From T2K to Hyper-K

David Hadley, University of Warwick
RAL Seminar, February 2018
Outline

Long baseline neutrino oscillation at T2K

Hyper-K Detector

Systematic uncertainty challenges and solutions
Neutrino Oscillations

Weak flavour eigenstates ≠ Mass eigenstates
Neutrinos produced and detected in their weak flavour states

\[ \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\text{MNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \]

Unitary PMNS mixing matrix parameterised with 3 angles and CP violating phase \( \theta_{ij}, \delta_{CP} \)

Relative phase difference between due to mass difference, \( \Delta m^2 \)

Appearance probability:

\[ P_{\mu \to e} \approx \sin^2 \theta_{23} \cdot \sin^2 2\theta_{13} \cdot \sin^2 \left( \frac{\Delta m^2 L}{4E_\nu} \right) \]

+ higher order terms involving \( \delta_{CP} \)
Neutrino Oscillations

T2K

Hi, Super-Kamiokande!

Shoot muon neutrinos!

Super-Kamiokande-chan

Wow!

Something different...?

Miracle shoot!

No! It's Neutrino Oscillation!

J-PARC-chan
lives in Tokai-mura, Naka-gun, Ibaraki, Japan.

Super-Kamiokande-chan
lives in Kamioka-cho, Hida-city, Gifu, Japan.

Higgstan [http://higgstan.com/4koma-t2k/]
Neutrino Oscillations

Typically perform experiment at fixed L with wide range of E

CP violation ~ 20% effect at 1st oscillation maximum
Much larger effect at 2nd oscillation maximum
Neutrino Oscillations

Typically perform experiment at fixed L with wide range of E

CP violation ~ 20% effect at 1st oscillation maximum
Much larger effect at 2nd oscillation maximum
What we actually measure:

\( \nu_\mu \) disappearance  \( \nu_e \) appearance

\[ \theta_{23} : \text{dip amplitude} \quad \Delta m_{32}^2 : \text{dip energy} \]

**Reconstructed** Energy (GeV)

\[ \sin^2 \theta_{13} = 0.0251 \]

\[ \sin^2 \theta_{13} = 0.5 \]

Measurement precision limited by:

- Statistics
- Neutrino energy reconstruction
- Knowledge of unoscillated spectrum and background contamination
Accelerator based Neutrino Oscillation Experiments

Current

LBL

Future

LBL

ICARUS

μBooNE

DUNE
Super-Kamiokande

J-PARC

Near Detectors

Neutrino Beam

295 km

Mt. Noguchi-Goro
2,924 m

Mt. Ikeno-Yama
1,360 m

1,700 m below sea level

Far Detector
(Super-K)

Near Detectors
(ND280+INGRID)
2013: $v_e$ appearance established

28 events observed (4.3 expected background)

Effect is large, opens the way to leptonic CP violation $\delta_{CP}$.
T2K $\nu_e$ appearance

2013: $\nu_e$ appearance established → 2017: “indications” of CP violation

28 events observed (4.3 expected background)


- Small $\nu_e$ excess and $\bar{\nu}_e$ deficit
- Current measurement based on 74+7 events in single ring sample

Effect is large, opens the way to leptonic CP violation $\delta_{CP}$. 

Small $\nu_e$ excess and $\bar{\nu}_e$ deficit
Current measurement based on 74+7 events in single ring sample
First Indications of CP violation

CP conserving values excluded at $2\sigma$

Statistically limited
Dependent on reactor $\bar{\nu}_e$ disappearance measurement
$\Delta \chi^2$ vs POT for T2K Projected Sensitivity.

~2.5σ projected significance if maximal CP violation.

to firmly establish CP violation we will need Hyper-K!
Kamiokande Detectors

Kamiokande
680 tonne fiducial mass
(1983)
Kamiokande Detectors

Kamiokande
680 tonne fiducial mass
(1983)

Super-Kamiokande
22.5kt fiducial mass
(33x Kamiokande)
(1996)
Kamiokande Detectors

Kamiokande
680 tonne fiducial mass (1983)

Super-Kamiokande
22.5kt fiducial mass (33x Kamiokande) (1996)

Hyper-Kamiokande
187 kt fiducial mass per tank (2026?)
Growing international collaboration: 14 countries, ~300 people
Why Water Cherenkov?

**Scalability**
Water is cheap, non-toxic, liquid at room temperature. We already know how to build big water WC detectors.

**Proven technology**
Many years of experience from Super-K, low risk.

**Excellent performance**
Based on real Super-K and T2K performance.
Water Cherenkov Technique

Muon
Water Cherenkov Technique

Muon

Electron
Water Cherenkov Technique

Muon

Electron

Neutral Pion
Water Cherenkov Technique

Muon

Electron

$\pi^0$

Excellent PID performance

Accelerator $\nu_e$ background is dominated by irreducible intrinsic $\nu_e$. 
Neutrino Energy Measurement

Protons usually below Cherenkov threshold
Neutrons can be counted but no energy measurement

For quasi-elastic interactions neutrino energy can be reconstructed from lepton kinematics

\[ E_{\nu}^{\text{rec}} = \frac{m_p^2 - (m_n - E_b)^2 - m_e^2 + 2(m_n - E_b)E_e}{2(m_n - E_b - E_e + p_e \cos \theta_e)} \]

Background from inelastic scattering where energy is mis-measured

Interaction is on bound state
Nuclear effects are important
Tank Design

Old: Horizontal Egg-shaped Tank

New: Optimised Vertical Tank
Tank Design

ID: 40% photo-coverage
40,000 photo sensors per tank
Hyper-K Projected Sensitivity

10 years x 1 tank x 1.3 MW
$\nu_e \sim 2058$, $\bar{\nu}_e \sim 1906$ events

Assuming 3-4% systematic uncertainty (cf T2K present $\sim 6\%$)
Physics at Hyper-K

- Proton Decay
- Neutrinos
  - Solar
  - Supernova
- Accelerator
- Atmospheric

Broad physics programme.
## Statistics

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$\nu_e + \bar{\nu}_e$</th>
<th>$1/\sqrt{N}$</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>T2K (current)</td>
<td>74 + 7</td>
<td>12% + 40%</td>
<td>$2.2 \times 10^{21}$ POT</td>
</tr>
<tr>
<td>NOvA (current)</td>
<td>33</td>
<td>17%</td>
<td>FERMILAB-PUB-17-065-ND</td>
</tr>
<tr>
<td>NOvA (projected)</td>
<td>110 + 50</td>
<td>10% + 14%</td>
<td>arXiv:1409.7469 [hep-ex]</td>
</tr>
<tr>
<td>T2K-I (projected)</td>
<td>150 + 50</td>
<td>8% + 14%</td>
<td>$7.8 \times 10^{21}$ POT, arXiv:1409.7469 [hep-ex]</td>
</tr>
<tr>
<td>T2K-II</td>
<td>470 + 130</td>
<td>5% + 9%</td>
<td>$20 \times 10^{21}$ POT, arXiv:1607.08004 [hep-ex]</td>
</tr>
<tr>
<td>Hyper-K</td>
<td>2058 + 1906</td>
<td>2% + 2%</td>
<td>10 yrs 1-tank 2017 Design Report TBR</td>
</tr>
<tr>
<td>DUNE</td>
<td>1200 + 350</td>
<td>3% + 5%</td>
<td>3.5+3.5 yrs x 40kt @ 1.07 MW arXiv:1512.06148 [physics.ins-det]</td>
</tr>
</tbody>
</table>

Current appearance measurements stats dominate $O(10^3) \nu_e$ at future experiments $\rightarrow$ demands $\sim 2\%$ systematics $\rightarrow O(10^4) \nu_\mu$ $\rightarrow$ need systematics as good as we can get!
Worldwide R&D
Photo Sensors

- Super-K PMT: QE 22%, CE 80%
- High QE/CE PMT: QE 30%, CE 93%
- High QE/CE Hybrid PD: QE 30%, CE 95%
- Venetian blind dynode
- Box and Line dynode
- Avalanche diode
Photo Sensors

2x improvement in photon detection efficiency

Better timing and charge resolution
Photo Sensors

Optimised bulb design
High pressure and implosion tests show new PMTs safe for use in HK tank
Calibration

Precise PMT response testing

Automated source deployment

Fake muon source

“Neutristor” Neutron Generator
Scattered Light

Calibration

R&D for new optical calibration system in progress

Using Super-K 2018 shutdown for direct testing of newly developed calibration systems for Hyper-K
Simulation

High fidelity calibration data is only as useful if it can be input into equally high fidelity simulation
Near Detector Development

Carbon and Oxygen target materials

Acceptance differs from far detector

Magnetic field for sign selection

Near Detector (ND280)
Near Detector Development

Planned ND280 Near Detector Upgrade

Near detector upgrades for T2K-II and T2HK era
New target with increased angular acceptance
E61 Experiment

Linear combinations of measurements at various off-axis angles

Measure response for an arbitrary flux

Reduce dependence on nuclear models
E61 Experiment

Pseudo-monochromatic beams

Far detector prediction for oscillated flux
Project Timeline

HK selected in “Master Plan” of Science Council in Japan
HK selected as highest-priority large-scale projects MEXT
Roadmap 2017
Funding request in progress
Summary

T2K established $\nu_e$ appearance and sees hints of CP violation

Hyper-K well placed to build on the huge success of Super-K and T2K experiments

Capable of world leading measurements in neutrino oscillations, nucleon decay, neutrino astrophysics

Funding request in progress
If construction starts 2018, operation in 2026

References:

T2HKK White Paper, arXiv:1611.06118 [hep-ex]
HK Design Report, KEK Preprint 2016-21
HK Physics Sensitivity, PTEP (2015) 053C02
Hyper-K

David Hadley, University of Warwick
Backup
## T2K Systematic Uncertainties

<table>
<thead>
<tr>
<th>Error Source</th>
<th>$\mu$ sample [%]</th>
<th>$e$ sample [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\nu$</td>
<td>$\bar{\nu}$</td>
</tr>
<tr>
<td>SK Detector</td>
<td>1.9</td>
<td>1.6</td>
</tr>
<tr>
<td>SK FSI+SI+PN</td>
<td>2.2</td>
<td>2.0</td>
</tr>
<tr>
<td>ND280 Constraint (Flux + Cross Section)</td>
<td>3.3</td>
<td>2.7</td>
</tr>
<tr>
<td>$\sigma(\nu_e)/\sigma(\nu_\mu)$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>NC 1γ</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>NC other</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>Total Systematic</td>
<td>4.4</td>
<td>3.8</td>
</tr>
<tr>
<td>Statistical</td>
<td>6.5</td>
<td>12</td>
</tr>
</tbody>
</table>

T2K preliminary (final systematics pending)

- ND280 constraint: 13% → 3%
- Pion Final State Interactions (FSI) and Secondary Interactions (SI) modelling important
- Theoretical uncertainty $\nu_e$ to $\nu_\mu$
  - Difficult to constrain with near detector

Total systematic uncertainty ~4 - 6%
Smaller than stats. uncertainty (for now!)
E61 Experiment

Two competing collaborations

nuPRISM

“Water elevator”
Measure $\int \sigma(E)\phi(E)dE$ as a function of theta
[arXiv:1412.3086]

Merged into a single collaboration:
E61 Experiment

TITUS

same off-axis angle far detector
Gd, muon range detector
[arXiv:1606.08114]
Near Detector Development

TPC measurements precisely image $\nu$-nucleus interaction vertex $\rightarrow$ better constraints on models

Ultra-low thresholds with gaseous TPC
Neutrino Interaction Model Uncertainties

Wide range of processes need to be simulated
Require both lepton and hadronic side of the interaction
Nuclear effects important in the relevant energy regime

Experiments rely on MC generators
for $E_{\text{visible}} \rightarrow E_\nu$ extrapolation

Model parameter uncertainties from fits to external datasets
Sometimes parameter error must be inflated or ad-hoc parameters to account for discrepancies between model and data or known flaws in the model
T2K Cross-Section Model

- Implemented in NEUT MC generator

Quasi-elastic scattering most important process at T2K energies
- Long-range effects with Random Phase Approximation
- Parameters introduced to vary normalisation and shape
- Relativistic Fermi Gas (RFG) nuclear model
- Uncertainties from RFG ↔ Local Fermi Gas
- Final state interactions with cascade model

No priors on most CCQE parameters
Constraint from near detector

Impact of alternative models not implemented in oscillation analysis evaluated with fake data studies
Flux Uncertainties

T2K ~ 8-12% (based on thin target tuning)

Dominated by hadron interaction modelling

Alignment/focussing uncertainties are also important (especially for near to far extrapolation)
Flux Uncertainties

Significant reductions from thick/replica target

If high power beam requires different target material/geometry, new dedicated hadron production measurements will be necessary.
J-PARC Beam Upgrades

Current: ~470 kW
Short-term: 750 kW after 2018 long shutdown
Goal: 1.3 MW operation at HK operation
Detector Modelling Uncertainties

SK detector response evaluated with data-MC comparisons in atmospheric sample
May be limited by control sample statistics
Possible to move toward bottom-up detector systematic uncertainty
T2K / Hyper-K Flux

Narrow band beam off-axis

Flavour composition

nu-mode: \( \sim 94\% \nu_\mu \)

anti-nu mode: \( \sim 92\% \bar{\nu}_\mu \)

(for \( E < 1.25 \text{ GeV} \))
Physics at Hyper-K

Proton Decay
\[ p \rightarrow e^+ + \pi^0 \]
\[ > 1.3 \times 10^{35} \text{ years} \quad 90\% \text{ CL} \]
\[ p \rightarrow \bar{\nu} + K^+ \]
\[ > 3.2 \times 10^{34} \text{ years} \quad 90\% \text{ CL} \]

Neutrinos
- Solar
- Supernova
  SN \( \sim 200,000 @ 10\text{kPC} \)
  SN \( \sim 30-50 @ M31 \)

200 solar \( \nu \) per day
Indirect dark matter search

Accelerator
- Leptonic CP violation
  (see following slides)
- Mass Hierarchy determination
  \( \theta_{23} \) octant determination
  \( > 3\sigma \)
  \( 3\sigma \) for \( \sin^2 \theta_{23} > 0.56 \) or \( \sin^2 \theta_{23} < 0.46 \)

Broad physics programme.
Lots of Physics with Hyper-K

Proton Decay

Mass hierarchy with atm.

$O(10^5)$ events from typical Supernova @ 10 kpc
Korean Tank

Stronger CP effect at the second oscillation maximum

A second tank in Korea would be able to measure this effect
Carbon and Oxygen target materials

Acceptance differs from far detector

Magnetic field for sign selection

Near Detector (ND280)
Photo Sensors

Time Resolution

1p.e. charge distribution

<table>
<thead>
<tr>
<th></th>
<th>SK PMT</th>
<th>B&amp;L PMT</th>
<th>50cm HPD (20cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1PE T resolution σ (ns)</td>
<td>2.1</td>
<td>1.1</td>
<td>1.4 (1.1)</td>
</tr>
<tr>
<td>FWHM (ns)</td>
<td>7.3</td>
<td>4.1</td>
<td>3.4 (3.3)</td>
</tr>
<tr>
<td>1PE Q resolution σ/mean</td>
<td>53%</td>
<td>35%</td>
<td>16% (12%)</td>
</tr>
<tr>
<td>Peak-to-Valley ratio</td>
<td>2.2</td>
<td>4.3</td>
<td>3.9 (5.2)</td>
</tr>
</tbody>
</table>
Photo Sensors

![Graph showing quantum efficiency vs. wavelength for High-QE R12860 and Normal-QE R3600 sensors. The graph displays the percentage of quantum efficiency across different wavelength ranges.]
Near Detector Development
New/Upgraded Detectors in the Existing ND280 Complex

WAGASHI

Water dominated target
$4\pi$ acceptance

Water based liquid scintillator

An alternative approach is to improve knowledge of neutrino-nucleus interactions

e.g. High Pressure Gas TPC
Leptonic CP Violation

Measure $\delta_{CP}$ by comparing data with beam in $\nu$-mode with anti-$\nu$ mode

CP violation can be established at $3\sigma$ ($5\sigma$) for 76% (58%) of $\delta_{CP}$ space.

$\delta_{CP}$ measured to $< 20^\circ$ over entire space
Neutron Capture on Hydrogen

\[ \bar{\nu}_e + p \rightarrow e^+ + n \]

\[ n + p \rightarrow d + \gamma \]

200 µs capture time
\[ E_\gamma = 2.2 \text{ MeV} \]

Low light yield
Close to or below trigger threshold
Low detection efficiency (~18%)

 Initial charged lepton signal

Delayed γ signal

arXiv:1311.3738 [hep-ex]

SK with n-tag

SK w/o n-tag
Neutron Capture on Gadolinium

\[ \bar{\nu}_e + p \rightarrow e^+ + n \]

20 $\mu$s capture time

$E_\gamma \sim 8$ MeV cascade (~4 MeV visible)

Fast capture time (small $\Delta T$ window)

Higher energy $\gamma$ signal

\text{arXiv:0811.0735 [hep-ex]}
Neutron Capture on Gadolinium

Cross section for neutron capture: Gd (49,700 b), H (0.3 b)

0.1% Gd fraction gives 90% neutrons captured on Gd.
Applications: Supernova Relic Neutrinos

A low energy example

Directly observable local supernova are all too rare

Alternative is to measure diffuse supernova background DSNB/SRN

Very low rate

Large backgrounds

arXiv:1109.3262 [hep-ex]
Applications: Supernova Relic Neutrinos

A low energy example

Directly observable local supernova are all too rare

Alternative is to measure diffuse supernova background DSNB/SRN

Very low rate
Large backgrounds

Removed by requiring coincidence with neutron

A few clean events per year in SK
~100s per year in HK

arXiv:1109.3262 [hep-ex]
## Tank Parameters

<table>
<thead>
<tr>
<th></th>
<th>KAM</th>
<th>SK</th>
<th>HK-1TankHD</th>
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<tbody>
<tr>
<td><strong>Depth</strong></td>
<td>1,000 m</td>
<td>1,000 m</td>
<td>650 m</td>
</tr>
<tr>
<td><strong>Dimensions of water tank</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>diameter</td>
<td>15.6 m φ</td>
<td>39 m φ</td>
<td>74 m φ</td>
</tr>
<tr>
<td>height</td>
<td>16 m</td>
<td>42 m</td>
<td>60 m</td>
</tr>
<tr>
<td><strong>Total volume</strong></td>
<td>4.5 kton</td>
<td>50 kton</td>
<td>258 kton</td>
</tr>
<tr>
<td><strong>Fiducial volume</strong></td>
<td>0.68 kton</td>
<td>22.5 kton</td>
<td>187 kton</td>
</tr>
<tr>
<td><strong>Outer detector thickness</strong></td>
<td>~ 1.5 m</td>
<td>~ 2 m</td>
<td>1 ~ 2 m</td>
</tr>
<tr>
<td><strong>Number of PMTs</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>inner detector (ID)</td>
<td>948 (50 cm φ)</td>
<td>11,129 (50 cm φ)</td>
<td>40,000 (50 cm φ)</td>
</tr>
<tr>
<td>outer detector (OD)</td>
<td>123 (50 cm φ)</td>
<td>1,885 (20 cm φ)</td>
<td>6,700 (20 cm φ)</td>
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<td><strong>Photo-sensitive coverage</strong></td>
<td>20%</td>
<td>40%</td>
<td>40%</td>
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<tr>
<td><strong>Single-photon detection efficiency of ID PMT</strong></td>
<td>unknown</td>
<td>12%</td>
<td>24%</td>
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<tr>
<td><strong>Single-photon timing resolution of ID PMT</strong></td>
<td>~ 4 nsec</td>
<td>2-3 nsec</td>
<td>1 nsec</td>
</tr>
</tbody>
</table>
Three Flavor Mixing in Lepton Sector

Weak eigenstates

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}
= \mathbf{U}_{\text{MNS}}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
\]

mass eigenstates

\[
\begin{pmatrix}
m_1 \\
m_2 \\
m_3
\end{pmatrix}
\]

\[
\mathbf{U}_{\text{PMNS}} =
\begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} + s_{23} & s_{23} \\
0 & -s_{23} + c_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
+c_{13} & 0 & +s_{13}e^{-i\delta} \\
0 & 1 & 0 \\
-s_{13}e^{i\delta} & 0 & +c_{13}
\end{pmatrix}
\begin{pmatrix}
c_{12} & +s_{12} & 0 \\
-c_{12} & +s_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

\[ (c_{ij} = \cos \theta_{ij}, s_{ij} = \sin \theta_{ij} ) \]

\[ \theta_{12}, \theta_{23}, \theta_{13}, \delta, \]
\[ \Delta m_{21}^2, \Delta m_{32}^2, \Delta m_{31}^2 \]

*\[ \Delta m_{ij}^2 = m_i^2 - m_j^2 \]

Out of three \( \Delta m^2 \)'s, number of free parameters is two. (\( \Delta m_{31}^2 = \Delta m_{21}^2 + \Delta m_{32}^2 \))
$\nu_\mu$ disappearance probability

$\theta_{13}=0$ case

$$P_{\mu \rightarrow x} \approx 1 - \sin^2 2\theta_{23} \cdot \sin^2 \left( \frac{\Delta m_{32}^2 L}{4 E_\nu} \right)$$

For non-zero $\theta_{13}$

$$P_{\mu \rightarrow x} \approx 1 - \left( \cos^4 \theta_{13} \cdot \sin^2 2\theta_{23} + \sin^2 \theta_{23} \cdot \sin^2 2\theta_{13} \right) \sin^2 \left( \frac{\Delta m^2 L}{4 E_\nu} \right)$$

$\Delta m^2 \approx \Delta m_{32}^2 \approx \Delta m_{31}^2$

Maximal disappearance occurs at $\sin^2 \theta_{23} = \frac{1}{2\cos^2 \theta_{13}} = 0.513$
more on $\nu_\mu$ disappearance

- $\nu_\mu$ disappearance probability in vacuum

$$P(\nu_\mu \to \nu_\mu) = 1 - \left( c_{13}^2 \sin^2 \theta_{23} + s_{23}^2 \sin^2 \theta_{13} \right) \sin^2 \Delta_{atm}$$

$$+ \frac{1}{2} \left[ \sin 2 \Delta_{solar} \sin 2 \Delta_{atm} + 2 \sin^2 \Delta_{solar} \sin^2 \Delta_{atm} \right]$$

$$- \left[ \sin^2 2 \theta_{12} (c_{23}^2 - s_{13}^2 s_{23}^2)^2 + s_{13}^2 \sin^2 2 \theta_{23} (1 - c_{13}^2 \sin^2 2 \theta_{12}) + 2 s_{13} \sin 2 \theta_{12} \cos 2 \theta_{12} \sin 2 \theta_{23} \cos 2 \theta_{23} c_{13} \delta - \frac{1}{2} c_{13} \sin 2 \theta_{13} \sin 2 \theta_{23} \sin 2 \theta_{12} \cos \delta s_{23}^2 s_{12}^2 + \sin^2 2 \theta_{23} c_{13}^2 \left( s_{12}^2 - s_{13}^2 s_{23}^2 \right) + s_{13}^2 s_{23}^2 \sin^2 2 \theta_{13} \right] \times \sin^2 \Delta_{solar}$$

$$P(\nu_\mu \to \nu_\mu) \sim 1 - \left( \frac{\cos^4 \theta_{13} \cdot \sin^2 2 \theta_{23} + \sin^2 2 \theta_{13} \cdot \sin^2 \theta_{23}}{\sin^2 \Delta_{solar}} \right) \cdot \sin^2 \frac{\Delta m_{31}^2 \cdot L}{4E}$$

Leading-term

Next-to-leading

$\nu_\mu$ disapp. probability depends on $\sin^2 2 \theta_{13} \cdot \sin^2 \theta_{23}$ to second order

$\Rightarrow$ Can be used in combination with known $\sin^2 2 \theta_{13}$ to resolve the $\theta_{23}$ octant
$\nu_e$ appearance probability
Leading term only

$$P_{\mu \rightarrow e} \approx \sin^2 \theta_{23} \cdot \sin^2 2\theta_{13} \cdot \sin^2 \left( \frac{\Delta m^2 L}{4E_\nu} \right)$$

$$\Delta m^2 \approx \Delta m_{32}^2 \approx \Delta m_{31}^2$$
$\nu_e$ appearance probability
(exact formula in vacuum)

$$P(\nu_\mu \rightarrow \nu_e) = 4c_{13}^2 s_{13}^2 s_{23}^2 \sin^2 \Delta_{31}$$

Leading term

$$+ 8c_{13}^2 s_{13} s_{23} (c_{12} c_{23} \cos \delta - s_{12} s_{13} s_{23}) \cos \Delta_{32} \sin \Delta_{31} \sin \Delta_{21}$$

$$- 8c_{13}^2 c_{12} c_{23} s_{12} s_{13} s_{23} \sin \delta \sin \Delta_{32} \sin \Delta_{31} \sin \Delta_{21}$$

$$+ 4s_{12}^2 c_{13}^2 (c_{12}^2 c_{23}^2 + s_{12}^2 s_{23}^2 - 2c_{12} c_{23} s_{12} s_{23} s_{13} \cos \delta) \sin^2 \Delta_{21}$$

$c_{ij} = \cos \theta_{ij}, s_{ij} = \sin \theta_{ij}$

$$\Delta_{ij} = \Delta m_{ij}^2 \frac{L}{4E_\nu}$$

replace $\delta$ by $-\delta$ for $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$

CP violating term introduced by interference among three-flavor mixing
\( P(\nu_\mu \rightarrow \nu_e) \approx 4c_{13}^2 s_{13}^2 s_{23}^2 \sin^2 \Delta_{31} \left( 1 + \frac{2a}{\Delta m_{31}^2} \left( 1 - 2s_{13}^2 \right) \right) \)

- Leading including matter effect

\[ + 8c_{13}^2 s_{12} s_{13} s_{23} \left( c_{12} c_{23} \cos \delta - s_{12} s_{13} s_{23} \right) \cos \Delta_{32} \sin \Delta_{31} \sin \Delta_{21} \text{ CP conserving} \]

\[ - 8c_{13}^2 c_{12} c_{23} s_{12} s_{13} s_{23} \sin \delta \sin \Delta_{32} \sin \Delta_{31} \sin \Delta_{21} \text{ CP violating} \]

\[ + 4s_{12}^2 c_{13}^2 \left( c_{12}^2 c_{23}^2 + s_{12}^2 s_{23}^2 s_{13}^2 - 2c_{12} c_{23} s_{12} s_{23} s_{13} \cos \delta \right) \sin^2 \Delta_{21} \text{ Solar} \]

\[ - 8c_{13}^2 s_{13}^2 s_{23}^2 \left( 1 - 2s_{13}^2 \right) \frac{aL}{4E} \cos \Delta_{32} \sin \Delta_{31} \text{ Matter effect \, (small)} \]

\[ c_{ij} = \cos \theta_{ij}, s_{ij} = \sin \theta_{ij} \]

\[ \Delta_{ij} = \Delta m_{ij}^2 \frac{L}{4E} \]

\[ a = 2\sqrt{2}G_F n_e E = 7.56 \times 10^{-5} \text{ eV}^2 \frac{\rho}{gcm^{-3}} \frac{E}{GeV} \]

replace \( \delta \) by \(-\delta\) and \( a \) by \(-a\) for \( P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \)
$P(\nu_\mu \to \nu_e) \approx \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \left( \frac{\Delta m_{32}^2 L}{4E_\nu} \right) \left( 1 + \frac{2a}{\Delta m_{31}^2} (1 - 2\sin^2 \theta_{13}) \right)$

Leading including matter effect

$\sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta \sin^2 \left( \frac{\Delta m_{32}^2 L}{4E_\nu} \right) \sin \left( \frac{\Delta m_{21}^2 L}{4E_\nu} \right)$

CP violating

replace $\delta$ by $-\delta$ and $a$ by $-a$ for $P(\overline{\nu}_\mu \to \overline{\nu}_e)$

$a \equiv 2\sqrt{2}G_F n_e E = 7.56 \times 10^{-5} \text{eV}^2 \frac{\rho}{\text{gcm}^{-3}} \frac{E}{\text{GeV}}$
Normal Hierarchy

$\Delta m^2_{\text{atm}}$

$\Delta m^2_{\odot}$