy scan, $60 \mathrm{pb}^{-1}$ in 12 points


## CLEO-c: double tagged $\Psi(3770)$ events

CLEO-c has collected $\sim 800 \mathrm{fb}^{-1}$ at the $\psi(3770)$ DDbar produced in quantum entangled state:

$$
e^{+} e^{-} \rightarrow \psi^{\prime \prime} \rightarrow \frac{1}{\sqrt{2}}\left[D^{0} \bar{D}^{0}-\bar{D}^{0} D^{0}\right]
$$

Reconstruct one D in decay of interest for $\gamma$ analysis (eg. $\mathrm{K} \pi \pi$ ), \& other in CP eigenstate (eg. $\mathrm{KK}, \mathrm{K}_{\mathrm{s}} \pi^{0} \ldots$ ) then CP of other is fixed.


19th December 2007
$\Leftarrow$ Almost background free

Can use $\mathrm{K}_{\mathrm{L}} \Rightarrow$
From talk by E. White at Charm 07


## CP-tagged $K_{S} \Pi^{+} T^{-}$Dalitz Plots

## 1/3 of total data ( $<1 / 2$ the CP tags) <br> $K_{S} \rho^{\theta}$ resonance enhanced <br> in CP-odd Dalitz plot

CP-odd $K_{S} \rho^{\rho}$ resonance absent in CP-even Dalitz plot
Studies not complete but projected uncertainties on $\mathbf{c}$ and s will lead to 3-5 degree uncertainty on $\gamma$






## Inkblot test

Absolute value of strong phase diff. (BABAR model used in LHCb-48-2007)

- Bondar and Poluektov show that the rectangular binning is far from optimal for both CLEOc and $\gamma$ analyses
- 16 uniform bins has only 60\% of the B statistical sensitivity
- c and s errors would be 3 times larger from the $\psi^{\prime \prime}$
- Best B-data sensitivity when $\cos \left(\Delta \delta_{D}\right)$ and $\sin \left(\Delta \delta_{D}\right)$ are as uniform as possible within a bin


Good approximation and the binning that yields smallest s and c errors is equal $\Delta \delta_{\mathrm{D}}$ bins $-80 \%$ of the unbinned precision $2 \pi\left(i-\frac{1}{2}\right) / N<\Delta \delta_{D}\left(m_{+}^{2}, m_{-}^{2}\right)<2 \pi\left(i+\frac{1}{2}\right) / N$

## Implementation at LHCb

$$
\left(\gamma=60^{\circ}, r_{B}=0.1 \text { and } \delta_{B}=130^{\circ}\right)
$$

- Generate samples of $B^{ \pm} \rightarrow D\left(K^{0}{ }_{s} \pi \tau\right) K^{ \pm}$with a mean of 5000 events split between the charges
- Bin according to strong phase difference, $\Delta \delta_{\mathrm{D}} \Rightarrow$
- Minimise $\chi^{2}$
$\chi^{2}=\sum_{i=-8(i \neq)}^{8}\left[\frac{\left(n_{i}^{+}-N_{i}^{+}\left(x_{+}, y_{+}, h\right)\right)^{2}}{n_{i}^{+}}+\frac{\left(n_{i}^{-}-N_{i}^{-}\left(x_{-}, y_{-}, h\right)\right)}{n_{i}^{-}}\right]$
$n_{i}^{ \pm}=$number of $B^{ \pm} \rightarrow D\left(K_{S}^{0} \pi^{+} \pi^{-}\right) K^{ \pm}$events in $i^{\text {th }}$ bin
$N_{i}^{ \pm}\left(x_{ \pm}, y_{ \pm}, h\right)=h\left[K_{ \pm i}+r_{B}^{2} K_{\mp i}+2 \sqrt{K_{i} K_{-i}}\left(c_{i} x_{ \pm} \pm s_{i} y_{ \pm}\right)\right]$
$h=$ normalization factor
$K_{ \pm i}=\int_{D_{i}}\left|f\left(m_{+}^{2}, m_{-}^{2}\right)\right|^{2} d D$ [measured from flavour tag data]

- $\mathrm{K}_{\mathrm{i}}, \mathrm{C}_{\mathrm{i}}$ and $\mathrm{s}_{\mathrm{i}}$ amplitudes calculated from model
- In reality from flavour tagged samples and CLEO-c


## No background with predicted $2 \mathrm{fb}^{-1}$ yield

## 5000 experiments

Input parameters $\gamma=60^{\circ}, \mathrm{r}_{\mathrm{B}}=0.1$ and $\delta_{\mathrm{B}}=130^{\circ}$







Model independent average uncertainty $7.7^{\circ}$ (c.f. Mod el dependent $5.9^{9}$
19th December 2007

## $\gamma$ uncertainties with 5000 toy experiments

| Scenario | $2 \mathrm{fb}^{-1} \mathrm{Mod}$. Indep. | $10 \mathrm{fb}^{-1}$ Mod. Indep. | $2 \mathrm{fb}^{-1}$ Mod. Dep. <br> (LHCb-048-2007) |
| :---: | :---: | :---: | :---: |
| No background | $7.9^{\circ}$ | $3.5{ }^{\circ}$ | $5.9^{\circ}$ |
| Acceptance | $8.1^{\circ}$ | $3.5{ }^{\circ}$ | $5.5^{\circ}$ |
| $\mathrm{D} \pi(\mathrm{B} / \mathrm{S}=0.24)$ <br> (Best case scenario) | $8.8{ }^{\circ}$ | $4.0{ }^{\circ}$ | $7.3^{\circ}$ |
| $\begin{aligned} & \mathrm{DK}_{\text {comb }}(\mathrm{B} / \mathrm{S}=0.7) \\ & \text { (Worst case scenario) } \end{aligned}$ | $12.8{ }^{\circ}$ | $5.7^{\circ}$ | $11.7^{\circ}$ |

## $B^{ \pm} \rightarrow D\left(K^{0}{ }_{S} \pi^{+} \pi^{-}\right) K^{ \pm}$at LHCb

- Model independent fit with binning that yields smallest error from exploiting CLEO-c data
- Binning depends on model - only consequence of incorrect model is non-optimal binning and a loss
 of sensitivity
- Measurement has no troublesome and hard-to-quantify systematic and outperforms model-dependent approach with full LHCb dataset with currently assigned model error
- $10 \mathrm{fb}^{-1}$ statistical uncertainty $4-6^{\circ}$ depending on background
- CLEO-c measurements essential to validation of assumptions in model dependent measurement
- LHCb-2007-141 - Available via CERN document server


## ADS

## ADS method

- Look at DCS and CF decays of D to rates that have enhanced

$$
\begin{aligned}
& \Gamma\left(B^{-} \rightarrow\left(K^{-} \pi^{+}\right)_{D} K^{-}\right) \propto 1+\left(r_{B} r_{D}^{K \pi}\right)^{2}+2 r_{B} r_{D}^{K \pi} \cos \left(\delta_{B}-\delta_{D}^{K \pi}-\gamma\right), \\
& \Gamma\left(B^{-} \rightarrow\left(K^{+} \pi^{-}\right)_{D} K^{-}\right) \propto r_{B}^{2}+\left(r_{D}^{K \pi}\right)^{2}+2 r_{B} r_{D}^{K \pi} \cos \left(\delta_{B}+\delta_{D}^{K \pi}-\gamma\right), \\
& \Gamma\left(B^{+} \rightarrow\left(K^{+} \pi^{-}\right)_{D} K^{+}\right) \propto 1+\left(r_{B}^{K \pi} r_{D}\right)^{2}+2 r_{B}^{K \pi} r_{D}^{K \pi} \cos \left(\delta_{B}-\delta_{D}^{K \pi}+\gamma\right), \\
& \Gamma\left(B^{+} \rightarrow\left(K^{-} \pi^{+}\right)_{D} K^{+}\right) \propto r_{B}^{2}+\left(r_{D}^{K \pi}\right)^{2}+2 r_{B} r_{D}^{K \pi} \cos \left(\delta_{B}+\delta_{D}^{K \pi}+\gamma\right) \\
& \Gamma\left(B^{-} \rightarrow\left(h^{+} h^{-}\right)_{D} K^{-}\right) \propto 1+r_{B}^{2}+2 r_{B} \cos \left(\delta_{B}-\gamma\right) \\
& \Gamma\left(B^{+} \rightarrow\left(h^{+} h^{-}\right)_{D} K^{+}\right) \propto 1+r_{B}^{2}+2 r_{B} \cos \left(\delta_{B}+\gamma\right) \quad \mathrm{h}=\pi \text { or } \mathrm{K} \\
& \hline
\end{aligned}
$$

- Unknowns : $r_{B} \sim 0.1, \delta_{B}, \delta_{D}{ }^{K \pi}, \gamma, N_{K \pi}, N_{h h}$ ( $r_{D}=0.06$ well measured)
- With knowledge of the relevant efficiencies and BRs, the normalisation constants ( $N_{K \pi}, N_{h h}$ ) can be related to one another
- Important constraint from CLEOc $\sigma\left(\cos \delta_{D}{ }^{K} \pi\right)=0.1-0.2$
- Overconstrained: 6 observables and 5 unknowns


## Expected yields

- ADS measurement is a counting experiment - but suppressed modes have $\sim 10^{-7}$ BRs
- Principal challenge background suppression
- Detailed selections studies as for Dalitz analysis
- LHCb-2006-066

| Channel | Signal yield/2 <br> fb | $\mathrm{B} / \mathrm{S}$ | B-factory yields <br> $(\sim 1 / 4$ final data set $)$ |
| :--- | :--- | :--- | :--- |
| $\mathrm{B}^{ \pm} \rightarrow\left(\mathrm{K}^{ \pm} \pi_{\mp}\right)_{\mathrm{D}} \mathrm{K}^{ \pm}$ | 56,000 | $\mathbf{0 . 6}$ | $\mathbf{4 0 0 0}$ |
| $\mathrm{~B}^{ \pm} \rightarrow\left(\mathrm{h}^{-} \mathrm{h}^{+}\right)_{\mathrm{D}} \mathrm{K}^{ \pm}$ | $\mathbf{8 2 0 0}$ | $\mathbf{1 . 8}$ | $\mathbf{5 0 0}$ |

- The suppressed modes have yields varying from 0 to 500 depending on the strong parameters
- 780 background events predicted


## Sensitivity from 2-body

## $\delta_{D}=-25^{\circ}-$ fit results from 1000 toy $2 \mathrm{fb}^{-1}$ experiments :



$\delta_{\mathrm{D}}$ constraint leads to a 0.5-1.0 ${ }^{\circ}$ reduction in $\sigma_{\gamma}$

Also important for D mixing measurements

| $\delta_{\mathrm{D}}\left({ }^{\circ}\right)$ | -25 | -16.6 | -8.3 | 0 | 8.3 | 16.6 | 25 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\sigma_{\gamma}\left({ }^{\circ}\right)$ | 9.5 | 8.6 | 7.5 | 8.6 | 8.6 | 9.3 | 9.4 |

## Four-body ADS

n $B \rightarrow D(K \pi \pi \pi) K$ can also be used for ADS style analysis

- Similar yields to 2-body - slightly worse B/S
- LHCb-2007-004
- However, need to account for the resonant substructure in $D \rightarrow K \pi \pi \pi$
- made up of $D \rightarrow K^{*} p, K^{-} a_{I}(1260)^{+}$,
- in principle each point in the phase space has a different strong phase associated with it - 3 and 4 body Dalitz plot analyses exploit this very fact to extract $\gamma$ from amplitude fits
- Atwood and Soni (hep-ph/0304085) show how to modify the usual ADS equations for this case
- Introduce coherence parameter $\mathbf{R}_{\mathrm{K} 3 \pi}$ which dilutes interference term sensitive to $\gamma$
$\Gamma\left(B^{-} \rightarrow\left(K^{+} \pi^{-} \pi^{-} \pi^{+}\right)_{D} K^{-}\right) \propto r_{B}^{2}+\left(r_{D}^{K 3 \pi}\right)^{2}+2 r_{B} r_{D}^{K 3 \pi}\left(R_{K 3 \pi}\right) \cos \left(\delta_{B}+\delta_{D}^{K 3 \pi}-\gamma\right)$
- $R_{K 3 \pi}$ ranges from
- 1=coherent (dominated by a single mode) to
- $0=$ incoherent (several significant components)
- Can slice and dice phase space to find most coherent regions


## Determining the coherence factor

- Measurements of the rate of $\mathrm{K} 3 \pi$ versus different tags at CLEO-c allows direct access to $\mathrm{R}_{\mathrm{K} 3 \pi}$ and $\delta_{\mathrm{K} 3 \pi}$

1. Normalisation from CF $\mathrm{K}^{-} \pi^{+} \pi^{+} \pi^{-}$vs. $\mathrm{K}^{+} \pi^{-} \pi^{-} \pi^{+}$and $\mathrm{K}^{-} \pi^{+} \pi^{+} \pi^{-}$vs. $\mathrm{K}^{+} \pi^{-}$
2. CP eigenstates: $\quad \Gamma(K 3 \pi: C P \pm)=\Gamma_{K 3 \pi}^{C F} \Gamma_{C P}\left[1+\left(r_{D}^{K 3 \pi}\right)^{2} \mp 2 r_{D}^{K 3 \pi} R_{K 3 \pi} \cos \delta_{D}^{K 3 \pi}\right.$.
3. $\mathrm{K}^{-} \pi^{+} \pi^{+} \pi^{-}$vs. $\mathrm{K}^{-} \pi^{+} \pi^{+} \pi^{-}: \Gamma\left(K^{-} 3 \pi: K^{-} 3 \pi\right)=\Gamma_{K 3 \pi}^{C F} \Gamma_{K 3 \pi}^{D C S}\left[1-R_{K 3 \pi}^{2}\right]$
4. $\mathrm{K}^{-} \pi^{+} \pi^{+} \pi^{-}$vs. $\mathrm{K}^{-} \pi^{+}: \Gamma\left(K^{-} 3 \pi: K^{-} \pi\right) \approx \Gamma_{K 3 \pi}^{C F} \Gamma_{K 3 \pi}^{D C S}\left[1+\left(\frac{r_{D}^{K_{D} 3 \pi}}{r_{D}^{K \tau}}\right)^{2}+2 \frac{\sum_{D_{D}^{K 3 \pi}}^{r_{D}^{K \tau}}}{r_{K 3 \pi}} \cos \delta_{D}^{K 3 \pi}\right]$

| $\mathrm{K} 3 \pi$ tag side | Expected $800 \mathrm{pb}^{-1}$ yield |
| :--- | :---: |
| $\mathrm{K} 3 \pi \mathrm{CF}$ | 3700 |
| $\mathrm{~K} \pi \mathrm{CF}$ | 5000 |
| $\mathrm{~K} 3 \pi / \mathrm{K} \pi$ DCS | $0-40$ per mode |
| $\mathrm{K}^{0} \pi^{0}$ | 650 |
| $\mathrm{~K}^{+} \mathrm{K}^{-}$ | 500 |
| $\pi^{+} \pi^{-}$ | 200 |

## Determining the coherence factor

- Analysis underway $10 \%$ effects in CP modes so great care with
- Background subtraction
- Efficiency calculation
- Estimate of current sensitivity with the addition of $\mathrm{K}_{\mathrm{L}} \pi^{0}$ and further CP tags i.e. $\mathrm{K}_{\mathrm{S}} \eta$ to be added
- Further information in mixed CP SCS tags such as $\mathrm{K}_{\mathrm{s}} \pi^{+} \pi^{-}$

$$
\sigma_{\text {stat }} \sim 0.1 \text { with } 800 \mathrm{pb}^{-1}
$$

- Binned analysis to determine the most coherent regions



## Conclusion-LHCb

- LHCb has estimated $2 \mathrm{fb}^{-1}$ sensitivity to $\gamma$ in $B^{ \pm} \rightarrow D K^{ \pm}$with
$-D \rightarrow K^{0}{ }_{S} \pi^{+} \pi^{-}-\sigma_{\gamma}=7-12^{\circ}$
$-D \rightarrow K^{-} \pi^{+}$and $D \rightarrow h^{+} h^{-}-\sigma_{\gamma}=7.5-9.5^{\circ}$
- $D \rightarrow K^{-} \pi^{+} \pi^{+} \pi^{-}$will add additional information
- Not the whole story with theoretically clean measurements:
- $B^{0} \rightarrow D K^{*} \quad \sigma_{\gamma} \sim 9^{\circ}$ [LHCb-2007-050]
- $B_{s} \rightarrow D_{s} K \sigma_{\gamma+\phi s} \sim 10^{\circ}$ [LHCb-2007-041]
- A few degree precision on $\gamma$ by the end of LHCb


## Conclusion CLEOc

- CLEO-c measurements essential to fulfilling this goal
- But there is much more that can be done
- Full amplitude analysis of 4-body should yield ultimate precision
- Need DCS model, which can be accessed via CP tags at CLEOc
- Also will guide division of phase space for binned coherence factor analysis
- Other modes that can be used:
- $D \rightarrow K^{-} \pi^{+} \pi^{0}$ (Coherence analysis underway)
- $D \rightarrow K^{0}{ }_{S} K^{+} K^{-}$and $D \rightarrow K_{S}^{0} K^{+} \pi^{-}$
- $D \rightarrow K^{-} K^{+} \pi^{+} \pi^{-}$and $D \rightarrow K^{0}{ }_{s} \pi^{-} \pi^{+} \pi^{0}$


## Additional slides

## Aside: K-matrix

- Breit Wigner description of broad overlapping resonances violates unitarity and requires non-physical $\sigma^{\prime}$
- K-matrix description preserves unitarity
- First studies (Lauren Martin/JL) of LHCb $\gamma$ fit with one K-matrix
parameterisation of the $\pi \pi$ S-wave
- Difference between assuming Kmatrix and BW model consistent with B-factory observations
- Draft available from CPWG webpage
- Explore different physical K-matrix parameterisation to evaluate systematic rather than introduce $\sigma^{\prime}$ will reduce model uncertainty



## No background with predicted 2 fb $^{-1}$ yield






5000 experiments
Input parameters $\gamma=60^{\circ}, \mathrm{r}_{\mathrm{B}}=0.1$ and $\delta_{\mathrm{B}}=130^{\circ}$

The four Cartesian coordinates and normalization are free parameters

All pulls are normal therefore calculate $\gamma, \mathrm{r}_{\mathrm{B}}$ and $\delta_{B}$ with propagated
Cartesian uncertainties

## Toy experiment results: $\gamma\left(2 \mathrm{fb}^{-1}\right)$

| Scenario | Mean | RMS | Mean $\sigma$ | Mean pull | Pull RMS |
| :--- | :--- | :--- | :--- | :--- | :--- |
| No bck | $60.5 \pm 0.1$ | 7.9 | 7.8 | $0.045 \pm 0.015$ | 1.05 |
| Acc | $60.7 \pm 0.1$ | 8.1 | 7.8 | $0.075 \pm 0.015$ | 1.07 |
| D $\pi$ | $60.7 \pm 0.1$ | 8.8 | 8.8 | $0.088 \pm 0.015$ | 1.04 |
| D $\pi+$ DK <br> $(B / S=0.7)$ | $60.7 \pm 0.2$ | 12.8 | 12.2 | $0.049 \pm 0.016$ | 1.11 |
| D $\pi+$ PS <br> $(B / S=0.7)$ | $60.8 \pm 0.2$ | 12.8 | 12.5 | $0.064 \pm 0.015$ | 1.05 |
| D $\pi+$ DK + <br> PS $(50: 50)$ <br> $(B / S=0.7)$ | $60.7 \pm 0.2$ | 12.7 | 12.6 | $0.049 \pm 0.015$ | 1.04 |

## Toy experiment results: $\gamma\left(10 \mathrm{fb}^{-1}\right)$

| Scenario | Mean | RMS | Mean $\sigma$ | Mean pull | Pull RMS |
| :--- | :---: | :---: | :---: | :---: | :---: |
| No bck | $60.17 \pm 0.05$ | 3.5 | 3.4 | $0.050 \pm 0.015$ | 1.03 |
| Acc | $60.13 \pm 0.05$ | 3.5 | 3.4 | $0.036 \pm 0.015$ | 1.01 |
| D $\pi$ | $60.22 \pm 0.06$ | 4.0 | 3.9 | $0.054 \pm 0.015$ | 1.03 |
| D $\pi+$ DK <br> $(\mathrm{B} / \mathrm{S}=0.7)$ | $60.18 \pm 0.08$ | 5.7 | 5.7 | $0.030 \pm 0.015$ | 1.01 |
| D $\pi+$ PS <br> $(\mathrm{B} / \mathrm{S}=0.7)$ | $60.26 \pm 0.08$ | 5.5 | 5.5 | $0.045 \pm 0.015$ | 1.00 |
| $\mathrm{D} \pi+\mathrm{DK}+$ <br> $\mathrm{PS}(50: 50)$ <br> $(\mathrm{B} / \mathrm{S}=0.7)$ | $60.22 \pm 0.08$ | 5.4 | 5.6 | $0.038 \pm 0.015$ | 0.97 |

## Toy experiment results: $\mathrm{r}_{\mathrm{B}}\left(\mathbf{~ f b}^{\mathbf{- 1}}\right)$

| Scenario | Mean | RMS | Mean $\sigma$ | Mean pull | Pull RMS |
| :--- | :--- | :--- | :--- | :--- | :--- |
| No bck | $0.1017 \pm 0.0002$ | 0.013 | 0.013 | $0.143 \pm 0.015$ | 1.02 |
| Acc | $0.1017 \pm 0.0002$ | 0.014 | 0.013 | $0.175 \pm 0.016$ | 1.13 |
| D $\pi$ | $0.1015 \pm 0.0002$ | 0.014 | 0.014 | $0.123 \pm 0.015$ | 1.02 |
| D $\pi+$ DK <br> $(\mathrm{B} / \mathrm{S}=0.7)$ | $0.1031 \pm 0.0003$ | 0.020 | 0.020 | $0.215 \pm 0.016$ | 1.16 |
| D $\pi+$ PS <br> $(\mathrm{B} / \mathrm{S}=0.7)$ | $0.1035 \pm 0.0003$ | 0.020 | 0.019 | $0.175 \pm 0.015$ | 0.99 |
| $\mathrm{D} \pi+\mathrm{DK}+$ <br> PS $(50: 50)$ <br> $(\mathrm{B} / \mathrm{S}=0.7)$ | $0.1038 \pm 0.0003$ | 0.020 | 0.020 | $0.186 \pm 0.015$ | 0.98 |

## Toy experiment results: $\mathrm{r}_{\mathrm{B}}\left(\mathbf{1 0} \mathrm{fb}^{-1}\right)$

| Scenario | Mean | RMS | Mean $\sigma$ | Mean pull | Pull RMS |
| :--- | :--- | :--- | :--- | :--- | :--- |
| No bck | $0.1003 \pm 0.0001$ | 0.006 | 0.006 | $0.056 \pm 0.015$ | 1.00 |
| Acc | $0.1003 \pm 0.0001$ | 0.006 | 0.006 | $0.051 \pm 0.015$ | 1.01 |
| D $\pi$ | $0.1003 \pm 0.0001$ | 0.006 | 0.006 | $0.049 \pm 0.015$ | 0.98 |
| D $\pi+$ DK <br> $(\mathrm{B} / \mathrm{S}=0.7)$ | $0.1009 \pm 0.0001$ | 0.009 | 0.009 | $0.101 \pm 0.015$ | 0.97 |
| D $\pi+$ PS <br> $(\mathrm{B} / \mathrm{S}=0.7)$ | $0.1008 \pm 0.0001$ | 0.009 | 0.009 | $0.093 \pm 0.015$ | 0.99 |
| $\mathrm{D} \pi+\mathrm{DK}+$ <br> $\mathrm{PS}(50: 50)$ <br> $(\mathrm{B} / \mathrm{S}=0.7)$ | $0.1007 \pm 0.0001$ | 0.009 | 0.009 | $0.077 \pm 0.015$ | 0.98 |

## Acceptance

- Acceptance in each bin calculated as a weighted average of the acceptance function used for model dependent studies
- 15\% relative difference amongst bins
- Modifies the fit function:

Can be
$\left.N_{i}^{ \pm}\left(x_{ \pm}, y_{ \pm}, h\right)=h \varepsilon_{ \pm i} \mid K_{ \pm i}+r_{B}^{2} K_{\mp i}+2 \sqrt{K_{i} K_{-i}}\left(c_{i} x_{ \pm} \pm s_{i} y_{ \pm}\right)\right]$
$\varepsilon_{i}=\frac{\int_{D_{i}}\left|f\left(m_{+}^{2}, m_{-}^{2}\right)\right|^{2} \varepsilon\left(m_{+}^{2}, m_{-}^{2}\right) d D}{K_{i}}$ where $\varepsilon\left(m_{+}^{2}, m_{-}^{2}\right)=0.28 \times 10^{-3}\left(1-0.08\left(m_{+}^{2}+m_{-}^{2}\right)\right)$
calculated from D $\pi$

- Average $\gamma$ uncertainty increases to $8.1^{\circ}$



## Background

- 3 types of background to consider
- $B \rightarrow D\left(K_{S} \pi \pi\right) \pi(\mathrm{B} / \mathrm{S}=0.24)$
${ }^{-} \mathrm{r}_{\mathrm{B}}(\mathrm{D} \pi) \mathrm{O}\left(10^{-3}\right)$ so Dalitz plots are like $D^{0}$ and $\overline{D^{0}}$ for $B^{-}$and $B^{+}$, respectively
- Combinatoric ( $\mathrm{B} / \mathrm{S}<0.7$ )-mixtures of two types considered

1. $\mathrm{DK}_{\text {comb: real }} D \rightarrow D\left(K_{S} \pi \pi\right)$ combined with a bachelor $K$
aDalitz plot an even sum of $D^{0}$ and $\overline{D^{0}}$ decays
2. $\mathrm{PS}_{\text {comb: combinatoric }} D$ with a bachelor $K$ aFollows phase space

- Integrate background PDFs used in model-dependent analysis over each bin, then scaled to background level assumed:
$N(D \pi)_{i}^{ \pm} \propto \varepsilon_{ \pm i} K_{ \pm i}$
$N\left(D K_{\text {comb }}\right)_{i}^{ \pm} \propto \frac{1}{2}\left(\varepsilon_{ \pm i} K_{ \pm i}+\varepsilon_{\mp i} K_{\mp i}\right)$
$N\left(P S_{\text {comb }}\right)_{i}^{ \pm} \propto P_{i}$
fractional area of Dalitz space covered by bin


## Systematic related to acceptance

- The acceptance varies over the Dalitz plane
- The relative acceptance in each bin can be measured using the $B \rightarrow D \pi$ control sample with DK selection applied without bachelor K PID
$\varepsilon_{i}=\frac{\int_{D_{i}}\left|f\left(m_{+}^{2}, m_{-}^{2}\right)\right|^{2} \varepsilon\left(m_{+}^{2}, m_{-}^{2}\right) d D}{K_{i}} \propto \frac{\mathrm{~N}\left(\mathrm{~B} \rightarrow \mathrm{D}\left(\mathrm{K}_{\mathrm{S}}^{0} \pi \pi\right) \pi\right)_{i}}{K_{i}}$
- With the DC04 selection expect 60k events/2 fb ${ }^{-1}$
- Relative relative-efficiency uncertainty $1-4 \% / \Delta \delta_{D}$ bin with $2 \mathrm{fb}^{-1}$
- Increased statistics reduces error
- Toy MC study smearing bin efficiencies in event generation by this amount leads to an additional $1^{\circ}$ uncertainty without background and $3.2^{\circ}$ uncertainty with $\mathrm{DK}_{\text {comb }} \mathrm{B} / \mathrm{S}=0.7$
- Small effect compared to statistical uncertainty
- NB: the efficiency related to the PID of the bachelor $\pi / K$ can be factored out and will be determined from the $\mathrm{D}^{*} \rightarrow \mathrm{D}(\mathrm{K} \pi) \pi$ data to better than one percent-ignore at present


## Asymmetry in efficiency in Dalitz space

- Considered charge asymmetries in the efficiency across the Dalitz plane
$-\varepsilon\left(m^{2}, m^{2}\right) \neq \varepsilon\left(m^{2}, m^{2}\right)$
- Generated with the efficiency biased relative to one another depending on whether the event had $\mathrm{m}_{+}^{2}>\mathrm{m}^{2}{ }_{-}$or $\mathrm{m}^{2}<\mathrm{m}^{2}{ }_{-}$
- Maximum bias on $\gamma$ induced was $<1^{\circ}$ for 10\% relative effect and full background
- $10 \%$ effects would be evident in the $\mathrm{D} \pi$ sample


## Resolution

- $\Delta \delta_{\mathrm{D}}$ binning has some narrow regions in Dalitz space
- Investigation of how resolution on the Dalitz variables might affected the extraction of $\gamma$
- $10 \mathrm{MeV}^{2} / \mathrm{c}^{4}$ resolution (DC04) on Dalitz variables and generated toy experiments with this smearing
- Found that this led to a few bins with largest (red) and smallest (dark blue) phase difference having a 2-3\% relative changes in expected yields due to resolution induced migration
- Fit results on toy experiments where resolution included in generation but ignored in fit found
 no significant bias $\left(<0.5^{\circ}\right)$ on $\gamma$


## Background fractions

- Combinatoric background rate will be determined from $B$ and $D$ mass sidebands which will cover at least 2-3 times the area of the signal region
- Use 10x in DC04 background studies but this will probably be unrealistic with data
- If background distributions relatively flat in masses one can estimate that this leads to $\mathrm{B} / \mathrm{S}$ will be determined absolutely to around 0.01 or better
- Toy studies suggest that there is no impact on $\gamma$ precision with this kind of uncertainty
- Maybe complications depending on Dalitz space distribution of the PS background but can only speculate until we have the data in hand


## Background composition

- For favoured mode background dominated by $B \rightarrow D \pi$
$-14 \times$ larger BF
- Power of the RICH
- For suppressed combinatoric dominates (green)
- For $B \rightarrow D(h h) K$ more even mixture of comb. and $D \pi$
- $B \rightarrow D(K K) K$ has significant nonresonant $B \rightarrow K K K$ component



