Twistor inspired developments in perturbative QCD

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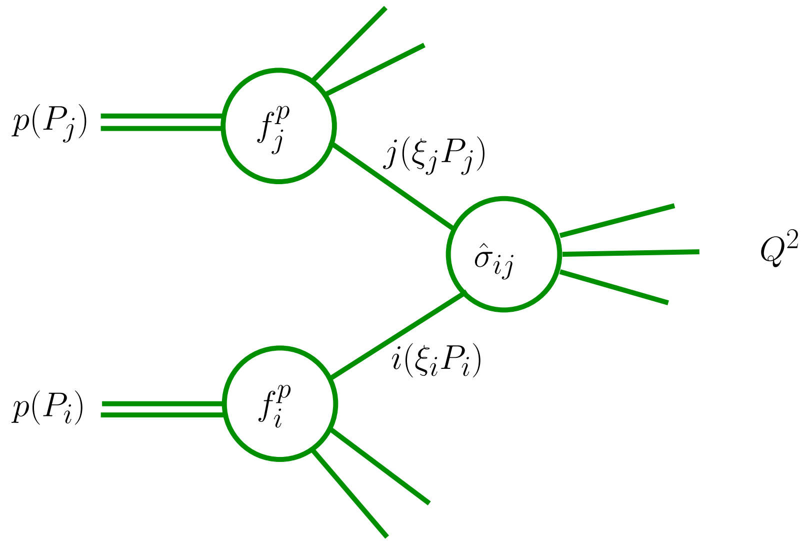
Hard processes in Hadron-Hadron collisions

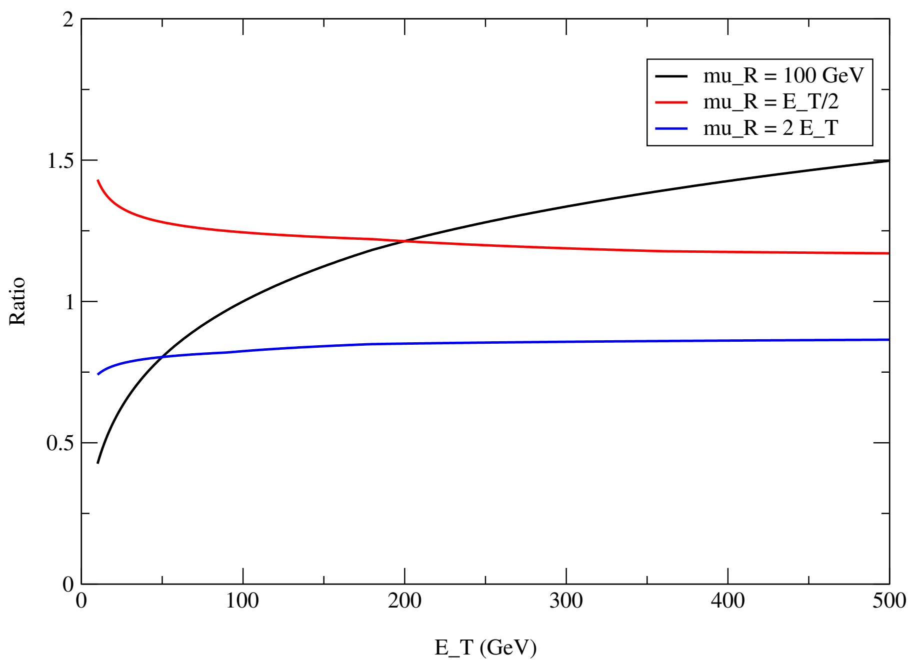


$$\sigma(Q^2) = \sum_{i,j} \left[\hat\sigma_{ij}(\alpha_s(\mu^2), \mu_R^2/Q^2, \mu_F^2/Q^2) \otimes f_i^p(\mu_F^2) \otimes f_j^p(\mu_F^2)\right]$$

- partonic cross sections $\hat\sigma_{ij}$
- parton distributions f_i
- renormalization/factorization scale μ_R/μ_F
- + parton shower + hadronisation model

The unphysical scales - μ_R

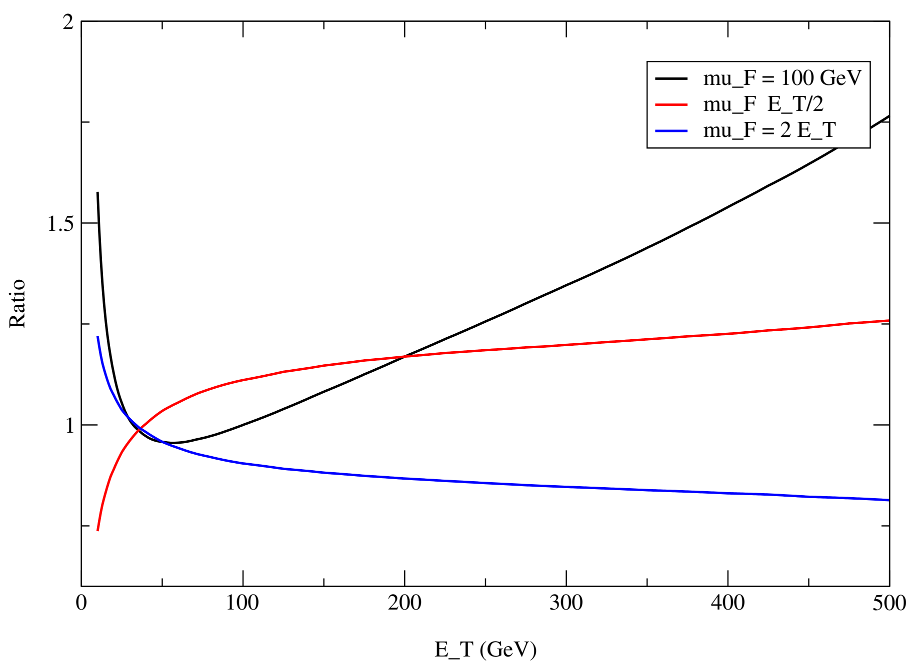
The renormalisation scale μ_R is introduced when redefining the bare fields in terms of the physical fields at scale μ_R. It is unphysical - and the answer shouldnt depend on it - but does because we work at a fixed order in perturbation theory. Therefore, you can choose any value (within reason). Typical values are the hard scale in the process $\mu_R \sim E_T$.



Example: $pp \to \text{jet} + X$ at LO
α_s^2 for various values of μ_R compared to $\mu_R = E_T$

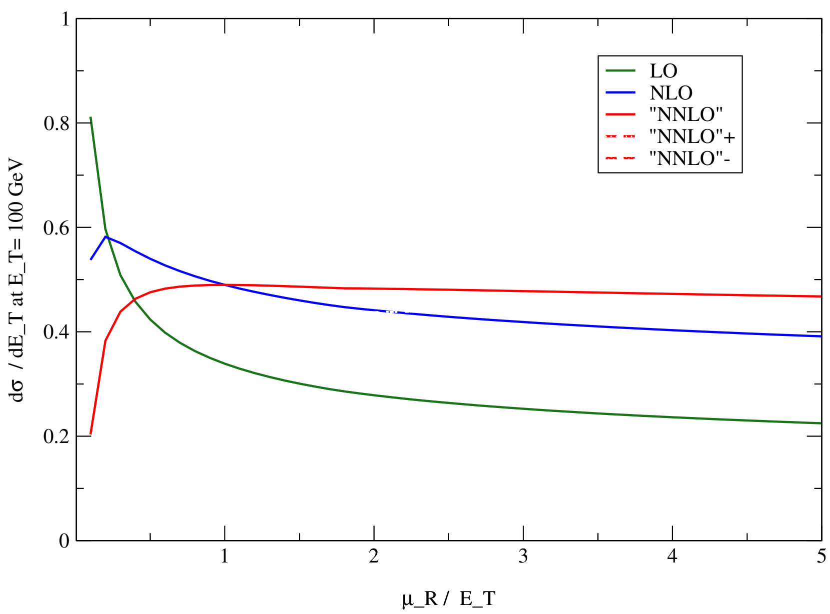
The unphysical scales - μ_F

The factorisation scale μ_F is introduced when absorbing the divergence from collinear radiation into the parton densities. It is unphysical - and the answer shouldnt depend on it - but does because we work at a fixed order in perturbation theory. Typically we think of radiation at a transverse energy $> \mu_F$ as being detectable so that $\mu_F \sim E_T$ is a reasonable choice.



Example: $pp \rightarrow \text{jet} + X$ at LO
The effective parton-parton luminosities for various values of μ_F compared to $\mu_F = E_T$ at $\eta_1 = \eta_2 = 0$

Unphysical scale dependence



- typically, NLO reduces scale uncertainty by factor 2 over LO
- ✓ maybe to $\pm 30\%$
- typically, NNLO reduces scale uncertainty by factor 2 over NLO
- ✓ maybe $\pm 10\%$
- ✗ won't know till you do it

✓ plus many other improvements in modelling hard scattering at NNLO

State of the Art

Relative Order	$2 \to 1$	$2 \to 2$	$2 \to 3$	$2 \to 4$	$2 \to 5$	$2 \to 6$
1	LO					
α_s	NLO	LO				
α_s^2	NNLO	NLO	LO			
α_s^3		NNLO	NLO	LO		
α_s^4				NLO	LO	
α_s^5					NLO	LO

LO
- ✓ matrix elements automatically generated up to $2 \to 8$ or more
- ✓ plus automatic integration over phase space HELAC/PHEGAS, MADGRAPH/MADEVENT, SHERPA/AMEGIC++, COMPHEP, GRACE, ...
- ✓ able to interface with parton showers - CKKW
- ✓ very good for estimating importance of various processes in different models - properly populate phase space with multiple hard objects
- ✗ rate very dependent on choice of renormalisation/factorisation scales

State of the Art

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1	LO					
α_s	NLO	LO				
α_s^2	NNLO	NLO	LO			
α_s^3		NNLO	NLO	LO		
α_s^4				NLO	LO	
α_s^5					NLO	LO

NLO
- ✓ parton level integrators available for most $2 \to 2$ Standard Model and MSSM processes for some time
- ✓ extensively used at LEP, TEVATRON and HERA
  EVENT, JETRAD, MCFM, DISENT, etc
- ✓ reduced renormalisation scale uncertainty
- ✓ can be matched with parton shower MC@NLO – Frixione, Webber

State of the Art

Relative Order	$2 \to 1$	$2 \to 2$	$2 \to 3$	$2 \to 4$	$2 \to 5$	$2 \to 6$
1	LO					
α_s	NLO	LO				
α_s^2	NNLO	NLO	LO			
α_s^3		NNLO	NLO	LO		
α_s^4				NLO	LO	
α_s^5					NLO	LO

NLO ✓ some $2 \to 3$ processes available at NLO
e.g. backgrounds $pp \to 3$ jets, $V + 2$ jets, $\gamma\gamma$ + jet, $V + b\bar{b}$
as well as signals $pp \to t\bar{t}H$, $b\bar{b}H$, qqH, HHH, $t\bar{t}j$

✗ many still missing VV + jet, $t\bar{t}+$ jet, etc

✗ understood how to do, but tedious and painstaking

State of the Art

Relative Order	$2 \to 1$	$2 \to 2$	$2 \to 3$	$2 \to 4$	$2 \to 5$	$2 \to 6$
1	LO					
α_s	NLO	LO				
α_s^2	NNLO	NLO	LO			
α_s^3		NNLO	NLO	LO		
α_s^4				NLO	LO	
α_s^5					NLO	LO

NLO ✗ no $2 \to 4$ LHC cross sections known

✗ need to extend range of available calculations to e.g. $pp \to W+$multijets that are backgrounds to New Physics

- ✓ 4 gluons@one-loop, Ellis, Sexton, 1986, σ_{2j}, 1992
- ✓ 5 gluons@one-loop, Bern, Dixon, Kosower, 1993, σ_{3j}, 2000
- ✓ 6 gluons@one-loop, many authors, 2006 σ_{4j}, 20??

✗ need a more efficient way of evaluating loop contributions and constructing σ

How to calculate scattering amplitudes

1. Off-shell methods

   Traditional Feynman diagram approach

2. On-shell methods

   Based on S-matrix ideas of 1960's but recently inspired by Witten's proposal to relate perturbative gauge theory amplitudes to topological string theory in twistor space

   Witten, hep-th/0312171

   $\Rightarrow$ new ways to calculate amplitudes in massless gauge theories:

Off-shell methods

Traditional Feynman diagram approach for off-shell Greens functions

- ✓ Direct link to Lagrangian
- ✓ Easy to adapt to any model
- ✓ Easy to include massive particles with/without spin
- ✓ Easy to automate
  $\Rightarrow$ tree-level packages Madgraph/Grace/CompHep/...
- ✓ Off-shell Berends-Giele recursion relations
  $\Rightarrow$ tree-level packages Alpgen/HELAC/PHEGAS/...
- ✗ Many Feynman diagrams
- ✗ Large cancellations between diagrams
- ✗ Loop amplitudes manpower intensive

Example

Multi-jet production at the LHC using HELAC/PHEGAS

Draggiotis, Kleiss, Papadopoloulos

# of jets	2	3	4	5	6	7	8
# of dist.processes	10	14	28	36	64	78	130
total # of processes	126	206	621	861	1862	2326	4342
$\sigma(nb)$	-	91.41	6.54	0.458	0.030	0.0022	0.00021
$\%$ Gluonic	-	45.7	39.2	35.7	35.1	33.8	26.6

- The number of Feynman diagrams for an n gluon process increases very quickly with n

$\Rightarrow$ for the 10 gluon amplitude there are 10,525,900 diagrams

$\Rightarrow$ Feynman diagrams very inefficient for many legs

- Control the quantum numbers of the scattering particles

On-shell methods

✓ New (and puzzling) insights into field theory amplitudes
$\Rightarrow$ new ways to calculate amplitudes in massless gauge theories:

- ✓ MHV rules

  Cachazo, Svrcek and Witten

  $\Rightarrow$ NEW analytic results for some QCD tree amplitudes with any number of legs

- ✓ BCF on-shell recursion relations — Britto, Cachazo and Feng (and Witten)

  $\Rightarrow$ NEW compact results for some multileg QCD tree amplitudes

- ✓ Unitarity and cut-constructibility

  Bern, Dixon, Dunbar, Kosower; Britto, Cachazo and Feng; . . .

  $\Rightarrow$ NEW analytic one-loop amplitudes in massless supersymmetric theories

- ✓ Recursive derivation of rational terms

  Bern, Dixon, Kosower + Berger, Forde; Xiao,Yang, Zhu

  $\Rightarrow$ NEW analytic one-loop amplitudes for multigluon amplitudes

Spinor Helicity Formalism

- In Weyl (chiral) representation, each helicity state is represented by a bi-spinor $(a = 1, 2)$

$$u_+(p) = \lambda_{pa}, \qquad u_-(p) = \tilde{\lambda}_p^{\dot{a}},$$
$$\overline{u_+(p)} = \tilde{\lambda}_{p\dot{a}}, \qquad \overline{u_-(p)} = \lambda_p^a$$

so that

$$\langle ij \rangle = \overline{u_-(p_i)} u_+(p_j) = \lambda_i^a \lambda_{ja} = \epsilon_{ab} \lambda_i^a \lambda_j^b$$
$$[ij] = \overline{u_+(p_i)} u_-(p_j) = \tilde{\lambda}_{i\dot{a}} \tilde{\lambda}_j^{\dot{a}} = -\epsilon_{\dot{a}\dot{b}} \tilde{\lambda}_i^{\dot{a}} \tilde{\lambda}_j^{\dot{b}}$$

- We can write massless vector

$$p_{a\dot{a}} \equiv p_\mu \sigma^\mu_{a\dot{a}} = \lambda_{pa} \tilde{\lambda}_{p\dot{a}}$$

Spinor Helicity Formalism

- Polarisation vectors for particle i:

$$\varepsilon^{-}_{ia\dot{a}} = \frac{\lambda_{ia}\tilde{\eta}_{\dot{a}}}{[\tilde{\lambda}_i\tilde{\eta}]}, \qquad \varepsilon^{+}_{ia\dot{a}} = \frac{\eta_a\tilde{\lambda}_{i\dot{a}}}{\langle\eta\lambda_i\rangle}$$

- For real momenta in Minkowski space,

$$\tilde{\lambda} = \lambda^*$$

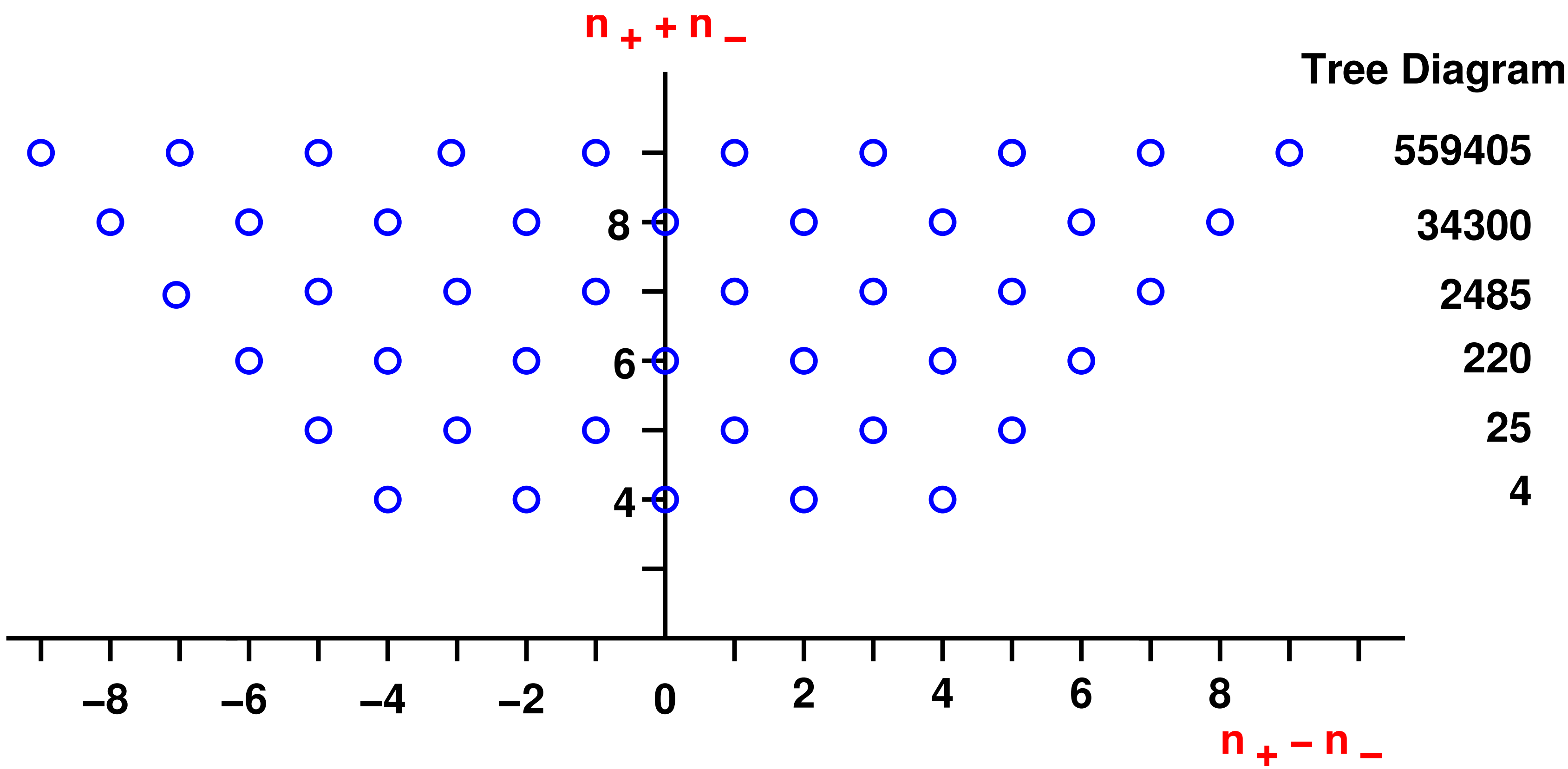
$$\langle ij\rangle^* = -[ij]$$

- For space-time signature $(+,+,-,-)$, $\tilde{\lambda}, \lambda$ are real and independent
- Amplitudes are functions of the λ_i and $\tilde{\lambda}_i$

Gluonic helicity amplitudes

- A gluon has either positive or negative helicity (right-handed or left-handed)
- A multigluon amplitude can be characterised by the helicity of the gluons
- There will n_+ positive helicities and n_- negative helicities.
- The order of helicities matters:
  $- - + + + + + + \ldots$ is not the same as $- + - + + + + + \ldots$ etc.

Gluonic helicity amplitudes



Each row describes scattering with n_+ positive helicities and n_- negative helicities.
Each circle represents an allowed helicity configuration - from all positive on the right to all negative on the left

Gluonic helicity amplitudes

For example, the result of computing the 25 diagrams for the colour-ordered five-gluon process yields

$$A_5(1^{\pm}, 2^{+}, 3^{+}, 4^{+}, 5^{+}) = 0$$

$$A_5(1^{-}, 2^{-}, 3^{+}, 4^{+}, 5^{+}) = \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}$$

In fact, for n point colour-ordered amplitudes,

$$A_n(1^{\pm}, 2^{+}, 3^{+}, , \ldots, n^{+}) = 0$$

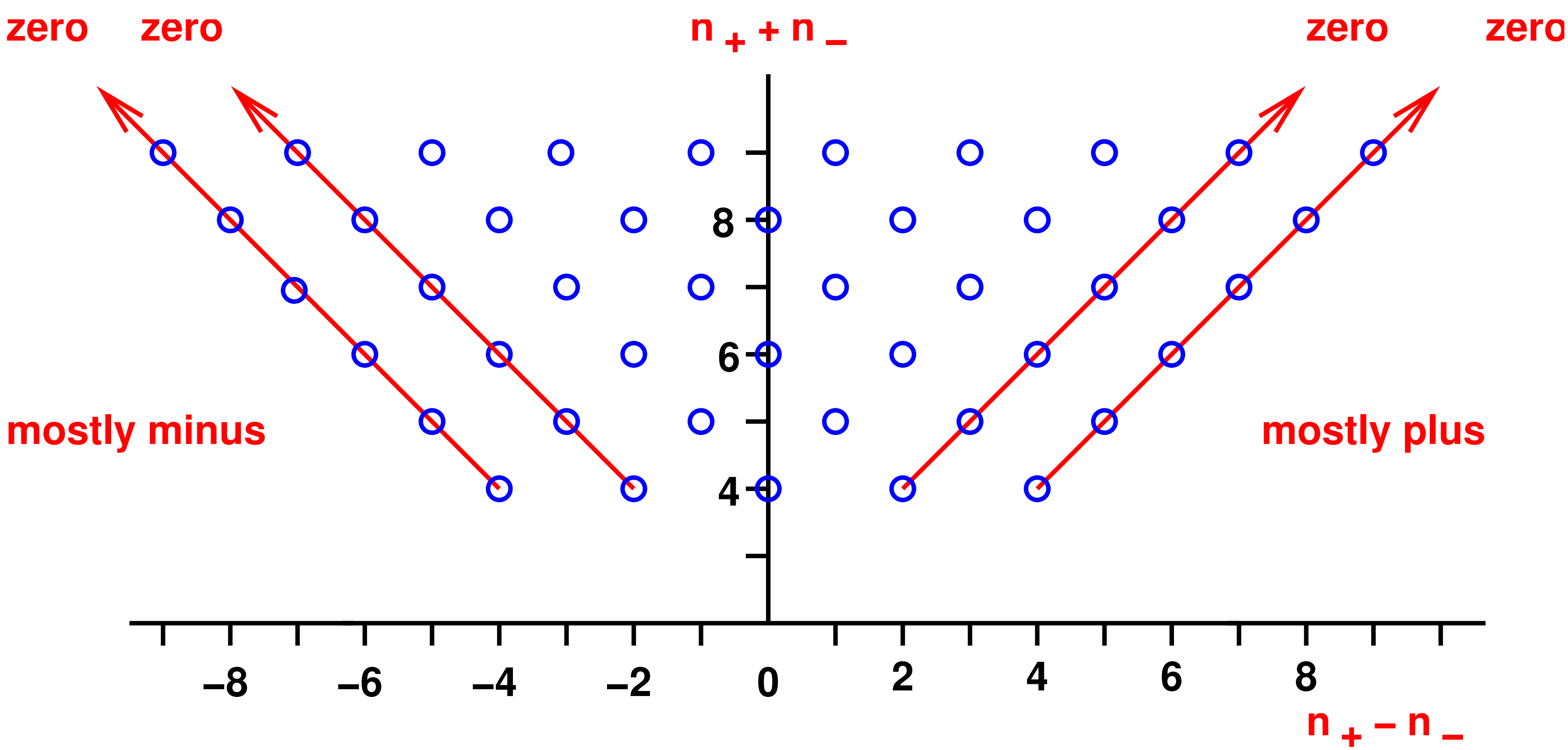
$$A_n(1^{-}, 2^{-}, 3^{+}, \ldots, n^{+}) = \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

$$A_n(1^{-}, 2^{+}, 3^{-}, \ldots, n^{+}) = \frac{\langle 13\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

Maximally helicity violating (MHV) amplitudes

Parke, Taylor; Berends, Giele

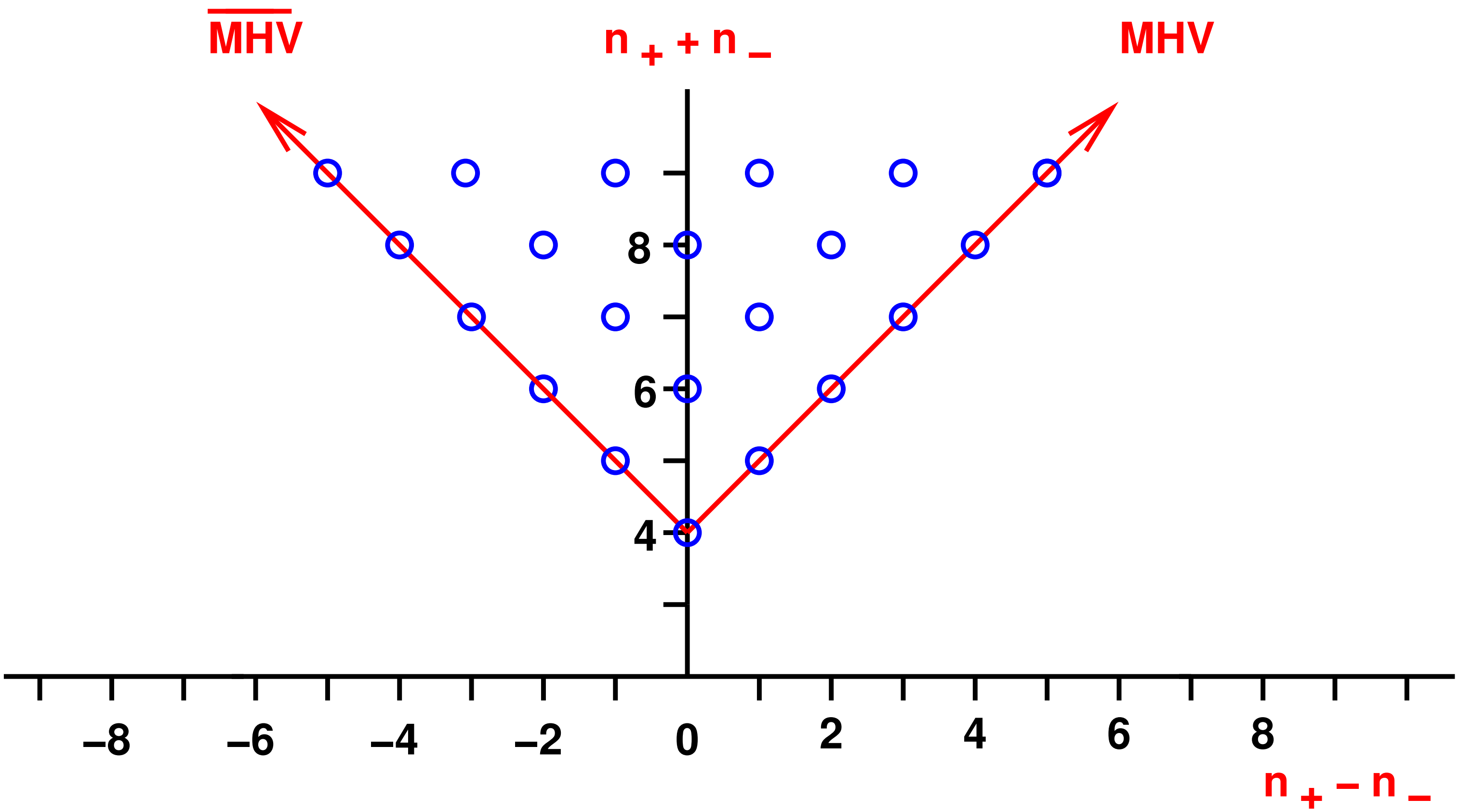
Gluonic helicity amplitudes



$$A_n(1^\pm, 2^+, 3^+, , \ldots, n^+) = 0$$

effective tree-level supersymmetry

Gluonic helicity amplitudes



$$A_n(1^-, 2^-, 3^+, \ldots, n^+) = \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

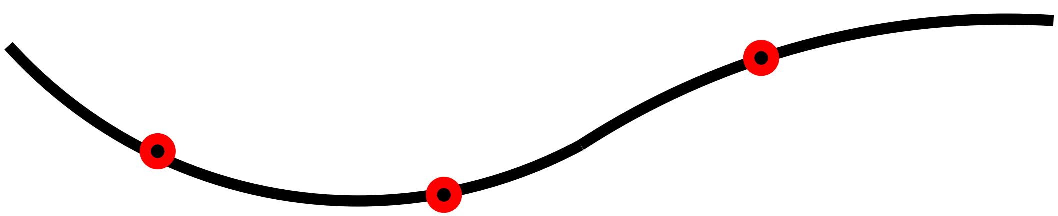
Twistor Space

Witten, hep-th/0312171

Witten observed that in twistor space external points lie on certain algebraic curves

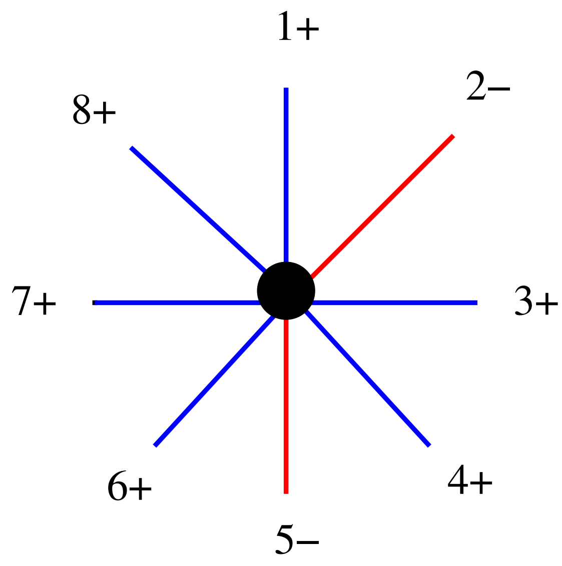
⇒ degree of curve is related to the number of negative helicities and loops

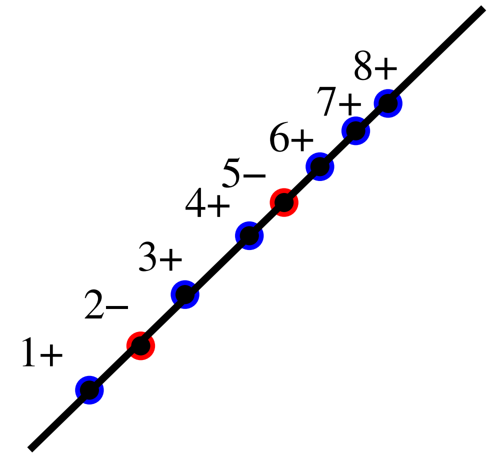
$$d = n_- - 1 + l$$



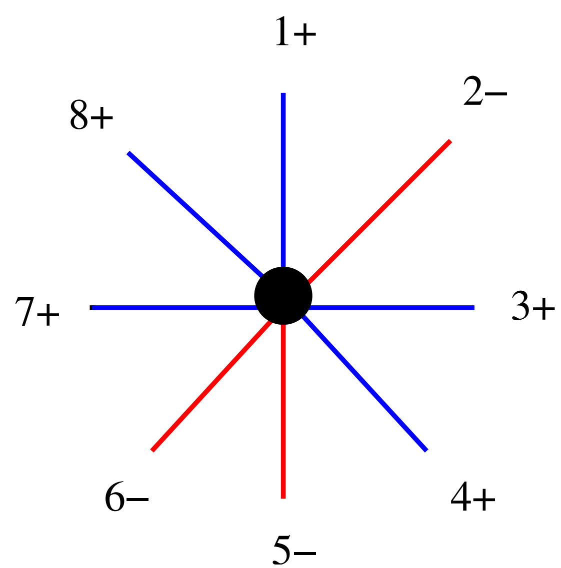
Twistor Space

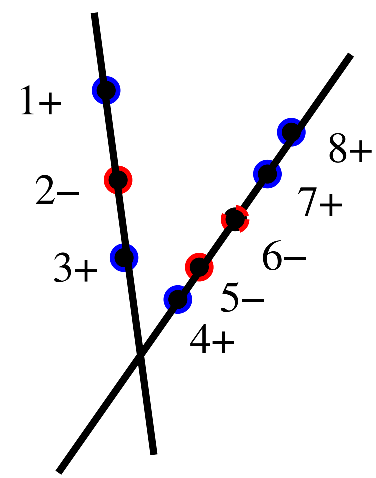
MHV





NMHV



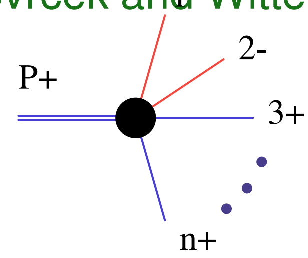


MHV rules

Start from on-shell MHV amplitude and define off-shell vertices

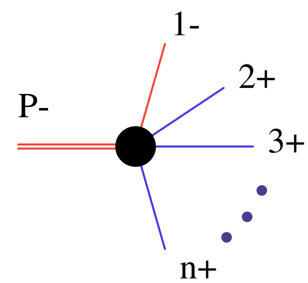
Cachazo, Svrcek and Witten

$$V(1^-,2^-,3^+,\ldots,n^+,P^+) = \frac{\langle 12\rangle^4}{\langle 12\rangle\cdots\langle n-1n\rangle\langle nP\rangle\langle P1\rangle}$$



and

$$V(1^-,2^+,3^+,\ldots,n^+,P^-) = \frac{\langle 1P\rangle^4}{\langle 12\rangle\cdots\langle n-1n\rangle\langle nP\rangle\langle P1\rangle}$$



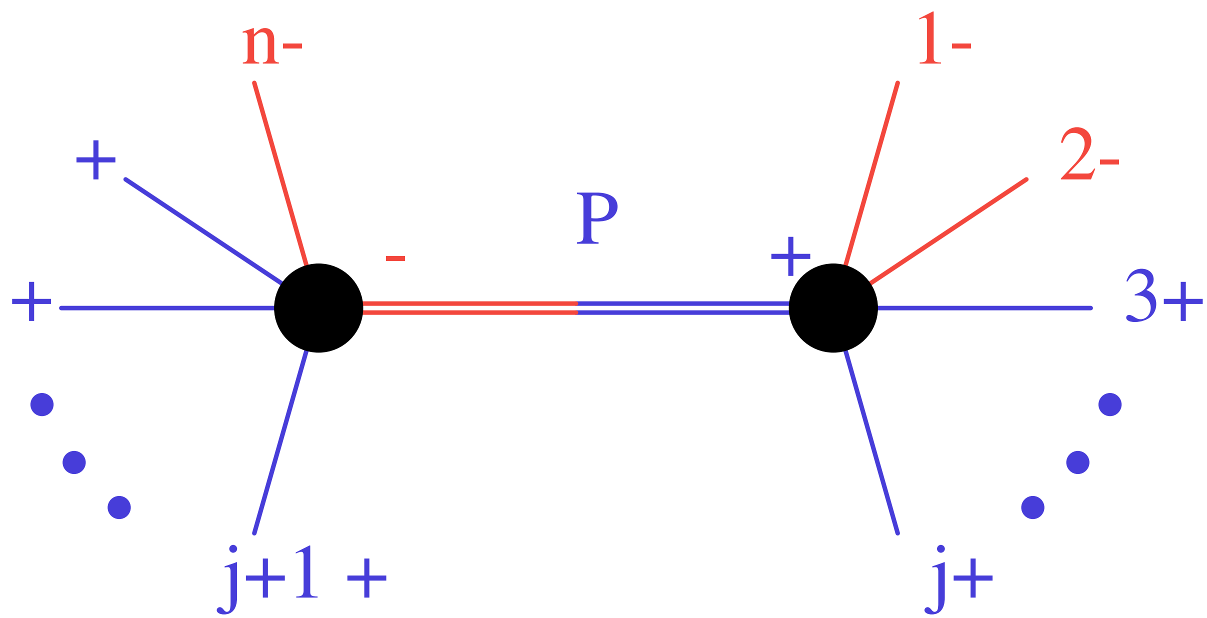
Crucial step is off-shell continuation $P^2 \neq 0$:

$$\langle iP\rangle = \frac{\langle i^-|\not{P}|\eta^-]}{[P\eta]} = \sum_j \frac{\langle i^-|\not{j}|\eta^-]}{[P\eta]}$$

where $P = \sum_j j$ and η is lightlike auxiliary vector

MHV rules

Must connect up a positive helicity off-shell line to a negative helicity off-shell line with a scalar propagator



Connecting two MHV's $\Rightarrow$ amplitude with 3 negative helicities
Connecting three MHV's $\Rightarrow$ amplitude with 4 negative helicities
etc.

Example: six gluon scattering

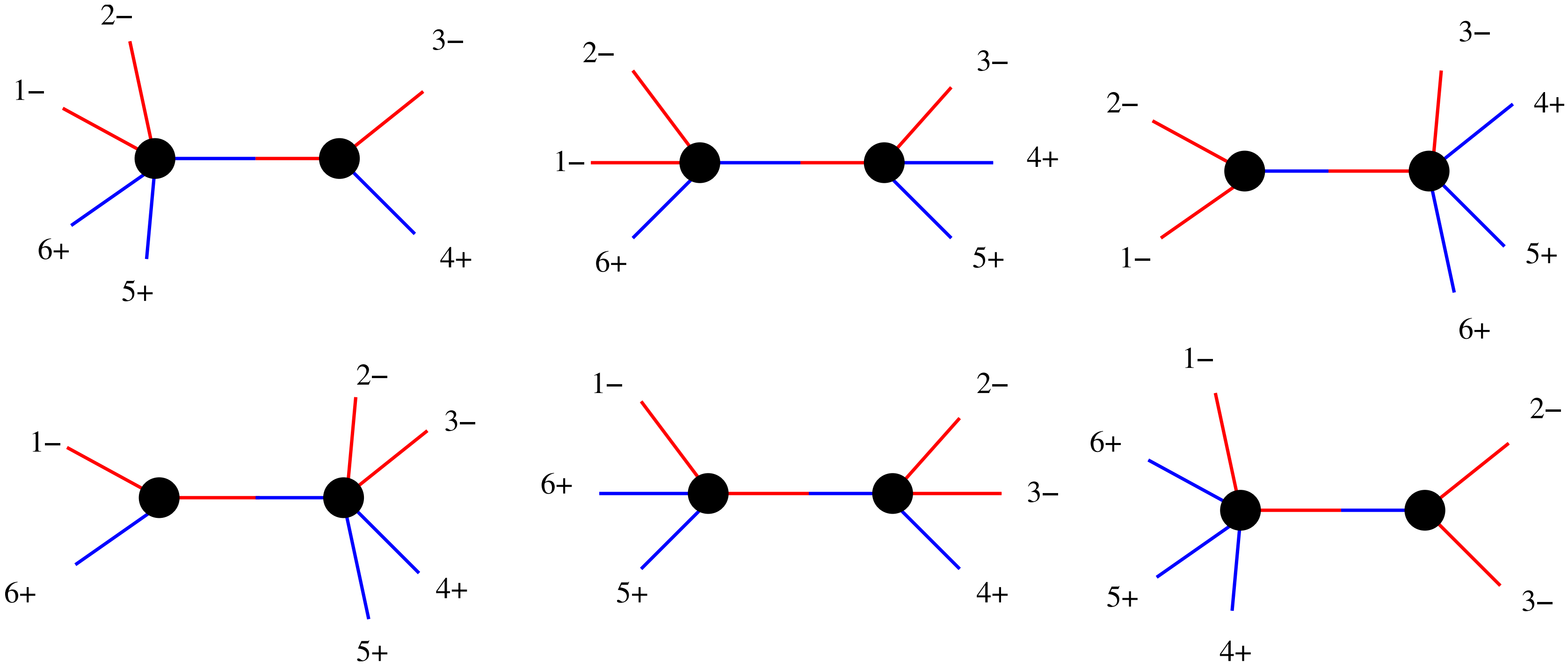
As an example, lets use the MHV rules to calculate one of the first non-MHV amplitudes

$$A_6(1^-, 2^-, 3^-, 4^+, 5^+, 6^+)$$

Step 1 Draw all the allowed MHV diagrams

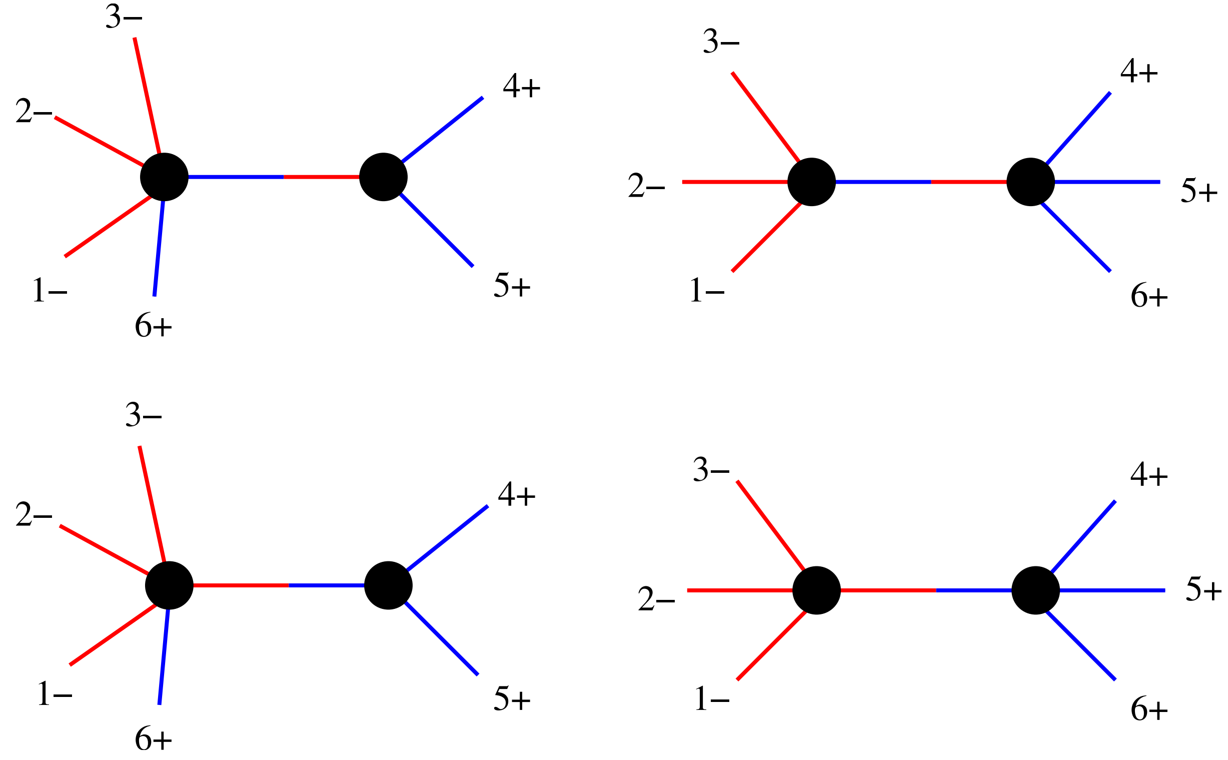
Example: six gluon scattering

There are six MHV graphs



Example: six gluon scattering

Some graphs are not allowed e.g.



Example: six gluon scattering

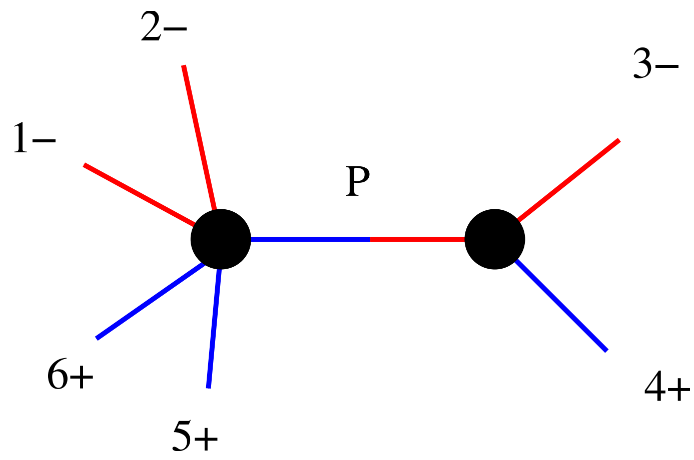
As an example, lets use the MHV rules to calculate one of the first non-MHV amplitudes

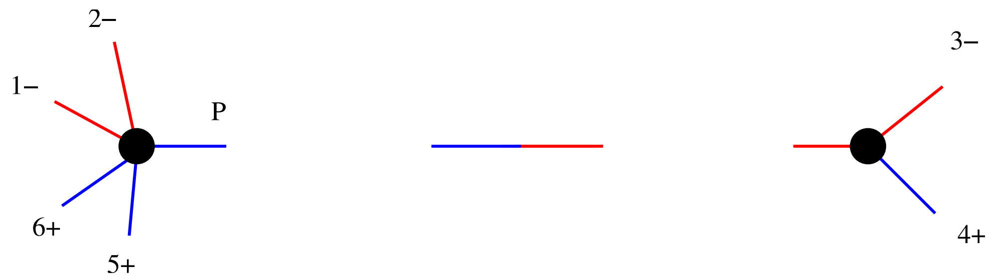
$$A_6(1^-, 2^-, 3^-, 4^+, 5^+, 6^+)$$

Step 1 Draw all the allowed MHV diagrams

Step 2 Apply MHV rules to each diagram

Example: six gluon scattering: diagram 1

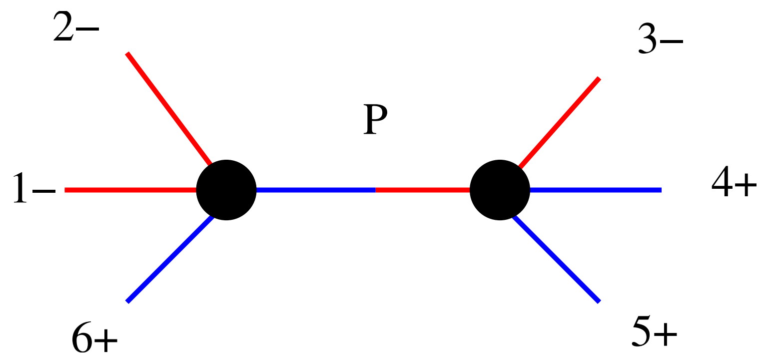


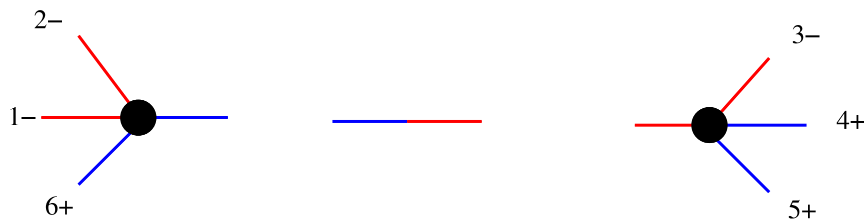


$$\frac{\langle 12\rangle^4}{\langle 56\rangle\langle 61\rangle\langle 12\rangle\langle 2|P|\eta]\langle 5|P|\eta]} \times \frac{1}{s_{34}} \times \frac{\langle 3|P|\eta]^4}{\langle 34\rangle\langle 4|P|\eta]\langle 3|P|\eta]}$$

with $P = 3 + 4 = -(1 + 2 + 5 + 6)$

Example: six gluon scattering: diagram 2





$$\frac{\langle 12\rangle^4}{\langle 61\rangle\langle 12\rangle\langle 2|P|\eta]\langle 6|P|\eta]} \times \frac{1}{s_{345}} \times \frac{\langle 3|P|\eta]^4}{\langle 34\rangle\langle 45\rangle\langle 5|P|\eta]\langle 3|P|\eta]}$$

with $P = 3 + 4 + 5 = -(1 + 2 + 6)$

Example: six gluon scattering

As an example, lets use the MHV rules to calculate one of the first non-MHV amplitudes

$$A_6(1^-, 2^-, 3^-, 4^+, 5^+, 6^+)$$

Step 1 Draw all the allowed MHV diagrams

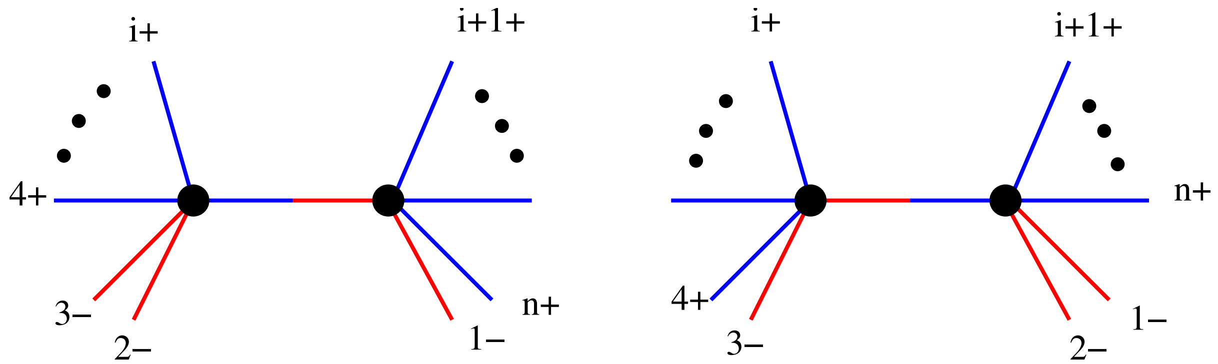
Step 2 Apply MHV rules to each diagram

Step 3 Add up diagrams and check η independence

Next-to MHV amplitude for n gluons

Simplest case: $A_n(1^-, 2^-, 3^-, 4^+, \ldots, n^+)$ $\qquad$ $2(n-3)$ graphs

Cachazo, Svrcek and Witten



$$A = \sum_{i=3}^{n-1} \frac{\langle 1|(2,i)|\eta]^3}{\langle (i+1)|(2,i)|\eta]\langle i+1 i+2\rangle \ldots \langle n1\rangle} \frac{1}{s_{2,i}^2} \frac{\langle 23\rangle^3}{\langle 2|(2,i)|\eta]\langle 34\rangle \cdots \langle i|(2,i)|\eta]}$$

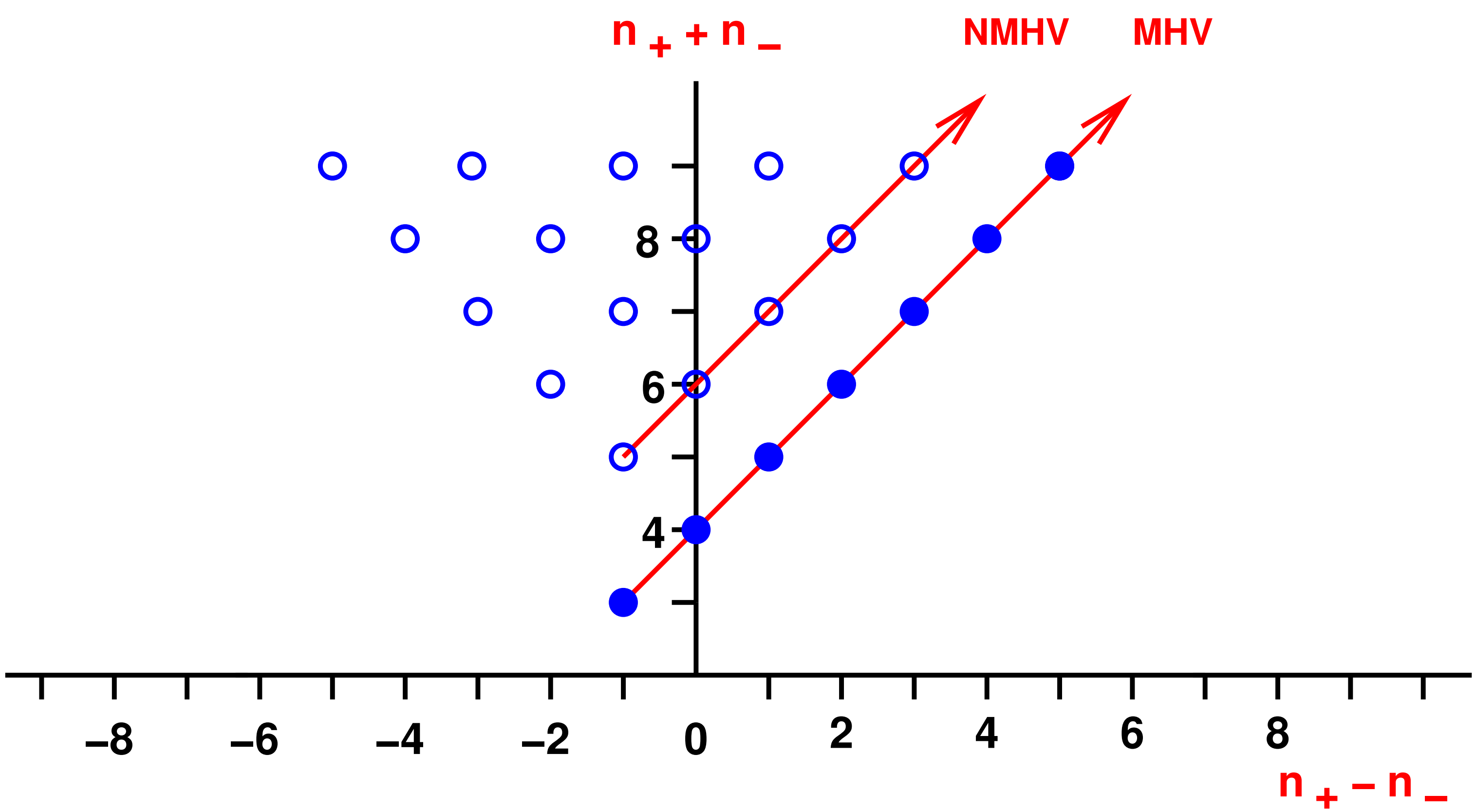
$$+ \sum_{i=4}^{n} \frac{\langle 12\rangle^3}{\langle 2|(3,i)|\eta]\langle (i+1)|(3,i)|\eta] \ldots \langle n1\rangle} \frac{1}{s_{3,i}^2} \frac{\langle 3|(3,i)|\eta]^3}{\langle 34\rangle \cdots \langle i-1 i\rangle\rangle\langle i|(3,i)|\eta]}.$$

where $(k,i) = k + \cdots + i$ is the off-shell momentum

$\Rightarrow$ Lorentz invariant and gauge invariant expressions

Generating all the tree amplitudes

Amplitudes with $i-$ and $j+$ helicities



- MHV rules always adds one negative helicity and any number of positive helicities
  $\Rightarrow$ maps out all allowed tree amplitudes

Other processes

MHV rules have been generalised to many other processes

- ✓ with massless fermions - quarks, gluinos

  Georgiou and Khoze; Wu and Zhu; Georgiou, EWNG and Khoze

- ✓ with massless scalars - squarks

  Georgiou, EWNG and Khoze; Khoze

- ✓ with an external Higgs boson

  Dixon, EWNG, Khoze; Badger, EWNG, Khoze

- ✓ with an external weak boson

  Bern, Forde, Kosower and Mastrolia

Has provided new analytic results for n-particle amplitudes
Also useful for studying infrared properties of amplitudes

Birthwright, EWNG, Khoze and Marquard

BCFW on-shell recursion relations

Britto, Cachazo, Feng; Roiban, Spradlin, Volovich

Based on elementary complex analysis - Cauchy Integral Formula

$$\frac{1}{2\pi i}\oint \frac{dz}{z}\,A(z) = \text{sum of residues}$$

provided that $A(z) \to 0$ as $z \to \infty$

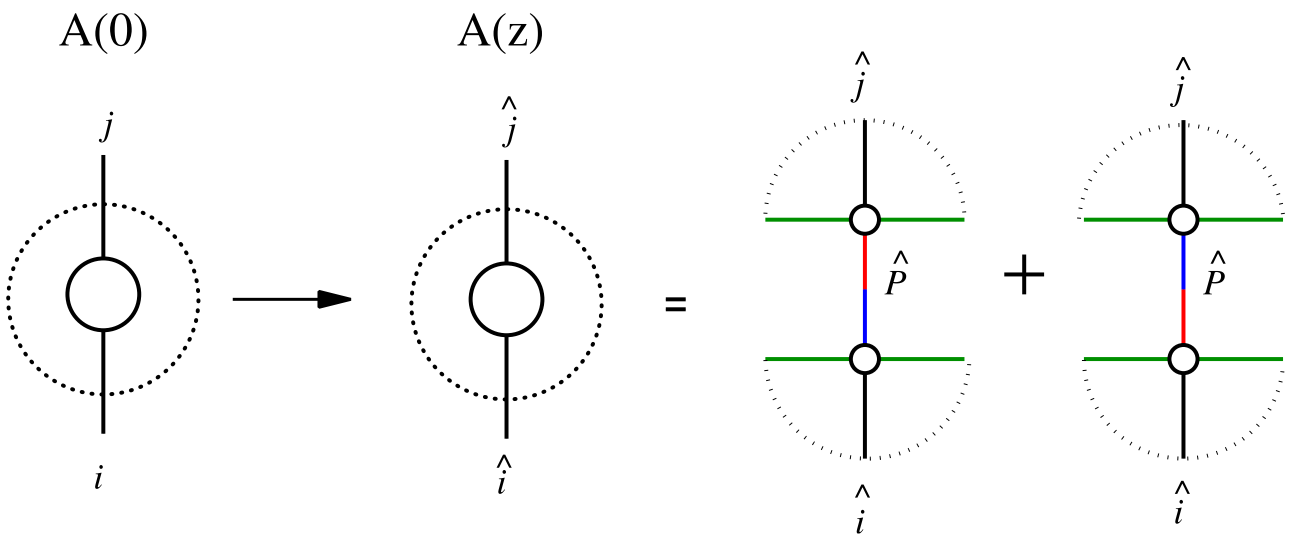
$$\text{sum of residues} = A(0) + \ldots$$

Simple enough, but how is this related to scattering amplitudes?

BCFW on-shell recursion relations

Britto, Cachazo, Feng; Roiban, Spradlin, Volovich

Lets consider an n particle amplitude $A(0)$.



hatted momenta are shifted to put on-shell

$$\hat{i} = i + z\eta, \qquad \hat{j} = j - z\eta, \qquad \hat{P} = P + z\eta$$

$\Rightarrow$ each vertex is an on-shell amplitude

BCFW recursion relations

- It turns out that the shift η is not a momentum, but

$$\eta = \lambda_i \tilde{\lambda}_j \qquad OR \qquad \eta = \lambda_j \tilde{\lambda}_i$$

- The parameter z is fixed by $\hat{P}^2 = 0$

$$z = \frac{P^2}{\langle j|P|i]}$$

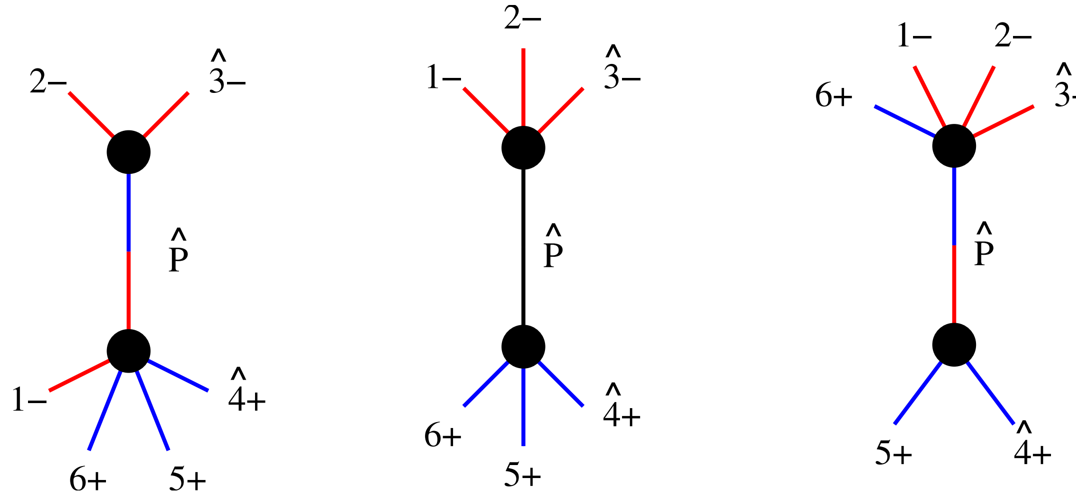
- Easy to prove that by complex analysis based on fact that only simple poles in z occur and that $A(z)$ vanishes as $z \to \infty$

Britto, Cachazo, Feng and Witten

- Requires on-shell three-point vertex contributions - both MHV and $\overline{\mathrm{MHV}}$.

BCFW - six gluon example

If we select 3 and 4 to be the special gluons, there are only three diagrams (for any helicities)



For this helicity assignment, the middle diagram is zero!.

$A_6(1^-, 2^-, 3^-, 4^+, 5^+, 6^+)$

$$= \frac{1}{\langle 5|\not{3} + \not{4}|2\rangle} \left(\frac{\langle 1|\not{2} + \not{3}|4\rangle^3}{[23][34]\langle 56\rangle\langle 61\rangle s_{234}} + \frac{\langle 3|\not{4} + \not{5}|6\rangle^3}{[61][12]\langle 34\rangle\langle 45\rangle s_{345}} \right)$$

Extremely compact analytic results for up to 8 gluons

Other processes

BCF recursion relations have been generalised to other processes

✓ with massless fermions - quarks, gluinos

Luo and Wen

✓ gravitons

Bedford, Brandhuber, Spence and Travaglini; Cachazo and Svrcek

There is nothing (in principle) to stop this approach being applied to particles with mass.

✓ massive coloured scalars

Badger, EWNG, Khoze and Svrcek

✓ massive vector bosons and heavy quarks

Badger, EWNG and Khoze

One loop amplitudes

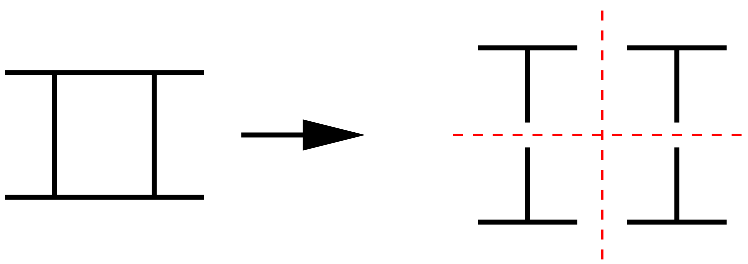
- So far, supersymmetry was not a major factor - tree level amplitudes same for $\mathcal{N}=4$ $\mathcal{N}=1$ and QCD
- Not true at the loop level due to circulating states

$$
\begin{aligned}
A_n^{\mathcal{N}=4} &= A_n^{[1]} + 4A_n^{[1/2]} + 3A_n^{[0]} \\
A_n^{\mathcal{N}=1,chiral} &= A_n^{[1/2]} + A_n^{[0]} \\
A_n^{glue} &= A_n^{\mathcal{N}=4} - 4A_n^{\mathcal{N}=1,chiral} + A_n^{[0]}
\end{aligned}
$$

- All plus and nearly all-plus amplitudes do not vanish for non-supersymmetric QCD
- A lot of progress by a lot of people

One loop amplitudes

- Key point is that loop amplitudes contain both poles and cuts - e.g. $\log(x)$ has cut for negative x
- Cut contributions are fully constructible by using unitarity
  - Cut lines are on-shell and 4-dimensional



Bern, Dixon, Dunbar, Kosower; Britto, Cachazo, Feng

- Pole contributions can be constructed using BCF type recursion and knowledge of factorisation properties

Forde, Zhu, . . .

Collectively this is the Unitarity Bootstrap

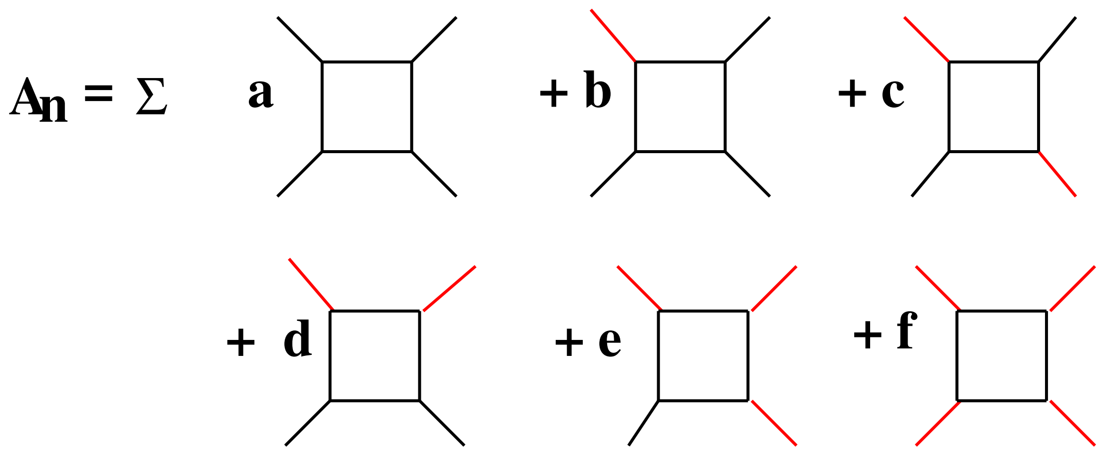
Kosower

SUSY QCD loops

- ✓ $\mathcal{N} = 4$ and $\mathcal{N} = 1$ one-loop amplitudes are constructible from their 4-dimensional cuts
  $\Rightarrow$ employ unitarity techniques

Bern, Dixon, Dunbar, Kosower

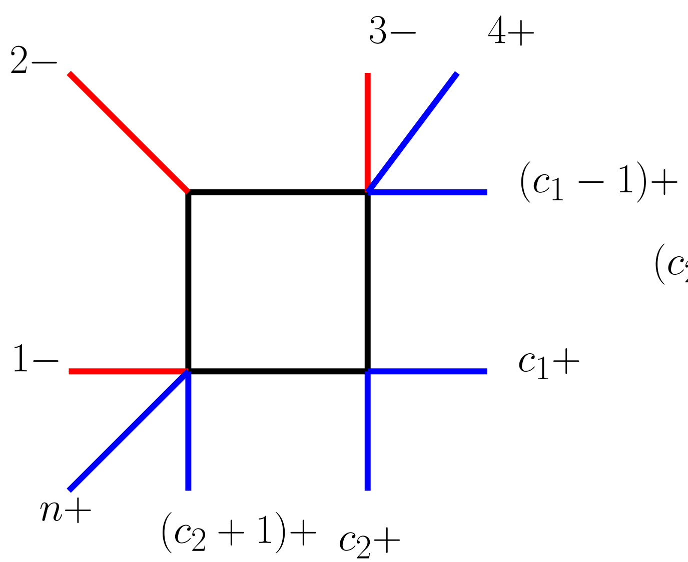
- ✓ For $\mathcal{N} = 4$ all amplitudes are a linear combination of known box integrals

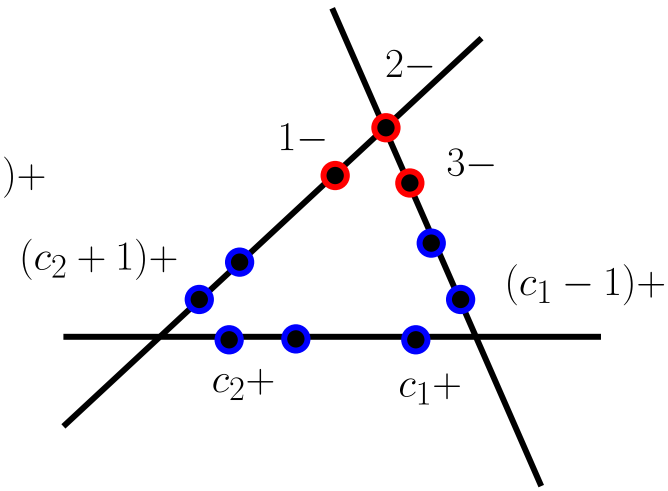


Twistor space interpretation

- Coefficients of boxes have very interesting structures.

Bern, Del Duca, Dixon, Kosower; Britto, Cachazo, Feng



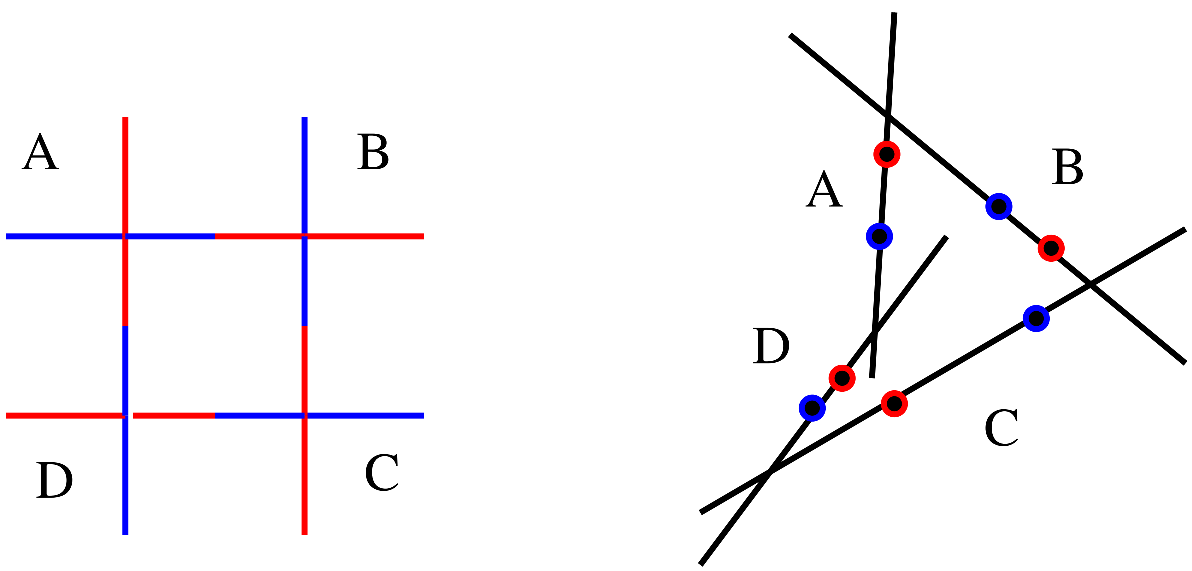


Twistor space interpretation

- Four mass box first appears in eight-point amplitude with four negative and four positive helicities

Bern, Dixon, Kosower

e.g.



- Still not fully understood

QCD loops

- ✗ QCD amplitudes more complicated because they are not 4-dimensional cut constructible.
  Rational contribution not probed by 4-d cut
- ✗ All plus and almost all plus amplitudes no longer zero - but pure rational functions. Not protected by SWI.
- ✓ Rational parts of infrared divergent amplitudes computed using
  - ✓ on-shell recursion relation

    Bern, Dixon and Kosower

    Recursion relations complicated by double pole terms and boundary terms
  - ✓ Direct Feynman diagram evaluation of rational part

    Xiao, Yang, Zhu
  - ✓ d-dimensional cuts

    Anastasiou, Britto, Feng, Kunszt, Mastrolia

Six gluon amplitude

✓ Analytic computation
Bedford, Berger, Bern, Bidder, Bjerrum-Bohr, Brandhuber, Britto, Buchbinder, Cachazo, Dixon, Dunbar, Feng, Forde, Kosower, Mastrolia, Perkins, Spence, Travaglini, Xiao, Yang, Zhu

Amplitude	$\mathcal{N}=4$	$\mathcal{N}=1$	$\mathcal{N}=0$ (cut)	$\mathcal{N}=0$ (rat)
$--++++$	BDDK (94)	BDDK (94)	BDDK (94)	BDK (94)
$-+-+++$	BDDK (94)	BDDK (94)	BBST (04)	BBDFK (06), XYZ (06)
$-++-++$	BDDK (94)	BDDK (94)	BBST (04)	BBDFK (06), XYZ (06)
$---+++$	BDDK (94)	BDDK (94)	BBDI (05), BFM (06)	BBDFK (06), XYZ (06)
$--+-++$	BDDK (94)	BBDP (05), BBCF (05)	BFM (06)	XYZ (06)
$-+-+-+$	BDDK (94)	BBDP (05), BBCF (05)	BFM (06)	XYZ (06)

✓ Numerical evaluation Ellis, Giele, Zanderighi (06)

Summary - I

- ✓ On-shell techniques are a very exciting and rapidly developing field
- ✓ MHV rules for tree-level
  Very simple way of deriving n-point amplitudes for massless partons
- ✓ BCFW recursion relations for tree-level
  Very powerful method for deriving amplitudes for both massless and massive particles
- ✗ Berends-Giele recursion still looks to be numerically faster
- ✓ Generalised unitarity and one-loop amplitudes
  SUSY amplitudes cut constructible - coefficients of loop integrals can be *read off* from graphs
  QCD amplitudes contain cut-non constructible parts. These simple pole terms can be attacked using the BCFW relations

  Bern, Dixon, Kosower

  Or by direct evaluation using Feynman diagrams

  Xiao, Yang, Zhu

Summary - II

- ✓ New methods already competitive with traditional methods for loop amplitudes with massless particles - gluons, quarks
- ✓ Will definitely see all six parton one-loop amplitudes in next few months
- ✗ Not necessarily the most interesting phenomenologically
- ? Will new methods be useful for amplitudes with heavy particles - top quarks, susy particles, Higgs bosons, vector bosons
- ✓ In principle heavy particles not a problem - but certainly a complication.
- ✓ yes for one vector boson plus multiparton e.g. V + multijet
- ✓ probable for two vector boson plus multiparton e.g. VV + multijet
- ? much more difficult for $pp \to t\bar{t}b\bar{b}$

SPARE SLIDES

Collider Physics

1. Predictions for multiparticle final states that occur at high rate and form background to New Physics

   High multiplicity, but low order - typically LO or NLO

   For example, $pp \to V + 4$ jets is background to $pp \to t\bar{t}$ and other new physics.

2. Precise predictions for hard pp processes involving “standard particles" like W, Z, jets, top, Higgs, ..

   Low multiplicity, but high order - NNLO is emerging standard

   For example, Drell Yan cross section.

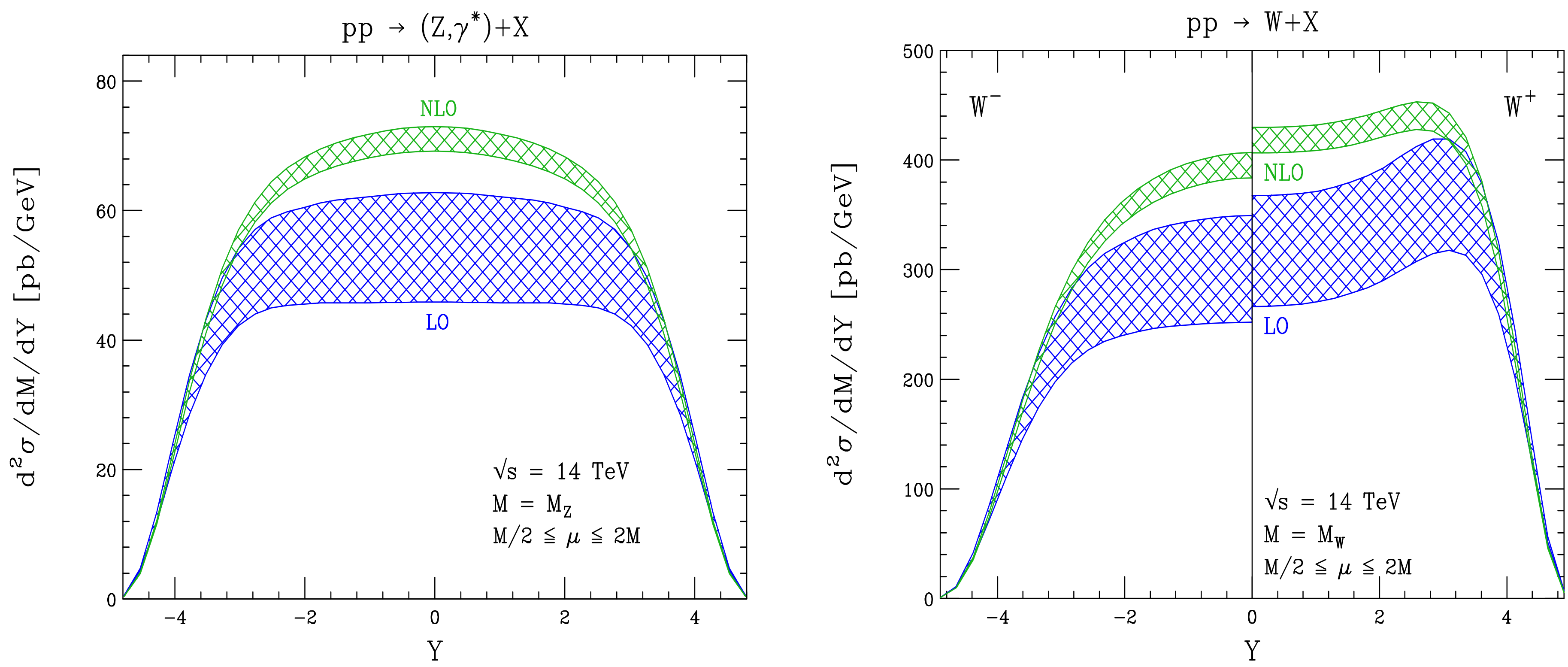
State of the Art

Relative Order	$2 \to 1$	$2 \to 2$	$2 \to 3$	$2 \to 4$	$2 \to 5$	$2 \to 6$
1	LO					
α_s	NLO	LO				
α_s^2	NNLO	NLO	LO			
α_s^3		NNLO	NLO	LO		
α_s^4				NLO	LO	
α_s^5					NLO	LO

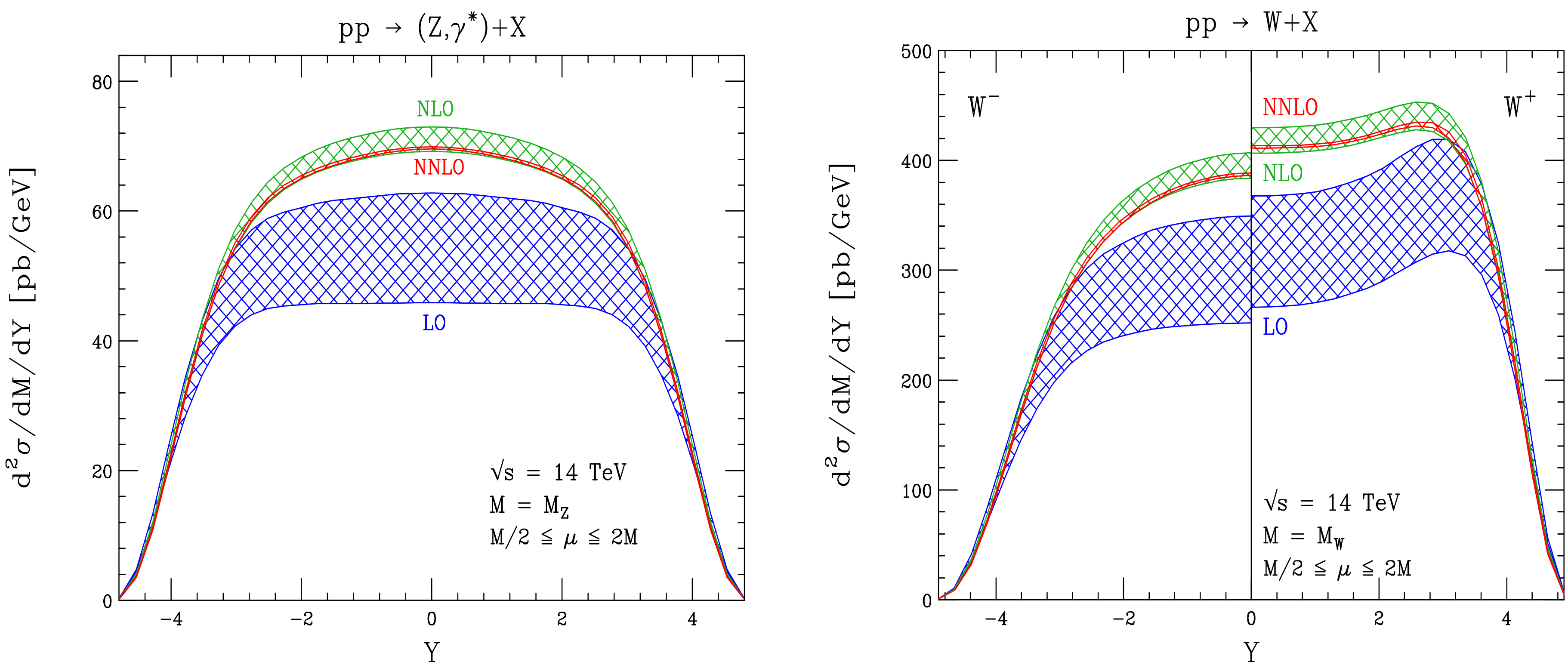
NNLO

- ✓ (inclusive) Drell-Yan and Higgs total cross sections – Anastasiou, Dixon, Melnikov, Petriello
- ✓ (inclusive) Drell-Yan and Higgs rapidity distributions – Anastasiou, Dixon, Melnikov, Petriello
- ✓ NNLO evolution – Moch, Vogt, Vermaseren
- ✗ need full set of NNLO observables for global fit. DIS and Drell-Yan will not be enough

Gauge boson production at the LHC



Gauge boson production at the LHC



Gold-plated process

Anastasiou, Dixon, Melnikov, Petriello

At LHC NNLO perturbative accuracy better than 1%

$\Rightarrow$ use to determine parton-parton luminosities at the LHC

State of the Art

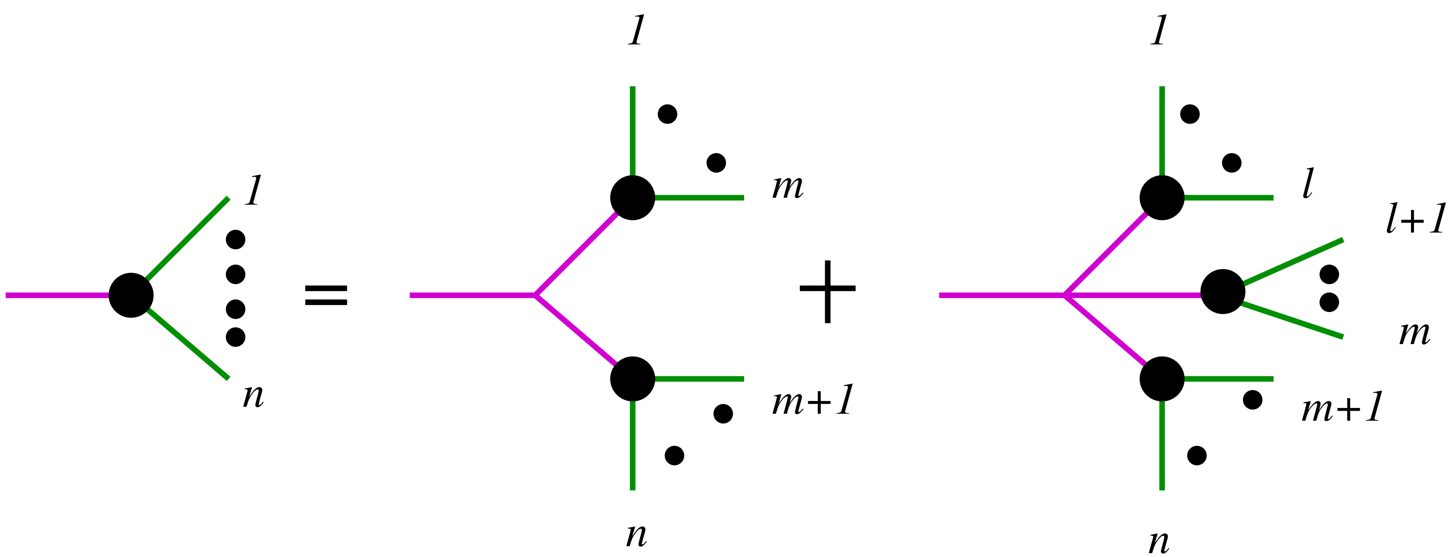
Relative Order	$2 \to 1$	$2 \to 2$	$2 \to 3$	$2 \to 4$	$2 \to 5$	$2 \to 6$
1	LO					
α_s	NLO	LO				
α_s^2	NNLO	NLO	LO			
α_s^3		NNLO	NLO	LO		
α_s^4				NLO	LO	
α_s^5					NLO	LO

NNLO

- ✓ want to calculate $2 \to 2$ to few percent accuracy and use as standard candle to determine pdfs and α_s more accurately
- ✓ with global pdf fit, gives impact on all observables
- ✗ still not available

Berends-Giele : Off-shell recursion relations

Full amplitudes can be built up from simpler amplitudes with fewer particles



Purple gluons are off-shell, green gluons are on-shell.
This is a recursion relation built from off-shell currents.

Berends, Giele

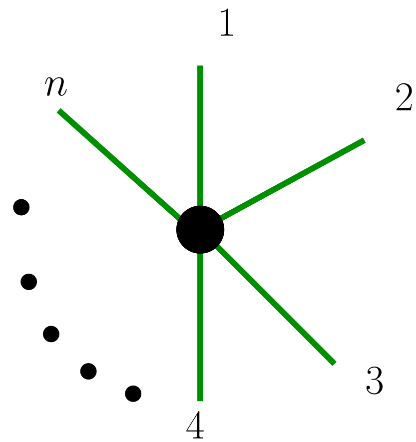
Particularly suited to numerical solution

`ALPGEN, HELAC/PHEGAS`

Common methods : Colour Ordered Amplitudes

$$\mathcal{A}_n(1,\ldots,n) = \sum_{perms} Tr(T^{a_1}\ldots T^{a_n}) A_n(1,\ldots,n)$$

Colour-stripped amplitudes A_n: cyclically ordered



Order of external gluons fixed

The subamplitudes A_n have nice properties in the infrared limits.

Can reconstruct the full amplitude $\mathcal{A}_n$ from A_n.
In the large N limit,

$$|\mathcal{A}_n(1,\ldots,n)|^2 \sim N^{n-2} \sum_{perms} |A_n(1,\ldots,n)|^2$$

Twistor Space

Penrose, 1967

Amplitudes in twistor space obtained by Fourier transform with respect to positive helicity spinors,

$$\tilde{\lambda}_{\dot{a}} = i\frac{\partial}{\partial \mu^{\dot{a}}}, \qquad \mu^{\dot{a}} = i\frac{\partial}{\partial \tilde{\lambda}_{\dot{a}}}$$

Momentum conservation yields

$$\delta\left(\sum k_j\right) = \int d^4x \exp\left(i\sum_j x\cdot k_j\right) = \int d^4x \exp\left(ix^{a\dot{a}}\sum_j \lambda_{ja}\tilde{\lambda}_{j\dot{a}}\right)$$

so that the amplitude in twistor space is

$$\tilde{A}(\lambda_i, \mu_i) = \int d^4x \int \prod_i \frac{d^2\tilde{\lambda}_i}{(2\pi)^2} \exp\left(i\sum_j \left(\mu_j^{\dot{a}} + x^{a\dot{a}}\lambda_{ja}\right)\tilde{\lambda}_{j\dot{a}}\right) A(\lambda_i, \tilde{\lambda}_i)$$