Determination of $\gamma$ from $B^{\pm} \rightarrow DK^{\pm}$: LHCb and CLEOc

Jim Libby (University of Oxford)
Outline

- Motivation for the precise determination of $\gamma$
- LHCb
  - Overview
  - Status
- Measuring $\gamma$ with $B^\pm \to DK^\pm$ at LHCb
- Complementary measurements of $D$ decay at CLEO-c
CP violation in weak decays of quarks

- CP violation implies differences between matter and antimatter
- In the Standard Model the weak and flavour eigenstates of the three generations of quarks are related by a unitary matrix
- A complex phase introduces CP violating effects
- Represented in terms of the Unitarity Triangle

\[
V_{CKM} V_{CKM}^* = I \Rightarrow V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0
\]

All sides and angles can be measured in \( b \)-hadron decay
Searching for new physics

- Non Standard Model particles contribute within the virtual loops
- Differences between tree-level and loop-level triangles
  - Signature of new physics
- Complements direct searches

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LHC Status

Last dipole lowered April 26th this year!
(First was in March 2005)

Last interconnect – Nov 2007

Quad triplet remediation – Sep 2007

Latest official schedule (August ’07)
had beam commissioning beginning in
May ’08, with then 2 months estimated
before first 14 TeV collisions

Since then, there have been problems,
eg. with shielding bellows in cold interconnects
The LHCb Experiment

- Dedicated experiment for precision measurement of CP violation and rare decays of b-hadrons (and charm) at the LHC
- Collider-mode operation at same time as the general-purpose detectors, with less-focused beams \( \rightarrow \) most events have a single pp interaction

Dipole magnet

Interaction point

10 – 300 mrad forward acceptance
LHCb in a slide

- *pp* collisions at a centre of mass energy of 14 TeV
  - $10^{12}$ $b\bar{b}$/year

- Ring Imaging Cherenkov detectors
  - hadron ID for momentum from 2 to 100 GeV/c

- First level hardware trigger rate from 10→1 MHz
  - 10 MHz the rate of bunch crossings with 1 or more interaction
  - Bunch crossing rate 30 MHz (offset interaction point)

- Software Higher Level Trigger (HLT):
  - inclusive and exclusive selections to reduce storage rate to 2 kHz
**Si Vertex Locator (VELO)**

- 21 stations of Si wafer pairs with $r$ and $\phi$ strip readout
- Split in two halves to allow retraction from beam line
  - When closed 8 mm from beam
- Both detector halves now completed, installed in the pit
Readout: Hybrid PhotoDiodes HPD – 1024 pixels – LHCb development
RICH Ddetectors
3 radiators: RICH1 Aerogel (2-10 GeV), C₄F₁₀ (10-60 GeV)
RICH2 CF₄ (16-100 GeV)

Status: RICH2 ready: full DAQ exercised
RICH1: full commissioning early 2008

$K^\pm$ identification efficiency $\sim 97\%$

$\pi^\pm$ misid rate $\sim 5\%$

With PID

$\pi\pi$ invariant mass

No PID
**Trigger**

Full bandwidth for flavour unlike GPDs

Hardware trigger (L0)
- Fully synchronized (40 MHz), 4 μs fixed latency
- High p_T particles: μ, μμ, e, γ and hadron
  - (typically p_T ~1-4 GeV/c)

Software trigger (HLT)
- Full detector info available, only limit is CPU time
- Use more tracking info to re-confirm L0+high IP
- Full event reconstruction: exclusive and inclusive streams tuned for specific final states

1 MHz (readout of all detector components)

PC farm of ~1000 nodes
≤ 2 kHz (storage: event size ~35kB) (multicore)

L0 HLT and L0×HLT efficiency

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It’s full!
Installation of major structures is essentially complete
Will be ready for collision mid-2008
**Introduction** $B^\pm \rightarrow DK^\pm$

- $B \rightarrow DK$ decays involve $b \rightarrow c$ and $b \rightarrow u$ transitions

![Diagram showing $B^- \rightarrow D^0 K^-$ and $B^- \rightarrow \bar{D}^0 K^-$ decay processes.]

- Access $\gamma$ via interference if $D^0$ and $\bar{D}^0$ decay to the same final state
- These measurements are theoretically clean
  - No penguin $\Rightarrow$ CKM standard candle
  - largest correction is sub-degree from D-mixing
- LHCb looking at a number of strategies to study such decays
  - $B^+$: Atwood-Dunietz-Soni ('ADS'), 3 and 4 body Dalitz Plot Anal.
$B^\pm \rightarrow D(K^0 S \pi^+ \pi^-)K^\pm$

- For $B^+ \rightarrow D(K^0 \pi^+ \pi^-)K^+$

$$A^- = f(m_-,m_+) + r_B e^{i(-\gamma + \delta)} f(m_+,m_-)$$
$$A^+ = f(m_+,m_-) + r_B e^{i(\gamma + \delta)} f(m_-,m_+)$$

$m_\pm = K^0 S \pi^\pm$ invariant mass and $f(m_+,m_-)$ Dalitz amplitudes

- Assume isobar model (sum of Breit-Wigners)

  Number of resonances

  Rel. BW

  Amplitude and phase extracted from $D^+ \rightarrow D^0 \pi^+$ sample at B-factories

  Fit $D$-Dalitz plots from $B$-decay to extract $\gamma$, $r_B$ and $\delta_B$

$m_+^2$ (GeV/c$^2$)

$\bar{D}^0$  

$\rho(770)$

$K^*(892)$

$M_+^2$ (GeV/c$^2$)

Non-resonant
Current $e^+e^-$ results

- Current best direct constraints on $\gamma$:
  \[
  \phi_3 = (53^{+15}_{-18} \text{ (stat)} \pm 3 \text{ (syst)} \pm 9 \text{ (model)})^\circ \quad [\text{Belle}]
  \]
  \[
  \gamma = (92 \pm 41 \text{ (stat)} \pm 11 \text{ (syst)} \pm 12 \text{ (model)})^\circ \quad [\text{BABAR}]
  \]

- Based on $\sim$300 events each (1/3 of final data set)

- However, large error from isobar model assumptions

- BABAR and Belle use large samples of flavour tagged $D^{*+} \rightarrow D^0\pi^+$ events to find parameters of the isobar model
  - Excellent knowledge of $|f|^2$ but phases less well known

- Model uncertainties from assumptions about the resonance structures in the model
Isobar model uncertainty

Most challenging aspects of the model uncertainty come from $K\pi$ and $\pi\pi$ S-wave

Fit to flavour tag sample

<table>
<thead>
<tr>
<th>Resonance</th>
<th>Amplitude</th>
<th>Phase (deg)</th>
<th>Fit fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^*(892)^-$</td>
<td>1.781 ± 0.018</td>
<td>131.0 ± 0.8</td>
<td>0.586</td>
</tr>
<tr>
<td>$K^0_0(1430)^-$</td>
<td>2.45 ± 0.08</td>
<td>-8.3 ± 2.5</td>
<td>0.083</td>
</tr>
<tr>
<td>$K^+_2(1430)^-$</td>
<td>1.05 ± 0.06</td>
<td>-54.3 ± 2.6</td>
<td>0.027</td>
</tr>
<tr>
<td>$K^+_0(1410)^-$</td>
<td>0.52 ± 0.09</td>
<td>154 ± 20</td>
<td>0.004</td>
</tr>
<tr>
<td>$K^+_2(1680)^-$</td>
<td>0.89 ± 0.30</td>
<td>-139 ± 14</td>
<td>0.003</td>
</tr>
<tr>
<td>$K^+(892)^+$</td>
<td>0.180 ± 0.008</td>
<td>-44.1 ± 2.5</td>
<td>0.006</td>
</tr>
<tr>
<td>$K^0_0(1430)^+$</td>
<td>0.37 ± 0.07</td>
<td>18 ± 9</td>
<td>0.002</td>
</tr>
<tr>
<td>$K^+_2(1430)^+$</td>
<td>0.075 ± 0.038</td>
<td>-104 ± 23</td>
<td>0.000</td>
</tr>
<tr>
<td>$\rho(770)$</td>
<td>1 (fixed)</td>
<td>0 (fixed)</td>
<td>0.224</td>
</tr>
<tr>
<td>$\omega(782)$</td>
<td>0.0391 ± 0.0016</td>
<td>115.3 ± 2.5</td>
<td>0.006</td>
</tr>
<tr>
<td>$f_0(980)$</td>
<td>0.482 ± 0.012</td>
<td>-141.8 ± 2.2</td>
<td>0.061</td>
</tr>
<tr>
<td>$f_0(1370)$</td>
<td>2.25 ± 0.30</td>
<td>113.2 ± 3.7</td>
<td>0.032</td>
</tr>
<tr>
<td>$f_2(1270)$</td>
<td>0.922 ± 0.041</td>
<td>-21.3 ± 3.1</td>
<td>0.030</td>
</tr>
<tr>
<td>$\rho(1450)$</td>
<td>0.52 ± 0.09</td>
<td>38 ± 13</td>
<td>0.002</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.36 ± 0.05</td>
<td>-177.9 ± 2.7</td>
<td>0.093</td>
</tr>
<tr>
<td>$\sigma'$</td>
<td>0.340 ± 0.026</td>
<td>153.0 ± 3.8</td>
<td>0.013</td>
</tr>
<tr>
<td>Non Resonant</td>
<td>3.53 ± 0.44</td>
<td>128 ± 6</td>
<td>0.073</td>
</tr>
</tbody>
</table>

BABAR (PRL 95 121802,2005)

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$B^\pm \rightarrow D(K^0_S \pi^+ \pi^-)K^\pm$ at LHCb

- Simulation studies performed to determine the expected yields and backgrounds at LHCb
  - One ‘nominal’ year of data-taking 2 fb$^{-1}$
  - Total luminosity goal 10 fb$^{-1}$
- Selection studies performed on PYTHIA/EVTGEN/GEANT4 simulated samples of signal and background events
- Limited statistics available for background estimates
  - 34 million $b$-inclusive events corresponds to $\sim$15 minutes of data taking at nominal luminosity
- Trigger simulation is applied for Level-0 and large impact parameter with $p_t$, HLT
$B^{\pm} \rightarrow D(K^0_S \pi^+ \pi^-)K^{\pm}$ at LHCb

- Selection based on large impact parameter, RICH particle ID and good $p$ resolution
- Efficiency = $0.7 \times 10^{-3}$
- Backgrounds:
  - $B^{\pm} \rightarrow D(K^0_S \pi^+ \pi^-)\pi^{\pm}$
    B/S = 0.24
  - Combinatoric
    B/S < 0.7 at 90% c.l.

5000 events/2fb$^{-1}$

(LHCb-048-2007)
Model uncertainty impact on LHCb

- The model-dependent likelihood fit yields an uncertainty on $\gamma$ between $7-12^\circ$ for an $r_B = 0.1$
  - Range represents differing assumptions about the background
- However, the current model uncertainty is $10-15^\circ$ with an $r_B = 0.1$
  - Uncertainties $\propto 1/r_B$
- **Without improvements LHCb sensitivity will be dominated by model assumptions within 1 year of data taking**
- Motivates a model-independent method that relies on a binned analysis of the Dalitz plot
  - Disadvantage is that information is lost via binning
Binned method

- Proposed in the original paper by Giri, Grossman, Soffer and Zupan and since been extended significantly by Bondar and Poluektov

- Bin the Dalitz plot symmetrically about $m_+^2 = m_-^2$ then number of entries in $B^-$ decay given by:

\[
N_i^- \propto \int_{D_i} \left| f(m_-, m_+) \right|^2 dD + r_B^2 \int_{D_i} \left| f(m_+, m_-) \right|^2 dD
\]

\[
+ 2\sqrt{\int_{D_i} \left| f(m_-, m_+) \right|^2 dD \int_{D_i} \left| f(m_+, m_-) \right|^2 dD} \left( x_{ci} + y_{si} \right)
\]

\[
x_\pm = r_B \cos(\delta_B \pm \gamma) \quad y_\pm = r_B \sin(\delta_B \pm \gamma)
\]

'Cartesian coordinates'

Average cosine and sine of strong phase difference between $D^0$ and $\bar{D}^0$ decay amplitudes ($\Delta\delta_{D^0}$) in this bin

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Binned method continued

- Can determine $s_i$ and $c_i$ at the same time as extracting $\gamma$, $r_B$ and $\delta_B$ from $B$ data
  - $3 + N_{\text{bins}}$ free parameters ($c_i = c_{-i}$ and $s_i = -s_{-i}$)
  - Huge loss in $\gamma$ sensitivity not practical until you have $O(10^6)$ events (2500/fb\(^{-1}\) @ LHCb)

- However, **CP-correlated** $e^+e^- \rightarrow \psi'' \rightarrow D^0 \bar{D}^0$ data where one decay is to $K_S\pi\pi$ and the other decays to a CP eigenstate and $K_S\pi\pi$ allows $c_i$ and $s_i$ to be determined, respectively
Enter CLEO-c

CLEO is the grand-daddy of flavour physics, with history of achievement dating back over 25 years

CLEO-c is latest incarnation. Dedicated programme of data-taking at and above the $c\bar{c}$ threshold

**Important studies for LQCD and B physics**

Oxford LHCb physicists (with Bristol) have joined CLEO-c in order to measure quantities essential for the $\gamma$ studies
CLEO-c detector
CLEO-c data samples


3686 MeV, 54 pb⁻¹, \( N(\psi(2S)) \approx 27 \text{M} \) \( e^+e^- \rightarrow \psi(2S) \rightarrow \pi\pi \ J/\psi, \ \gamma_{\text{c}} \) etc.

3773 MeV, 800 pb⁻¹ delivered, \( \sim 3 \text{ milion } \psi(3770) \rightarrow D^0 \bar{D}^0 \)

4170 MeV, 195 pb⁻¹ \( \rightarrow \sim 300 \text{pb}^{-1} \rightarrow \text{more} \rightarrow \sim 720 \text{pb}^{-1}, \)

3970–4260 MeV energy scan, 60 pb⁻¹ in 12 points
CLEO-c: double tagged $\psi(3770)$ events

CLEO-c has collected ~ 800 fb$^{-1}$ at the $\psi(3770)$
DDbar produced in quantum entangled state:

$$e^+e^- \rightarrow \psi'' \rightarrow \frac{1}{\sqrt{2}} [D^0\overline{D}^0 - \overline{D}^0D^0]$$

Reconstruct one D in decay of interest for $\gamma$
analysis (eg. $K\pi\pi$), & other in CP eigenstate
(eg. $KK, K_s\pi^0$ ...) then CP of other is fixed.

$\equiv$ Almost
background free

Can use $K_L$⇒

From talk by E. White
at Charm 07
CP-tagged $K_S\pi^+\pi^-$ Dalitz Plots

1/3 of total data
(<1/2 the CP tags)

$K_S\rho^0$ resonance enhanced
in CP-odd Dalitz plot

CP-odd $K_S\rho^0$ resonance absent
in CP-even Dalitz plot

Studies not complete but projected uncertainties on c and s will lead to 3-5 degree uncertainty on $\gamma$
**Inkblot test**

- Bondar and Poluektov show that the rectangular binning is far from optimal for both CLEOc and $\gamma$ analyses
  - 16 uniform bins has only 60% of the B statistical sensitivity
  - c and s errors would be 3 times larger from the $\psi$

- Best B-data sensitivity **when** $\cos(\Delta\delta_D)$ and $\sin(\Delta\delta_D)$ are **as uniform as possible** within a bin

**Absolute value of strong phase diff.**

(BABAR model used in LHCb-48-2007)

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Good approximation and the binning that yields smallest s and c errors is equal

$\Delta\delta_D$ bins-80% of the unbinned precision

$$2\pi(i - \frac{1}{2}) / N < \Delta\delta_D(m_1^2, m_2^2) < 2\pi(i + \frac{1}{2}) / N$$
Implementation at LHCb

- Generate samples of $B^\pm \rightarrow D(K^0_S \pi \pi)K^\pm$ with a mean of 5000 events split between the charges
- Bin according to strong phase difference, $\Delta\delta_D \Rightarrow$
- Minimise $\chi^2$

$$\chi^2 = \sum_{i=-8(i\neq 0)}^{8} \left[ \frac{(n_i^+ - N_i^+(x_+, y_+, h))^2}{n_i^+} + \frac{(n_i^- - N_i^-(x_-, y_-, h))^2}{n_i^-} \right]$$

$n_i^\pm$ = number of $B^\pm \rightarrow D(K^0_S \pi^+ \pi^-)K^\pm$ events in $i^{th}$ bin

$N_i^\pm (x_\pm, y_\pm, h) = h \left[ K_{\pm i} + r_B^2 K_{\mp i} + 2\sqrt{K_i K_{-i}} (c_i x_\pm \pm s_i y_\pm) \right]$

$h$ = normalization factor

$K_{\pm i} = \int_D \left| f(m_+^2, m_-^2) \right|^2 dD$ [measured from flavour tag data]

- $K_i, c_i$ and $s_i$ amplitudes calculated from model
- In reality from flavour tagged samples and CLEO-c

$\Delta\delta_B (\text{rad})$

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No background with predicted 2 fb\(^{-1}\) yield

5000 experiments

Input parameters
\(\gamma=60^\circ\), \(r_B=0.1\) and
\(\delta_B=130^\circ\)

Model independent average uncertainty 7.7\(^\circ\) (c.f. Model dependent 5.9\(^\circ\))
### $\gamma$ uncertainties with 5000 toy experiments

<table>
<thead>
<tr>
<th>Scenario</th>
<th>2 fb$^{-1}$ Mod. Indep.</th>
<th>10 fb$^{-1}$ Mod. Indep.</th>
<th>2 fb$^{-1}$ Mod. Dep. (LHCb-048-2007)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No background</td>
<td>7.9°</td>
<td>3.5°</td>
<td>5.9°</td>
</tr>
<tr>
<td>Acceptance</td>
<td>8.1°</td>
<td>3.5°</td>
<td>5.5°</td>
</tr>
<tr>
<td>D$\pi$ (B/S = 0.24) (Best case scenario)</td>
<td>8.8°</td>
<td>4.0°</td>
<td>7.3°</td>
</tr>
<tr>
<td>DK$_{comb}$ (B/S=0.7) (Worst case scenario)</td>
<td>12.8°</td>
<td>5.7°</td>
<td>11.7°</td>
</tr>
</tbody>
</table>
\[ B^\pm \rightarrow D(K^0 S \pi^+ \pi^-)K^\pm \] at LHCb

Model independent fit with binning that yields smallest error from exploiting CLEO-c data

- **Binning depends on model - only consequence of incorrect model is non-optimal binning and a loss of sensitivity**

Measurement has no troublesome and hard-to-quantify systematic and outperforms model-dependent approach with full LHCb dataset with currently assigned model error

- **10 fb^{-1} statistical uncertainty 4-6° depending on background**

**CLEO-c measurements essential to validation of assumptions in model dependent measurement**

**LHCb-2007-141 – Available via CERN document server**
ADS
Look at DCS and CF decays of D to rates that have enhanced rates that have enhanced interferenced terms

\[
\Gamma(B^- \to (K^-\pi^+)_D K^-) \propto 1 + (r_B r_{D}^{K\pi})^2 + 2r_B r_{D}^{K\pi} \cos(\delta_B - \delta_{D}^{K\pi} - \gamma),
\]

\[
\Gamma(B^- \to (K^+\pi^-)_D K^-) \propto r_B^2 + (r_{D}^{K\pi})^2 + 2r_B r_{D}^{K\pi} \cos(\delta_B + \delta_{D}^{K\pi} - \gamma),
\]

\[
\Gamma(B^+ \to (K^+\pi^-)_D K^+) \propto 1 + (r_B r_{D}^{K\pi})^2 + 2r_B r_{D}^{K\pi} \cos(\delta_B - \delta_{D}^{K\pi} + \gamma),
\]

\[
\Gamma(B^+ \to (K^-\pi^+)_D K^+) \propto r_B^2 + (r_{D}^{K\pi})^2 + 2r_B r_{D}^{K\pi} \cos(\delta_B + \delta_{D}^{K\pi} + \gamma)
\]

\[
\Gamma(B^- \to (h^+h^-)_D K^-) \propto 1 + r_B^2 + 2r_B \cos(\delta_B - \gamma)
\]

\[
\Gamma(B^+ \to (h^+h^-)_D K^+) \propto 1 + r_B^2 + 2r_B \cos(\delta_B + \gamma)
\]

- Unknows: \( r_B \sim 0.1, \delta_B, \delta_{D}^{K\pi}, \gamma, N_{K\pi}, N_{hh} \) (\( r_D = 0.06 \) well measured)
- With knowledge of the relevant efficiencies and BRs, the normalisation constants (\( N_{K\pi}, N_{hh} \)) can be related to one another
- Important constraint from CLEOc \( \sigma(\cos \delta_{D}^{K\pi}) = 0.1 - 0.2 \)
- Overconstrained: 6 observables and 5 unknowns
Expected yields

- **ADS measurement is a counting experiment - but suppressed modes have $\sim 10^{-7}$ BRs**
  - Principal challenge background suppression

- **Detailed selections studies as for Dalitz analysis**
  - LHCb-2006-066

<table>
<thead>
<tr>
<th>Channel</th>
<th>Signal yield/2 fb$^{-1}$</th>
<th>B/S</th>
<th>B-factory yields ($\sim 1/4$ final data set)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^\pm \to (K^\pm \pi^\mp)_D K^\pm$</td>
<td>56,000</td>
<td>0.6</td>
<td>4000</td>
</tr>
<tr>
<td>$B^\pm \to (h^+ h^-)_D K^\pm$</td>
<td>8200</td>
<td>1.8</td>
<td>500</td>
</tr>
</tbody>
</table>

- The suppressed modes have yields varying from 0 to 500 depending on the strong parameters
  - 780 background events predicted
Sensitivity from 2-body

\( \delta_D = -25^\circ \) – fit results from 1000 toy 2 fb\(^{-1} \) experiments:

\( \delta_D \) constraint leads to a 0.5-1.0\(^\circ \) reduction in \( \sigma_\gamma \)

Also important for D mixing measurements

<table>
<thead>
<tr>
<th>( \delta_D ) ((^\circ))</th>
<th>-25</th>
<th>-16.6</th>
<th>-8.3</th>
<th>0</th>
<th>8.3</th>
<th>16.6</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_\gamma ) ((^\circ))</td>
<td>9.5</td>
<td>8.6</td>
<td>7.5</td>
<td>8.6</td>
<td>8.6</td>
<td>9.3</td>
<td>9.4</td>
</tr>
</tbody>
</table>

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Four-body ADS

- $B \to D(K \pi \pi \pi)K$ can also be used for ADS style analysis
- Similar yields to 2-body – slightly worse B/S
  - LHCb-2007-004
- However, need to account for the resonant substructure in $D \to K \pi \pi \pi$
  - made up of $D \to K^*\rho$, $K^-a_1(1260)^+$
  - in principle each point in the phase space has a different strong phase associated with it - 3 and 4 body Dalitz plot analyses exploit this very fact to extract $\gamma$ from amplitude fits
- Atwood and Soni (hep-ph/0304085) show how to modify the usual ADS equations for this case
  - Introduce coherence parameter $R_{K3\pi}$ which dilutes interference term sensitive to $\gamma$

$$\Gamma(B^- \to (K^+\pi^-\pi^-\pi^+)_D K^-) \propto r_B^2 + (r_D^{K3\pi})^2 + 2r_B r_D^{K3\pi} R_{K3\pi} \cos(\delta_B + \delta_D^{K3\pi} - \gamma)$$

- $R_{K3\pi}$ ranges from
  - 1=coherent (dominated by a single mode) to
  - 0=incoherent (several significant components)
- Can slice and dice phase space to find most coherent regions
Determining the coherence factor

- Measurements of the rate of $K3\pi$ versus different tags at CLEO-c allows direct access to $R_{K3\pi}$ and $\delta_{K3\pi}$.

1. Normalisation from CF $K^-\pi^+\pi^+\pi^-$ vs. $K^+\pi^-\pi^-\pi^+$ and $K^-\pi^+\pi^+\pi^-$ vs. $K^+\pi^-$

2. CP eigenstates:
   \[ \Gamma(K3\pi : CP \pm) = \Gamma_{K3\pi}^{CF} \Gamma_{CP} \left[ 1 + \left( r_{D}^{K3\pi} \right)^2 \mp 2r_{D}^{K3\pi} R_{K3\pi} \cos \delta_{D}^{K3\pi} \right] \]

3. $K^-\pi^+\pi^+\pi^-$ vs. $K^-\pi^+\pi^+\pi^-$:
   \[ \Gamma(K^-3\pi : K^-3\pi) = \Gamma_{K3\pi}^{CF} \Gamma_{K3\pi}^{DCS} \left[ 1 - R_{K3\pi}^2 \right] \]

4. $K^-\pi^+\pi^+\pi^-$ vs. $K^-\pi^-$:
   \[ \Gamma(K^-3\pi : K^-\pi) \approx \Gamma_{K3\pi}^{CF} \Gamma_{K3\pi}^{DCS} \left[ 1 + \left( \frac{r_{D}^{K3\pi}}{r_{D}^{K3\pi}} \right)^2 + 2 \frac{r_{D}^{K3\pi}}{r_{D}^{K3\pi}} R_{K3\pi} \cos \delta_{D}^{K3\pi} \right] \]

Assume $\delta_{D}^{K3\pi} \sim \pi$

<table>
<thead>
<tr>
<th>K3\pi tag side</th>
<th>Expected 800 pb$^{-1}$ yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>K3\pi CF</td>
<td>3700</td>
</tr>
<tr>
<td>K3\pi CF</td>
<td>5000</td>
</tr>
<tr>
<td>K3\pi/K3\pi DCS</td>
<td>0-40 per mode</td>
</tr>
<tr>
<td>K^0_S\pi^0</td>
<td>650</td>
</tr>
<tr>
<td>K^+K^-</td>
<td>500</td>
</tr>
<tr>
<td>\pi^+\pi^-</td>
<td>200</td>
</tr>
</tbody>
</table>
Determining the coherence factor

- Analysis underway 10% effects in CP modes so great care with
  - Background subtraction
  - Efficiency calculation

- Estimate of current sensitivity with the addition of $K^0_L\pi^0$ and further CP tags i.e. $K^0_S\eta$ to be added

- Further information in mixed CP SCS tags such as $K^0_S\pi^+\pi^-$
  \[ \sigma_{stat} \sim 0.1 \text{ with } 800 \text{ pb}^{-1} \]

- Binned analysis to determine the most coherent regions
Conclusion-LHCb

- LHCb has estimated 2 fb\(^{-1}\) sensitivity to \(\gamma\) in \(B^{\pm} \to DK^{\pm}\) with
  - \(D \to K^0_S \pi^+ \pi^-\) - \(\sigma_\gamma = 7-12^\circ\)
  - \(D \to K^- \pi^+ \) and \(D \to h^+ h^-\) - \(\sigma_\gamma = 7.5-9.5^\circ\)
  - \(D \to K^- \pi^+ \pi^+ \pi^-\) will add additional information

- Not the whole story with theoretically clean measurements:
  - \(B^0 \to DK^*\) \(\sigma_\gamma \sim 9^\circ\) [LHCb-2007-050]
  - \(B_s \to D_s K\) \(\sigma_\gamma + \phi_s \sim 10^\circ\) [LHCb-2007-041]

- A few degree precision on \(\gamma\) by the end of LHCb
Conclusion CLEOc

- CLEO-c measurements essential to fulfilling this goal
- But there is much more that can be done
- Full amplitude analysis of 4-body should yield ultimate precision
  - Need DCS model, which can be accessed via CP tags at CLEOc
  - Also will guide division of phase space for binned coherence factor analysis
- Other modes that can be used:
  - \( D \rightarrow K^- \pi^+ \pi^0 \) (Coherence analysis underway)
  - \( D \rightarrow K^0_s K^+ K^- \) and \( D \rightarrow K^0_s K^+ \pi^- \)
  - \( D \rightarrow K^- K^+ \pi^+ \pi^- \) and \( D \rightarrow K^0_s \pi^- \pi^+ \pi^0 \)
Additional slides
Aside: K-matrix

- Breit Wigner description of broad overlapping resonances violates unitarity and requires non-physical $\sigma'$
- K-matrix description preserves unitarity

- First studies (Lauren Martin/JL) of LHCb $\gamma$ fit with one K-matrix parameterisation of the $\pi\pi$ S-wave
  - Difference between assuming K-matrix and BW model consistent with B-factory observations
  - Draft available from CPWG webpage
- Explore different physical K-matrix parameterisation to evaluate systematic rather than introduce $\sigma'$ 
  **will reduce model uncertainty**
No background with predicted 2 fb⁻¹ yield

5000 experiments

Input parameters
\( \gamma=60^\circ, r_B=0.1 \) and \( \delta_B=130^\circ \)

The four Cartesian coordinates and normalization are free parameters

All pulls are normal therefore calculate \( \gamma, r_B \) and \( \delta_B \) with propagated Cartesian uncertainties
## Toy experiment results: $\gamma$ (2 fb$^{-1}$)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Mean</th>
<th>RMS</th>
<th>Mean $\sigma$</th>
<th>Mean pull</th>
<th>Pull RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>No bck</td>
<td>60.5±0.1</td>
<td>7.9</td>
<td>7.8</td>
<td>0.045±0.015</td>
<td>1.05</td>
</tr>
<tr>
<td>Acc</td>
<td>60.7±0.1</td>
<td>8.1</td>
<td>7.8</td>
<td>0.075±0.015</td>
<td>1.07</td>
</tr>
<tr>
<td>$D\pi$</td>
<td>60.7±0.1</td>
<td>8.8</td>
<td>8.8</td>
<td>0.088±0.015</td>
<td>1.04</td>
</tr>
<tr>
<td>$D\pi + DK$ (B/S=0.7)</td>
<td>60.7±0.2</td>
<td>12.8</td>
<td>12.2</td>
<td>0.049±0.016</td>
<td>1.11</td>
</tr>
<tr>
<td>$D\pi + PS$ (B/S=0.7)</td>
<td>60.8±0.2</td>
<td>12.8</td>
<td>12.5</td>
<td>0.064±0.015</td>
<td>1.05</td>
</tr>
<tr>
<td>$D\pi + DK+$ PS (50:50) (B/S=0.7)</td>
<td>60.7±0.2</td>
<td>12.7</td>
<td>12.6</td>
<td>0.049±0.015</td>
<td>1.04</td>
</tr>
</tbody>
</table>
## Toy experiment results: $\gamma$ (10 fb$^{-1}$)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Mean</th>
<th>RMS</th>
<th>Mean $\sigma$</th>
<th>Mean pull</th>
<th>Pull RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>No bck</td>
<td>60.17±0.05</td>
<td>3.5</td>
<td>3.4</td>
<td>0.050±0.015</td>
<td>1.03</td>
</tr>
<tr>
<td>Acc</td>
<td>60.13±0.05</td>
<td>3.5</td>
<td>3.4</td>
<td>0.036±0.015</td>
<td>1.01</td>
</tr>
<tr>
<td>$D\pi$</td>
<td>60.22±0.06</td>
<td>4.0</td>
<td>3.9</td>
<td>0.054±0.015</td>
<td>1.03</td>
</tr>
<tr>
<td>$D\pi + DK$ (B/S=0.7)</td>
<td>60.18±0.08</td>
<td>5.7</td>
<td>5.7</td>
<td>0.030±0.015</td>
<td>1.01</td>
</tr>
<tr>
<td>$D\pi + PS$ (B/S=0.7)</td>
<td>60.26±0.08</td>
<td>5.5</td>
<td>5.5</td>
<td>0.045±0.015</td>
<td>1.00</td>
</tr>
<tr>
<td>$D\pi + DK+ PS$ (50:50) (B/S=0.7)</td>
<td>60.22±0.08</td>
<td>5.4</td>
<td>5.6</td>
<td>0.038±0.015</td>
<td>0.97</td>
</tr>
</tbody>
</table>
## Toy experiment results: $r_B$ (2 fb$^{-1}$)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Mean</th>
<th>RMS</th>
<th>Mean $\sigma$</th>
<th>Mean pull</th>
<th>Pull RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>No bck</td>
<td>0.1017±0.0002</td>
<td>0.013</td>
<td>0.013</td>
<td>0.143±0.015</td>
<td>1.02</td>
</tr>
<tr>
<td>Acc</td>
<td>0.1017±0.0002</td>
<td>0.014</td>
<td>0.013</td>
<td>0.175±0.016</td>
<td>1.13</td>
</tr>
<tr>
<td>D$\pi$</td>
<td>0.1015±0.0002</td>
<td>0.014</td>
<td>0.014</td>
<td>0.123±0.015</td>
<td>1.02</td>
</tr>
<tr>
<td>D$\pi$ + DK (B/S=0.7)</td>
<td>0.1031±0.0003</td>
<td>0.020</td>
<td>0.020</td>
<td>0.215±0.016</td>
<td>1.16</td>
</tr>
<tr>
<td>D$\pi$ + PS (B/S=0.7)</td>
<td>0.1035±0.0003</td>
<td>0.020</td>
<td>0.019</td>
<td>0.175±0.015</td>
<td>0.99</td>
</tr>
<tr>
<td>D$\pi$ + DK+ PS (50:50) (B/S=0.7)</td>
<td>0.1038±0.0003</td>
<td>0.020</td>
<td>0.020</td>
<td>0.186±0.015</td>
<td>0.98</td>
</tr>
</tbody>
</table>
# Toy experiment results: $r_B$ (10 fb$^{-1}$)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Mean</th>
<th>RMS</th>
<th>Mean $\sigma$</th>
<th>Mean pull</th>
<th>Pull RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>No bck</td>
<td>0.1003±0.0001</td>
<td>0.006</td>
<td>0.006</td>
<td>0.056±0.015</td>
<td>1.00</td>
</tr>
<tr>
<td>Acc</td>
<td>0.1003±0.0001</td>
<td>0.006</td>
<td>0.006</td>
<td>0.051±0.015</td>
<td>1.01</td>
</tr>
<tr>
<td>$D\pi$</td>
<td>0.1003±0.0001</td>
<td>0.006</td>
<td>0.006</td>
<td>0.049±0.015</td>
<td>0.98</td>
</tr>
<tr>
<td>$D\pi$ + DK (B/S=0.7)</td>
<td>0.1009±0.0001</td>
<td>0.009</td>
<td>0.009</td>
<td>0.101±0.015</td>
<td>0.97</td>
</tr>
<tr>
<td>$D\pi$ + PS (B/S=0.7)</td>
<td>0.1008±0.0001</td>
<td>0.009</td>
<td>0.009</td>
<td>0.093±0.015</td>
<td>0.99</td>
</tr>
<tr>
<td>$D\pi$ + DK+ PS (50:50)</td>
<td>0.1007±0.0001</td>
<td>0.009</td>
<td>0.009</td>
<td>0.077±0.015</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Acceptance

- Acceptance in each bin calculated as a weighted average of the acceptance function used for model dependent studies
  - 15% relative difference amongst bins
- Modifies the fit function:
  \[
  N_{i}^{\pm}(x_{\pm}, y_{\pm}, h) = h \varepsilon_{\pm i} \left[ K_{\pm i} + r_{B}^{2} K_{\mp i} + 2 \sqrt{K_{i} K_{\mp i}} (c_{i} x_{\pm} \pm s_{i} y_{\pm}) \right]
  \]
  \[
  \varepsilon_{i} = \frac{\int_{D_{i}} \left| f(m_{+}^{2}, m_{-}^{2}) \right|^{2} \varepsilon(m_{+}^{2}, m_{-}^{2}) dD}{K_{i}}
  \]
  where \( \varepsilon(m_{+}^{2}, m_{-}^{2}) = 0.28 \times 10^{-3} (1 - 0.08(m_{+}^{2} + m_{-}^{2})) \)
- Average \( \gamma \) uncertainty increases to 8.1°
Background

- 3 types of background to consider
  - \( B \rightarrow D(K_S \pi \pi) \pi (B/S = 0.24) \)
    - \( r_B(D\pi) \sim O(10^{-3}) \) so Dalitz plots are like \( D^0 \) and \( \bar{D}^0 \) for \( B^- \) and \( B^+ \), respectively
  - Combinatoric (\( B/S < 0.7 \))-mixtures of two types considered
    1. \( D_{\text{comb}} \): real \( D \rightarrow D(K_S \pi \pi) \) combined with a bachelor \( K \)
      - Dalitz plot an even sum of \( D^0 \) and \( \bar{D}^0 \) decays
    2. \( P_{\text{comb}} \): combinatoric \( D \) with a bachelor \( K \)
      - Follows phase space

- Integrate background PDFs used in model-dependent analysis over each bin, then scaled to background level assumed:

\[
\begin{align*}
N(D\pi)_i^{\pm} &\propto \varepsilon_{\pm i} K_{\pm i} \\
N(DK_{\text{comb}})_i^{\pm} &\propto \frac{1}{2} (\varepsilon_{\pm i} K_{\pm i} + \varepsilon_{\mp i} K_{\mp i}) \\
N(PS_{\text{comb}})_i^{\pm} &\propto P_i
\end{align*}
\]

fractional area of Dalitz space covered by bin
Systematic related to acceptance

- The acceptance varies over the Dalitz plane
- The relative acceptance in each bin can be measured using the $B \rightarrow D\pi$ control sample with DK selection applied without bachelor K PID

$$\varepsilon_i = \frac{\int_{D_i} \left| f(m_+^2, m_-^2) \right|^2 \varepsilon(m_+^2, m_-^2) dD}{K_i} \propto \frac{N(B \rightarrow D(K_S^0\pi\pi)\pi)_i}{K_i}$$

- With the DC04 selection expect 60k events/2 fb$^{-1}$
  - Relative relative-efficiency uncertainty 1-4%/\Delta\delta_D bin with 2 fb$^{-1}$
  - Increased statistics reduces error
- Toy MC study smearing bin efficiencies in event generation by this amount leads to an additional 1° uncertainty without background and 3.2° uncertainty with $DK_{comb}$ B/S=0.7
  - Small effect compared to statistical uncertainty
- NB: the efficiency related to the PID of the bachelor $\pi/K$ can be factored out and will be determined from the $D^{*}\rightarrow D(K\pi)\pi$ data to better than one percent-ignore at present
Asymmetry in efficiency in Dalitz space

- Considered charge asymmetries in the efficiency across the Dalitz plane
  - \( \varepsilon(m^2_+, m^2_-) \neq \varepsilon(m^2_-, m^2_+) \)

- Generated with the efficiency biased relative to one another depending on whether the event had \( m^2_+ > m^2_- \) or \( m^2_+ < m^2_- \)

- **Maximum bias on \( \gamma \) induced was <1° for 10% relative effect and full background**

- 10% effects would be evident in the \( D\pi \) sample
Resolution

- $\Delta \delta_0$ binning has some narrow regions in Dalitz space
- Investigation of how resolution on the Dalitz variables might affect the extraction of $\gamma$
- 10 MeV$^2$/c$^4$ resolution (DC04) on Dalitz variables and generated toy experiments with this smearing
- Found that this led to a few bins with largest (red) and smallest (dark blue) phase difference having a 2-3% relative changes in expected yields due to resolution induced migration
- **Fit results on toy experiments where resolution included in generation but ignored in fit found no significant bias (<0.5°) on $\gamma$**
Background fractions

- Combinatoric background rate will be determined from $B$ and $D$ mass sidebands which will cover at least 2-3 times the area of the signal region
  - Use $10\times$ in DC04 background studies but this will probably be unrealistic with data
- If background distributions relatively flat in masses one can estimate that this leads to B/S will be determined absolutely to around 0.01 or better
  - Toy studies suggest that there is no impact on $\gamma$ precision with this kind of uncertainty
- Maybe complications depending on Dalitz space distribution of the PS background but can only speculate until we have the data in hand
Background composition

- For favoured mode background dominated by $B \to D\pi$
  - 14 $\times$ larger BF
  - Power of the RICH
- For suppressed combinatoric dominates (green)
- For $B \to D(\text{hh})K$ more even mixture of comb. and $D\pi$
  - $B \to D(KK)K$ has significant non-resonant $B \to KKK$ component