Extra Dimensions at the LHC

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Introduction

Hierarchy problem: large separation between:

- \bullet weak interaction scale: W mass $\sim 100~{\rm GeV}$
- gravitation scale: Plank mass $M_{\rm Pl} \equiv (G_F)^{-1/2} \sim 10^{19} {\rm ~GeV}$

Possible approach: exploit geometry of space-time:

Postulate that we live in 3-d "brane" embedded in higher dimensional space Hierarchy is generated by geometry of extra dimensions (ED)

Possibility that matter and non-gravitational forces confined on 3-brane and gravity propagates through higher dimensional volume ("bulk")

Since we do not observe deviation from Newton's force at a distance \lesssim mm extra-dimensions must be compactified with radius $R\lesssim$ mm

Number of perceived dimensions depends on whether observer can resolve compactification radius R

Two ways of establishing hierarchy through extra-dimension:

• Arkhani Ahmed, Dimopoulos, Dvali (ADD):

Trough "Large" flat extra-dimensions, compactified on torus.

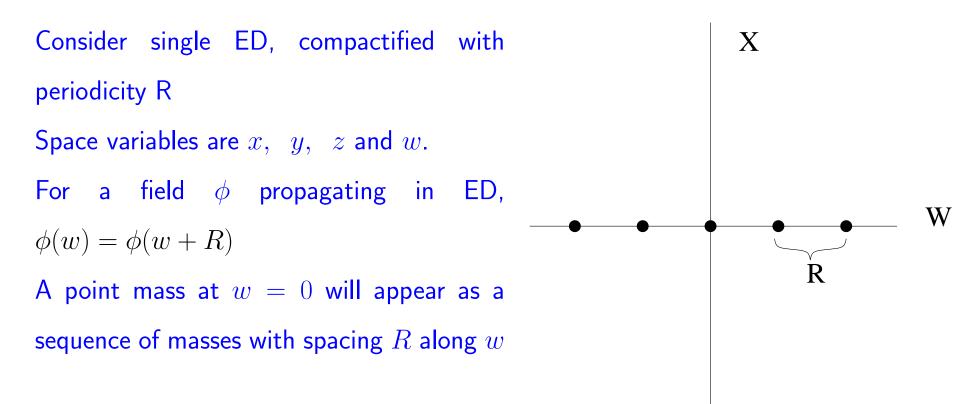
- Gravitational strength diluted by volume of n extra dimensions: $M_{Pl}^2 = M_D^{n+2} R^n$ with M_D scale of gravity in the bulk
- To account for hierarchy: $M_D \sim 1 \text{ TeV} \Rightarrow R \gg 1/\text{TeV}$ "Large" ED
- \bullet EW measurements test SM gauge fields to distances $\sim 1/{\rm TeV}$
- \Rightarrow SM fields localized on a brane
- Randall Sundrum (RS):

Through a curved geometry of the extra-dimension

- Only one extra dimension
- hierarchy from exponential warp factor in non factorizable geometry: $\Lambda_{\pi} = M_P e^{-k\pi r_c}$

In both cases, the presence of compactified dimensions gives rise to a Kaluza-Klein tower of excited states for the gravitons: Many striking signatures predicted at Colliders by these models ED theories have emerged after design of detectors completed \Rightarrow ideal way of verifying robustness of detector for unforeseen signatures

ADD: Lowering the scale of gravity



Calculate the gravitational force F felt by a unit mass at a distance r from the origin Use of Gauss' law in n dimensions which can be written as:

$$\int F da = S_n G_n M_{enc}$$

Where S_n is the surface area of a unit *n*-sphere, M_{enc} is the mass enclosed in the Gaussian volume and G_n is the *n*-dimensional Newton's constant

Surface area of unit sphere:

 Γ is gamma function:

$$S_n = \frac{2\pi^{n/2}}{\Gamma\left(\frac{n}{2}\right)}$$
$$S_3 = 4\pi \quad S_4 = 2\pi^2$$

$$\Gamma(n-1) = (n-1)\Gamma(n-1)$$

$$\Gamma(1) = 1 \quad \Gamma(\frac{1}{2}) = \sqrt{\pi}$$

$$\int F da = S_n G_n M_{enc}$$

Case 1: $r \ll R \Rightarrow$ only the mass at W = 0 contributes to the field Using the fact that $F \equiv F(r)$, and $\int da = r^{n-1}S_n$

3 dimensions :
$$\int da = 4\pi r^2 \Rightarrow F = G_3 \frac{M}{r^2}$$

 \Rightarrow Recover Newton's law

4 dimensions :
$$\int da = 2\pi^2 r^2 \rightarrow F = G_4 \frac{M}{r^3}$$

The form of Newton's law is modified at short distances Short-distance experiments can in principle verify this Case 2: $r \gg R$: mass source appears as a wire with uniform mass density M/RCylindrical geometry, for calculating field at distance r from the wire consider a 4-d cylinder with side length L and end caps composed of 3-d spheres of radius r.

In 4-d: $\int da = 4\pi r^2 L$, $S_4 = 2\pi^2$, $M_{enc} = M(L/R)$. Substituting into Gauss theorem:

$$F4\pi r^2 L = 2\pi^2 G_4 M \frac{L}{R} \Rightarrow F = G_4 M \frac{2\pi^2}{4\pi} \frac{1}{r^2 R}$$

At large distances we recover the $1/r^2$ dependence.

Generalising to n space dimensions:

$$\int da = 4\pi r^2 L^{n-3} \ M_{enc} = M(L/R)^{(n-3)} \quad \Rightarrow \quad F = \frac{S_n}{4\pi} \frac{G_n}{R^{n-3}} \frac{M}{r^2}$$

Identifying the *n*-dimensional formula to Newton's law with $G_N \equiv G_3$:

$$\frac{S_n}{4\pi} \frac{G_n}{R^{n-3}} \frac{M}{r^2} = G_N \frac{M}{r^2} \quad \Rightarrow \quad G_N = \frac{S_n}{4\pi} \frac{G_n}{V_{n-3}}$$

Where $V_{(n-3)} = R^{n-3}$ is the volume of the (n-3)-dimensional compactified space The strength of the gravitational interaction as felt at long distance (G_N) is equal to the strength in n dimension diluted by the extra dimension volume Define $\delta \equiv n - 3$ of previous derivation, number of extra-dimensions

Define the characteristic mass scales: $M_{\text{Pl}} = (G_N)^{-1/2} \sim 10^{19} \text{ GeV}$ and

$$\hat{M}_D^{\delta+2} = \frac{G_\delta^{-1}}{S_{3+\delta}}$$

We obtain the reduction formula:

$$M_{\rm Pl}^2 = 8\pi R^{\delta} M_D^{2+\delta}$$

Assume now $M_D \sim 1$ TeV: solve hierarchy problem $R \sim 10^{32/\delta+3} \text{ Gev}^{-1} \Rightarrow R \sim 10^{32/\delta-16} \text{ mm}$ $\delta = 1$ corresponds to astronomical distances: excluded

 $\delta=2$ at the limit of present tests of Newton's law

δ	R (mm)
1	10^{16}
2	1
3	5×10^{-6}
4	10^{-8}
5	10^{-10}

Experimental tests of Newton's law

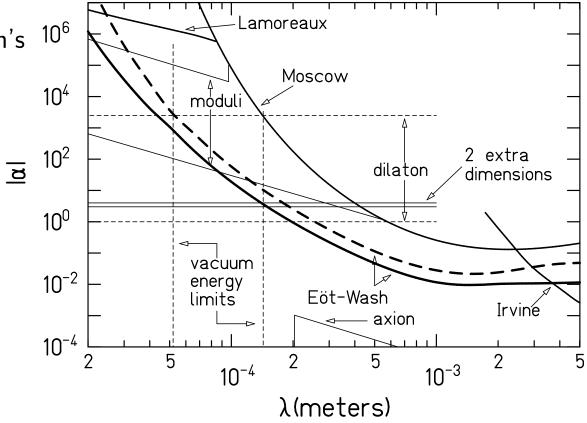
Cavendish-type experiments using torsion pendulum

Parametrize the deviation from Newton's 10^6 potential with an exponential law 10^4

$$V(r) = -\frac{1}{M_{\rm Pl}^2} \frac{m_1 m_2}{r} (1 + \alpha e^{-r/\lambda}) \label{eq:V}$$

For ED compactified on torus:

$$V(r) = -\sim \frac{G_N M}{r} (1 + 2ne^{-r/R_c})$$



 $\alpha = 4 \text{ for 2 ED on torus} \Rightarrow R_c < 0.19 \ mm$

Kaluza-Klein towers

Features of compactified extra-dimensions, due to periodicity condition of fields in extra dimension: $\phi(y + 2n\pi R) = \phi(y)$ where y is extra dimension

Spacing of Kaluza-Klein states can be understood with heuristic considerations Standing waves in box:

- Wavelengths λ such as the size $L\equiv 2\pi R$ of the box is a multiple of λ
- \bullet The wave number k satisfies $k\equiv 2\pi/\lambda=n/R$ with n integer
- Energy is quantized E = hk

Compact dimensions can be assimilated to a finite box.

• Expect in compactified dimension particles with mass spectrum characteristic of standing waves, i.e. quantized in units of 1/R

These oscillations are called Kaluza-Klein modes

Case of a single ED

Standard relativistic formula $E^2 = \mathbf{p}^2 + m_0^2$ reads:

$$E^2 = \mathbf{p}^2 + p_5^2 + m_0^2$$

Where p_5 is momentum in fifth dimension, quantised as $p_5 = hk_5 = nh/R$ Thus in center of mass ($\mathbf{p} = 0$) one obtains the following energy spectrum:

$$E^{2} = \left[m_{0}^{2} + \frac{n^{2}h^{2}}{R^{2}}\right]$$

A 5-dimensions field is identified in 4 dimensions to a tower of particles regularly spaced in mass squared, the gap being the inverse of the compact dimension size \Rightarrow For each field propagating in the bulk, with mass m_0 , if $m_0 \ll 1/R$ in the theory will appear an infinite sequence of states with masses 1/R, 2/R, 3/R..... Study whether, for the different implementations of the model these KK states can be detected at the LHC

ADD phenomenology

Two parameters defining the model: number of ED δ , compactification scale M_D ED compactified with radius R_c connected to δ and M_D by reduction formula

 $M_{\rm Pl}^2 = 8\pi R_c^\delta M_D^{2+\delta}$

For $M_D \sim \text{TeV}$, Extra Dimensions "Large" mm $\leq R_c \leq \text{fm}$ for δ from 2 to 6 EW+strong forces tested down to 10^{-15} mm: SM fields confined on a 3-brane \Rightarrow only gravity probes existence of Extra Dimensions

Gravity propagates in bulk: KK tower of spin-2 graviton fields

- Equally spaced masses with $m_{\vec{n}} = \sqrt{\vec{n}^2/R_c^2}$, where $\vec{n} = (n_1, n_2, ..., n_{\delta})$ labels the KK excitation level
- Coupling to the Standard Model with universal strength $M_{\rm Pl}^{-1}$

Two classes of possible collider signatures: real emission of KK gravitons, virtual graviton exchange

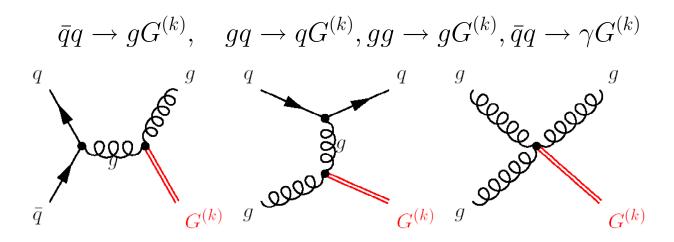
Direct graviton production

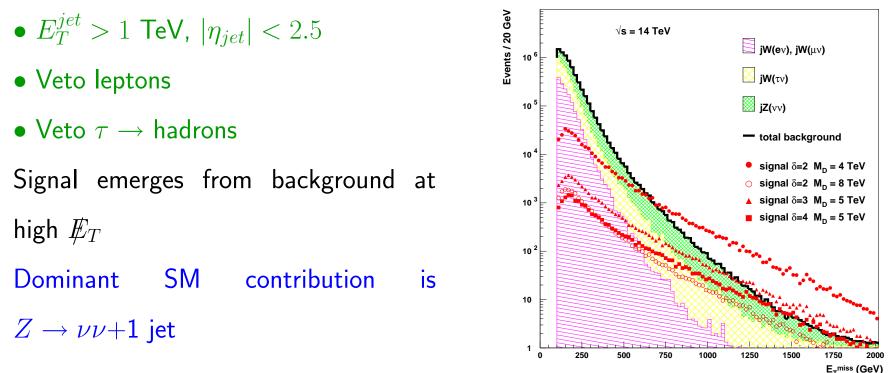
Graviton coupling strength ~ $1/M_p^2$, but large number of accessible KK modes Spacing of modes $1/R_c$, for $\delta = 2$, $M_D = 1$ TeV, $\Delta m_{KK} \ll eV$ In collider process with energy E, $(ER_c)^{\delta}$ massive KK modes accessible: For $\delta = 2$ and E = 1 TeV have 10^{30} modes

Using the reduction formula, sum over all modes exactly cancels $\sim 1/M_p l$ dependence

$$\sigma_{KK} \sim \frac{1}{M_{\rm Pl}^2} (\sqrt{s}R_c)^{\delta} \sim \frac{1}{M_D^2} \left(\frac{\sqrt{s}}{M_D}\right)^{\delta} \ .$$

 \Rightarrow Sizable cross-section for processes:

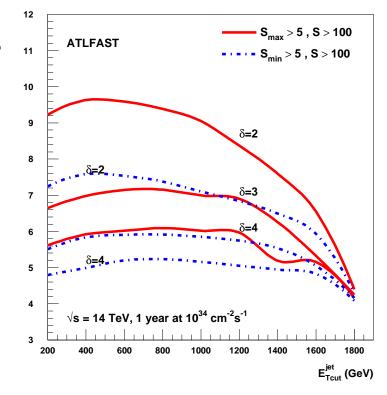




Apply standard cut at $\not\!\!E_T > 1$ TeV to evaluate discovery potential Effective low-energy theory: valid up to M_D . Truncate cross-section when $\hat{s} > M_D^2$

Sensitivity evaluation

M_D^{max}teV) Significance for jG production as a function of E_T^{jet} cut $S_{max} = S/\sqrt{B}$ $S_{min} = S/\sqrt{\alpha B}$ with $\alpha \sim 7$, accounts for the fact that the background calibration sample $Z \rightarrow \ell \ell$ is smaller than the dominant $Z \rightarrow \nu \nu$ background.

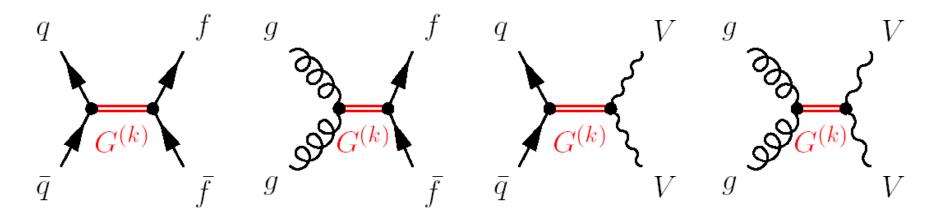


Reach in M_D : $S_{max} > 5$, ≥ 100 signal events, $E_T^{jet} > 1$ TeV $M_D^{max}(\text{TeV}) \mid M_D^{max} \text{ (TeV)} \mid M_d^{min}(\text{TeV})$ δ 100 fb^{-1} 100 fb^{-1} ~ 4 9.1 2 7.7 ~ 4.5 3 6.2 7.0 ~ 5 4 5.2 6.0

 M_d^{min} is M_d below which analysis results not reliable, because high-scale physics affects results

Virtual graviton exchange

Exchange of a virtual gravitons alters the cross section for particle pair production



Most promising channels $pp \to G^* \to \ell^+ \ell^-$, $pp \to G^* \to \gamma \gamma$ Interference with SM: effect parametrized in terms of $\eta = \frac{F}{M_e^4}$

$$\frac{d\sigma_{\eta}}{dMd\cos\theta^{*}} = \frac{d\sigma_{SM}}{dMd\cos\theta^{*}} + \eta f_{int}(M,\cos\theta) + \eta^{2} f_{KK}(M,\cos\theta^{*})$$

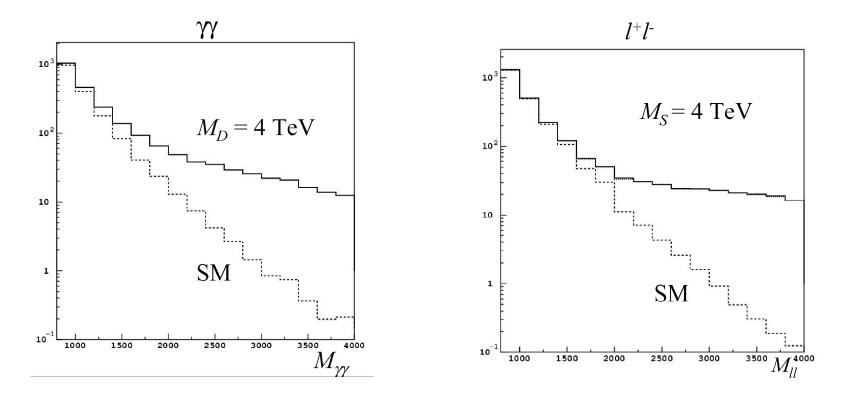
Sum over graviton states divergent, assume cut-off at M_s .

Explicit form of F depends on (unknown) quantum gravity theory, perturbative approach no more valid when $\hat{s} \sim M_s^2$

Gluon-gluon initial state contributes to f_{KK} for leptons

In convention where F = 1, independent from number of extra-dimensions:

Study invariant mass spectrum for both $\ell\ell$ and $\gamma\gamma$, $M_{\gamma\gamma,\ell\ell} < 0.9M_S$ (regularization)



Establish minimal cut on $M_{\gamma\gamma,\ell\ell}$ to optimize sensitivity For 10 (100) fb⁻¹:

 $M_s > 5.1(6.7) \text{ TeV} (\gamma \gamma)$ $M_s > 5.4(7.0) \text{ TeV} (\ell \ell)$ $M_s > 5.7(7.4) \text{ TeV} (combined)$ Reach depends crucially on systematic control of $m_{\gamma\gamma}$ and $m_{\ell\ell}$ at high masses

TeV^{-1} Extra Dimensions

Standard ADD model:

EW precision measurement test SM gauge fields to distances $\sim 1/\text{TeV} \Rightarrow$ SM fields can not propagate in "Large" ED and are localized on a brane Variation on the model: "asymmetric" models where different ED have different compactification radii. Two types of ED:

- "large" ED where only gravity propagates
- ullet "small" ($R\sim 1/$ TeV) extra dimensions where both gravity and SM fields propagate

This scheme could be pictured as a "thick" brane in side wihci SM fields propagate, immersed in the usual "large" ADD bulk

Various models, depending on which SM fields propagate in the bulk:

- Only gauge fields: describe it today
- Both fermion and gauge fields (UED)

General signature for models with compactified ED: regularly spaced Kaluza Klein excitations of fields propagating in the bulk

KK mass spectra and couplings given by compactification scheme and number of ED In case of one "small" ED with radius $R_c \equiv 1/M_c$:

• Excitations equally spaced with masses:

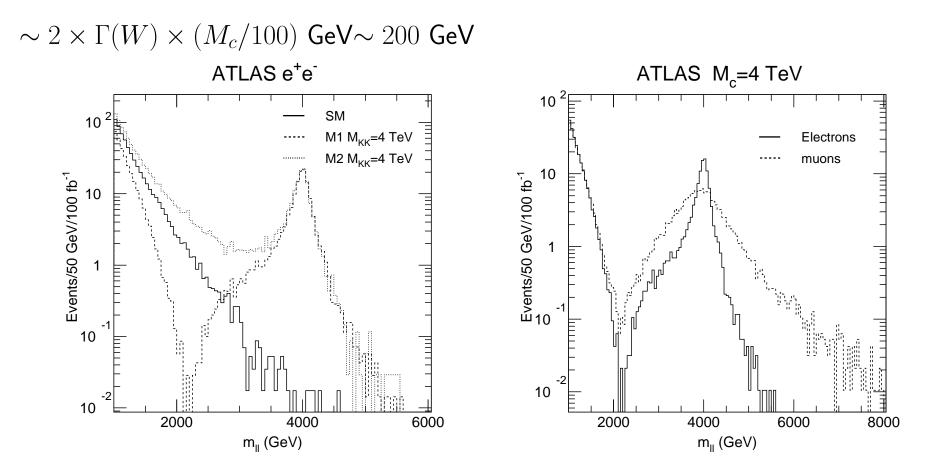
$$M_n^2 = M_0^2 + n^2 M_c^2$$

 \bullet Couplings equal to $\sqrt{2}\times$ gauge couplings

Minimum excitation mass compatible with EW precision measurement: 4 TeV Consider excitations for all SM bosons:

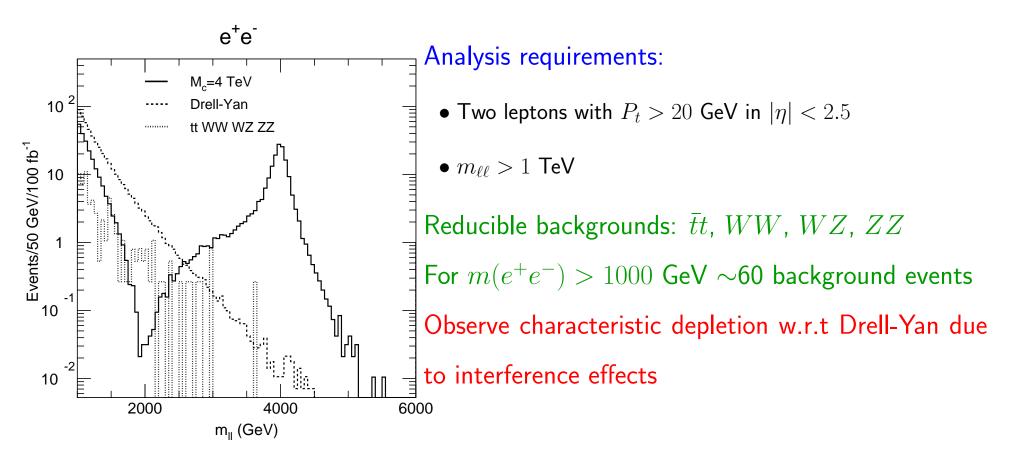
- Z/γ , discovery channel: decay into $\ell^+\ell^-$
- $\bullet~W$, discovery channel: decay into $\ell\nu$
- \bullet gluon, width $\sim 2\alpha_s M(g^{(n)}),$ difficult to observe above QCD background

Minimum excitation mass considered: 4 TeV: natural width



Natural width dominates for e^+e^- . Detailed knowledge of electron resolution not needed as long as $\sigma(E)/E$ better better than 2-3%. Experimental width dominates for $\mu^+\mu^- \Rightarrow$ use muons only for discovery, not for measurements

Data analysis: Z/γ



Resonance includes excitation of both γ and Z, two resonances can not be resolved Evaluate number of events in peak as a function of mass of first excitation (M_{kk}) Require: $S/\sqrt{B} > 5$ and > 10 events in peak, summed over two lepton flavours Reach for 100 fb⁻¹: ~ 5.8 TeV

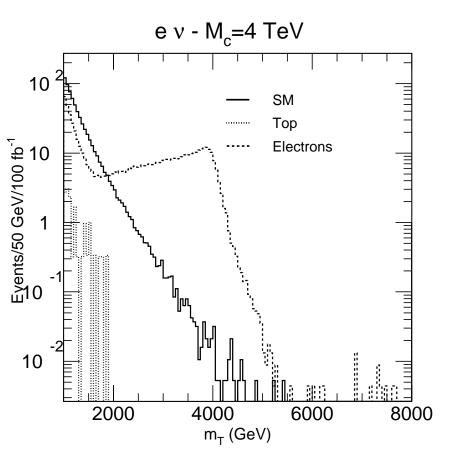
In no case second KK peak observable

Data analysis: W

Analysis requirements:

- One lepton with $P_t > 200 \text{ GeV}$ in $|\eta| < 2.5$
- $\not\!\!\!E_T > 200 \text{ GeV}$
- $m_T(\ell\nu)) > 1 \text{ TeV}$

Where $m_T = \sqrt{2p_T^\ell p_T^\nu (1 - \cos \Delta \phi)}$ If no new physics 500 events from off-shell SM W (100 fb⁻¹)



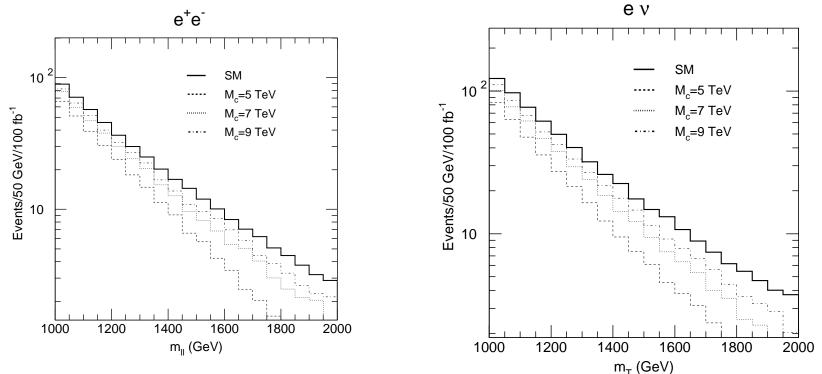
Reducible backgrounds considered: $\bar{t}t, WW, ZZ$

For $m_T(\ell\nu) > 1$ TeV ~75 background events, dominated by WW and WZWith moderate jet veto at 100 GeV, background reduced to ~ 20 events, but bias for study of Jacobian shape

Reach for 100 fb⁻¹: ~ 5.8 TeV

Even if no events in peak, can observe depletion in invariant (transverse) mass

distribution off-peak



Count events with respectively: $1000 < M_{\ell\ell} < 2500 \text{ GeV} (Z/\gamma)$

 $1000 < M_T < 2500 \text{ GeV} (W)$

Require: $(N(M_c) - N(SM))/\sqrt{N(SM)} > 5$ (two lepton flavours)

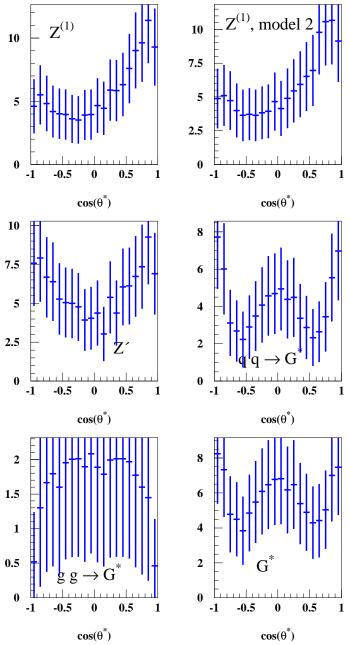
Reach for 100 fb⁻¹: ~ 8 TeV for Z/γ , ~ 9 TeV for W

Deviation from SM at sensitivity limit: $\sim 15\% \Rightarrow$ need systematic control on DY

If $Z^{(1)}/\gamma^{(1)}$ observed, study distribution of polar angle $\cos \theta^*$ for $M(Z^{(1)}) = 4$ TeV and different models:

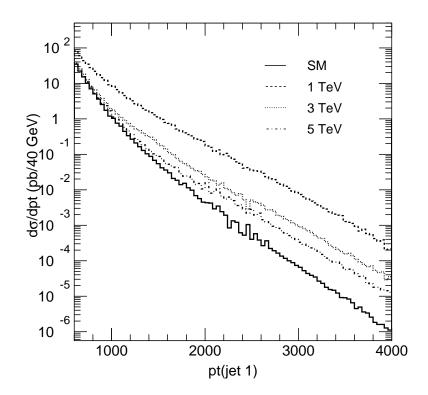
- Alternative $Z^{(1)}$ model
- Z' model with Standard Model couplings
- \bullet Graviton exchange with $G^* \to e^+ e^-$

Trough Kolmogorov test study discrimination power: Reject Z' hypothesis at 95% CL in 52% of cases Reject $G^* \rightarrow e^+e^-$ hypothesis at 95% CL in 94% of cases



KK excitations of the gluon

Require no third jet with P_T above 100 GeV



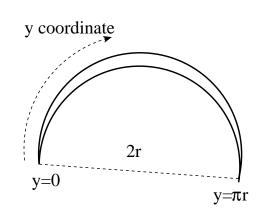
Large deviation from SM QCD spectrum, but Need to understand how well we know jet

 p_T spectrum:

- PDF uncertainites
- NLO corrections
- Detector linearity at high p_T

Also need to study if peak from s-channel g* exchange can be seen above smooth SM+KK background

Difficult due to large width of resonances and complex multi-resonance pattern



Randall-Sundrum model

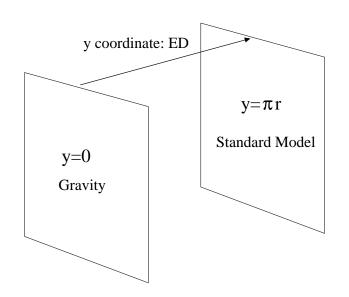
One additional dimension in which gravity propagates ED compactified on S^1/Z_2 (circle folded on itself \equiv orbifold)

Two branes at extremal values of compactification:

- Planck brane: y=0, where gravity localized
- Tev-brane where SM fields (us) constrained

Metric for this scenario is non-factorizable:

$$ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2 \,,$$



Exponential term: "warp factor". Parameter k of order Planck scale governs curvature of space

(1)

Consistency of low energy theory: $k/\overline{M}_{\text{Pl}} \lesssim 0.1$ with $\overline{M}_{\text{Pl}} = M_{\text{Pl}}/\sqrt{8\pi} = 2.4 \times 10^{18}$ being the reduced 4-d Planck scale. Write action for gravitational field in 4-d effective theory (like it was done for ADD), obtain form for 5-dim fundamental scale \overline{M}_5

$$\overline{M}_{\mathsf{Pl}}^2 = \frac{\overline{M}_5^3}{k} \tag{2}$$

Scale of all physical processes on the TeV brane described by:

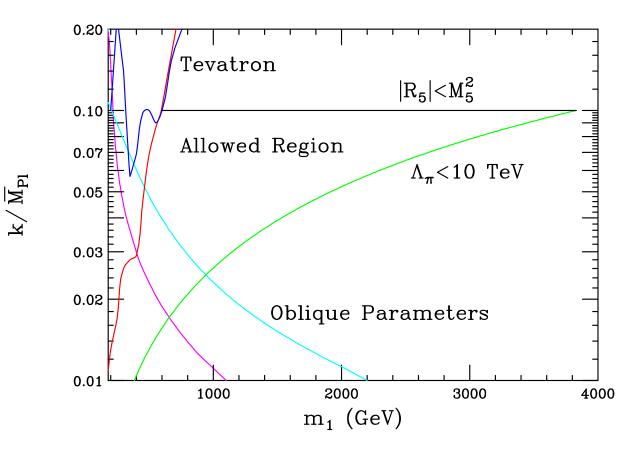
$$\Lambda_{\pi} \equiv \overline{M}_{\mathsf{PI}} e^{-kR_c\pi}$$

 $\Lambda_{\pi} \sim 1 \text{TeV}$ provided than kR = 10.

Two parameters define the model:

- Λ_{π}
- ratio k/\overline{M}_{Pl}

If require $\Lambda_{\pi} < 10$ TeV (hierarchy) closed region in parameter space $(m_1 = 3.83 \frac{k}{M_{Pl}} \Lambda_{\pi})$



Randall-Sundrum: Narrow graviton states

Masses of KK graviton obtained from Bessel expansion, replacing Fourier expansion of flat geometry

Mass m_n of excitation $G^{(n)}$ at:

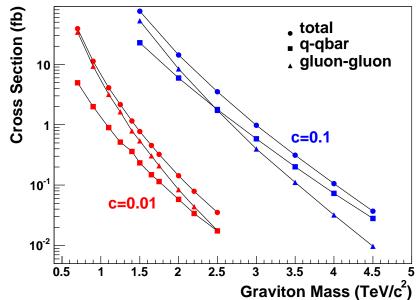
$$m_n = x_n k e^{-k\pi r_c} = x_n \frac{k}{M_{Pl}} \Lambda_\pi$$

where x_n are the roots of the first order Bessel function. $x_1 = 3.83 \Rightarrow \sim TeV$ scale for mass of first excitation

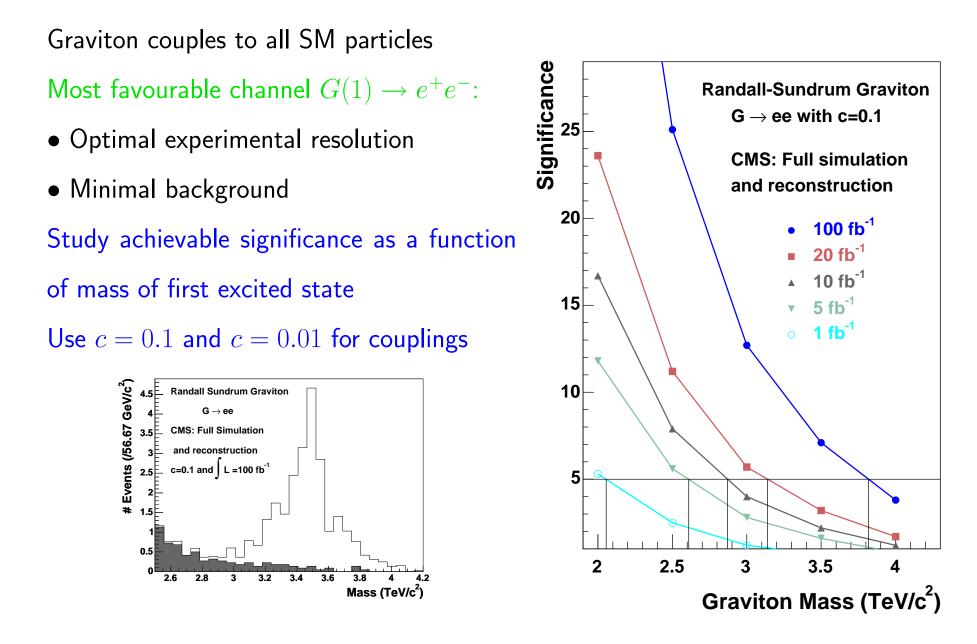
Couplings of $G^{(n)}$ to SM fields $\sim 1/\Lambda_{\pi} \Rightarrow$

- sizable cross-section at the LHC
- Narrow resonances

Coupling driven by factor $c = k/M_{Pl}$

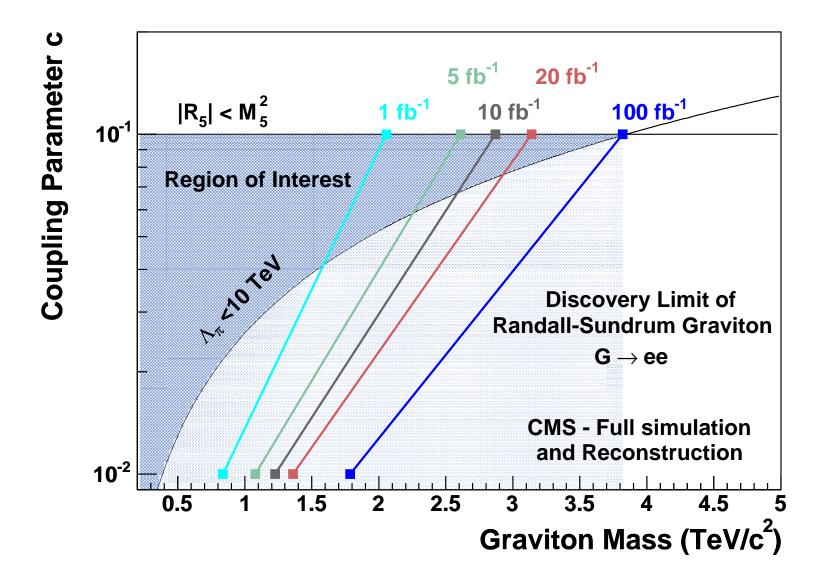


$G(1) \rightarrow e^+e^-$ in CMS (full simulation)



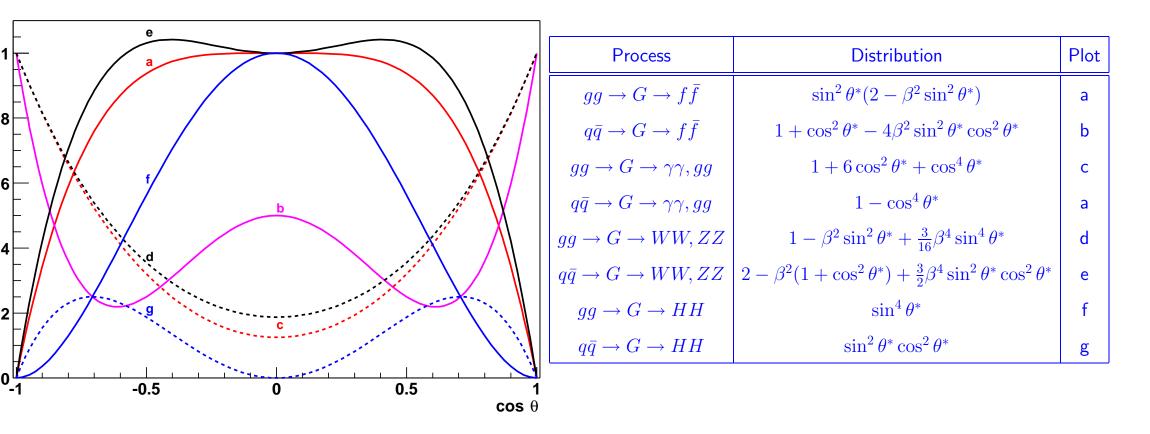
Coverage of parameter space

With one year at the LHC (high lumi) full coverage of parameter space



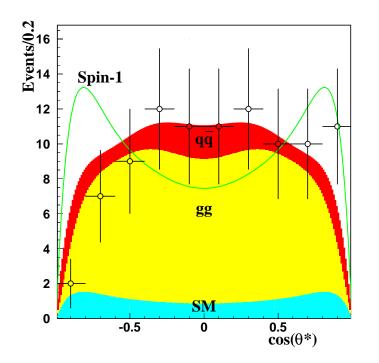
Spin determination of graviton resonance

Graviton is spin-2 particle. Angular distribution of decay products depends on production mechanism, and on spin and mass of decay products

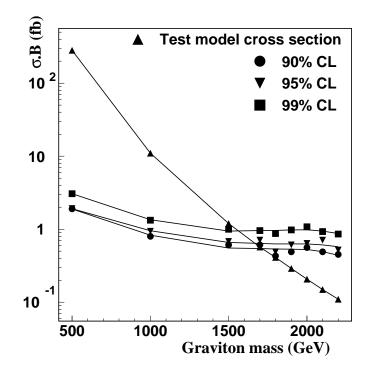


 β is v/c of decay products

Gluon fusion dominates, contribution from $\bar{q}q$ flattens distribution



Polar angle distribution of e^+e^- after the acceptance cuts are applied For $m_1 = 1500$ GeV and 100 fb⁻¹ can distinguish from spin 1 case



Test spin hypotheses with a likelihood technique

Spin-1 hypothesis can be ruled out at 90% CL up to $m_1 = 1720 \text{ GeV}$

Black Holes

Geometrical semi-classical reasoning:

Possibility of black hole formation when two colliding partons have impact parameter smaller than the radius of a black hole

Consider two colliding partons with CMS energy $\sqrt{\hat{s}} = M_{\text{BH}}$ Dimensional analysis: partonic X-section for formation of black hole of mass M_{BH} is

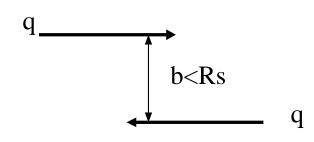
$$\sigma(\hat{s} = M_{BH}^2) \sim \pi R_s^2$$

Where R_S is Schwarzchild radius of black hole

$$R_S \sim \frac{1}{\sqrt{\pi}M_P} \left[\frac{M_{\rm BH}}{M_P}\right]^{\frac{1}{n+1}}$$

In extra-dimension theories $M_P \sim \text{Tev} \Rightarrow$, for $M_{\text{BH}} \sim M_P$, $\sigma \sim (\text{T}eV)^{-2} \sim 400 \text{ pb}$ Potentially large production cross-section

Theoretical debate on geometrical formation factors. Possible big suppression

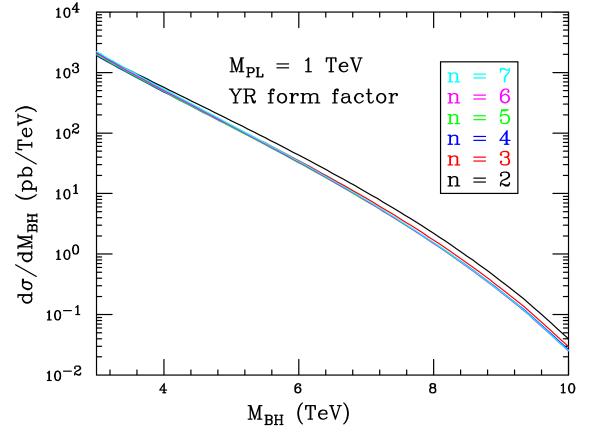


Black Hole production

Convolve the parton-level cross-section with parton distribution functions

For n > 2 dimensions little dependence on n because of assumed form of formation

factor in CHARYBDIS generator (Cambridge group)



At high luminosity, > 1 black hole per second with $M_BH > 5$ TeV

Black Hole decay

Decay through Hawking radiation

Details of decay extremely model-dependent.

Simplifying assumptions: all partonic energy goes into BH formation, all Hawking radiation through SM Particles on the brane

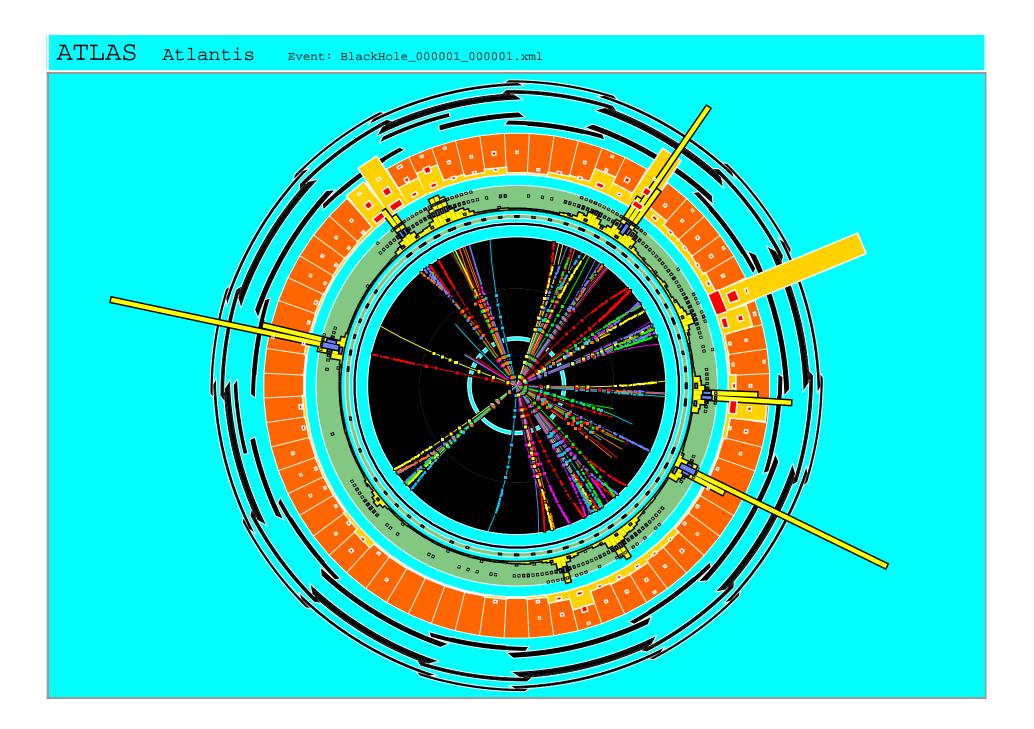
Thermal radiation: black body energy spectrum

$$\frac{dN}{dE} \propto \frac{\gamma E^2}{(e^{E/T_{\rm H}} \pm 1)} T_{\rm \tiny H}^{n+6} \tag{3}$$

 \pm applies to fermions and bosons, $T_{\scriptscriptstyle\rm H}$ is the Hawking temperature

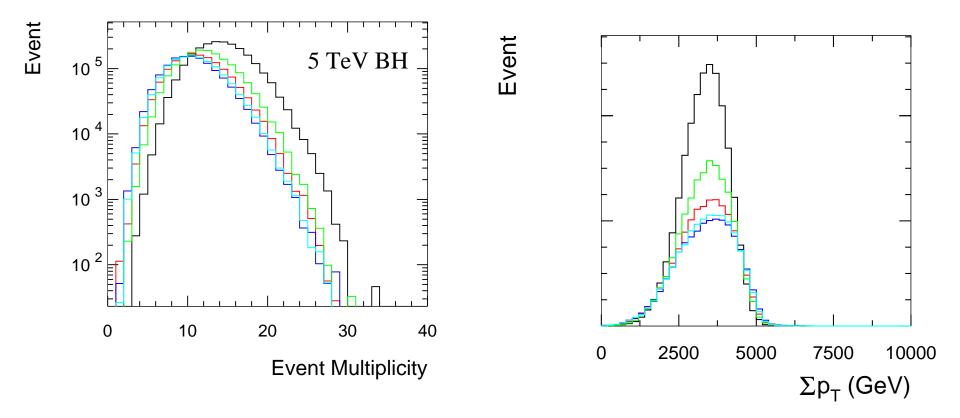
$$T_{\rm H} = \frac{n+1}{4\pi r_{\rm s}} \propto M_{\rm \tiny BH}^{-\frac{1}{n+1}}$$
(4)

 γ is a $(4+n)\mbox{-dimensional}$ grey-body factor: absorption factor from propagation in curved space



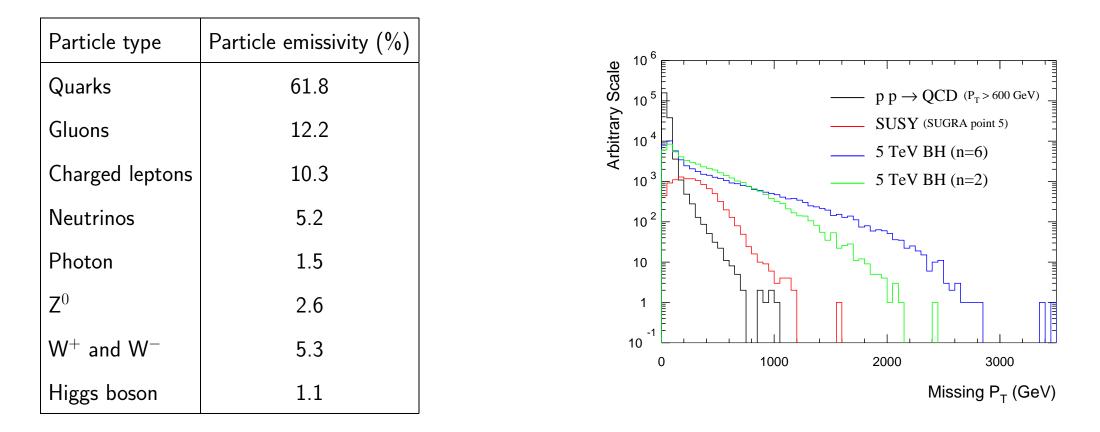
Event characteristics of BH decays

- From integrating flux: large multiplicities of particles in final state
- Hawking decay isotropic: spherical events (more spherical than SUSY)
- High mass: High Σp_T of final state particles



No significant SM background !

Democratic decay of BH into all types of SM particles Large number of events containing a high- P_T neutrino $\not\!\!E_T$ distribution even in excess of SUSY

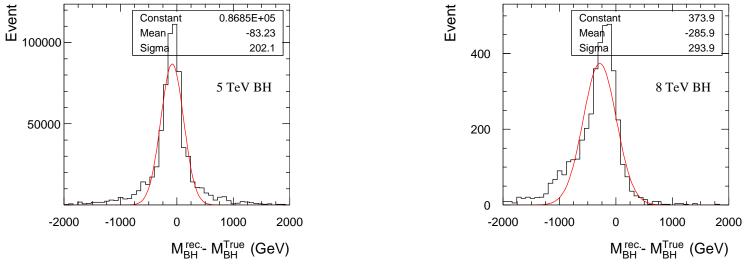


Also large production of gauge bosons and higgses, BH decay even be privileged production mode for higgs boson

Black hole mass measurement

Simply sum the 4-momentum of all reconstructed particles in the event Test procedure on two BH mass ranges around 5 and 8 TeV for n between 2 and 6 Require at least 4 jets with respectively PT > 500, 400, 300 GeV

Efficiency between 15 and 30% depending on mass and n



Achieve mass resolution of 3-5%

 m_{BH} correlated to T_H , but large theoretical uncertainties

Useful benchmark process for study of high multiplicities and energies in the detector Vigorous full simultion effort ongoing in ATLAS to verify these results

Conclusions

Extra Dimension theories offer an attractive way of solving the hierarchy problem basedon the space-time geometry of space

The presence of fields propagating in the extra-dimensions produces Kaluza Klein towers of particles

The mass scale of the lowest lying of the KK towers is typically approximately in th range of LHC

The details of the KK fields depend on the specific model implementation. For the main models available, detailed studies performed to test the LHC potential In general the LHC will be sensitive to the nex phenomenologies arising from ED theories for scales up to a few TeV