Family symmetries and fermion masses/mixings

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Outline

Standard model: Yukawa couplings

Yukawa Lagrangian

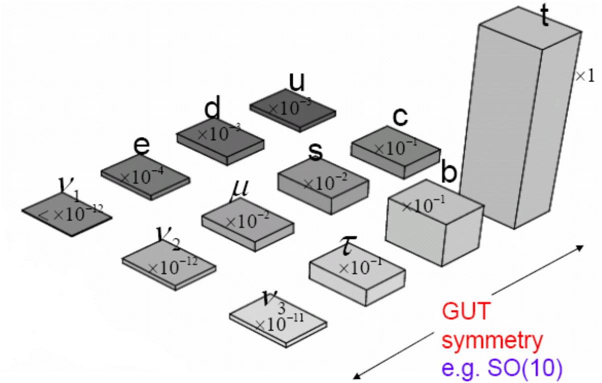
$$L_{Yukawa} = Y^u_{ij} Q^i u^{c,j} H + Y^u_{ij} Q^i u^{c,j} \bar{H}$$

Mass matrices

$$M^u_{ij} = Y^u_{ij} \langle H^0 \rangle$$

$$M^d_{ij} = Y^d_{ij} \langle \bar{H}^0 \rangle$$

Summary of data: masses



Summary of data: quark mixing

Wolfenstein parameterization

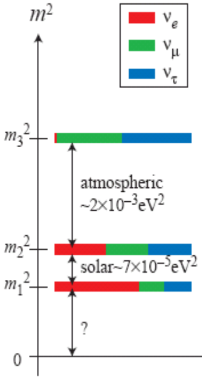
$$V_{CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ -\lambda & 1 & \lambda^2 \\ \lambda^3 & -\lambda^2 & 1 \end{pmatrix}$$

$\lambda \approx 0.23$

Summary of data: lepton mixing

Harrison-Perkins-Scott

$$V_{PMNS} = \begin{pmatrix} -\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{pmatrix}$$



GUT scale texture fits: Quarks

Symmetric fits

$$Y^u \sim \begin{pmatrix} 0 & i\,\epsilon_u^3 & \epsilon_u^3 \\ \cdot & \epsilon_u^2 & \epsilon_u^2 \\ \cdot & \cdot & 1 \end{pmatrix}$$

$$Y^d \sim \begin{pmatrix} 0 & 1.7\epsilon_d^3 & (0.8)e^{-i\,(45)^o}\,\epsilon_d^3 \\ \cdot & \epsilon_d^2 & (2.1)\epsilon_d^2 \\ \cdot & \cdot & 1 \end{pmatrix}$$

$$\epsilon_d \sim 0.13 \qquad \epsilon_u \sim 0.048$$

Paper out soon (today?)

GUT scale texture fits: Charged leptons

Georgi-Jarlskog relations

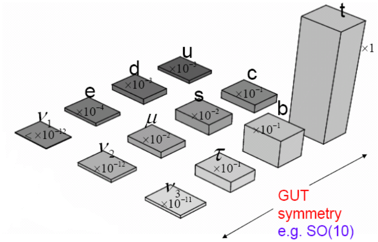
To good approximation:

- $\frac{m_b}{m_\tau}(M_X) = 1$
- 11 texture zero $\rightarrow \frac{det(M^q)}{det(M^l)}(M_X) = 1$

Hints of GUT? But:

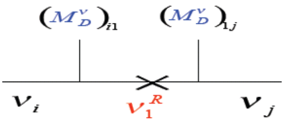
- $\frac{m_s}{m_\mu}(M_X) = \frac{1}{3}$

Neutrinos?



Seesaw mechanism

$$m_\nu = (M_D^\nu)\,(M_{RR})^{-1}\,(M_D^\nu)^T$$



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Seesaw mechanism: masses

Seesaw formula

$$m_\nu = (M_D^\nu)\,(M_{RR})^{-1}\,(M_D^\nu)^T$$

1 generation example

$$M^\nu = \begin{pmatrix} 0 & m_D \\ m_D & m_{RR} \end{pmatrix}$$

if $\det(M^\nu) = -m_D^2$; $\mathrm{tr}(M^\nu) = m_{RR} \gg m_D$:

$$M^\nu \sim \begin{pmatrix} -m_D^2/M_{RR} & 0 \\ 0 & M_{RR} \end{pmatrix}$$

Seesaw mechanism: mixing

2 generation SD example

$$m_\nu = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix} \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}^{-1} \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}^T$$

$$= 1/M_1 \begin{pmatrix} a_1^2 & a_1 a_2 \\ a_2 a_1 & a_2^2 \end{pmatrix} + 1/M_2 \begin{pmatrix} b_1^2 & b_1 b_2 \\ b_2 b_1 & b_2^2 \end{pmatrix}$$

if $M_1 \ll M_2$:

- heaviest eigenstate $\sim (a_1, a_2)$ (mass $\propto 1/M_1$)
- lightest eigenstate $\sim (b_1, b_2)$ mass $\propto 1/M_2$)

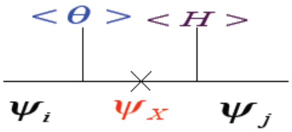
Broken family symmetry

Messengers

- Hierarchical structure suggests broken symmetry ($\langle\theta\rangle \neq 0$)
- Suppresions can arise through messengers (Ψ_X)

Froggatt-Nielsen

$$m_{ij} = \frac{\langle\theta\rangle\langle H\rangle}{M_X}$$



Abelian?

Simple $U(1)$ example

Field	$U(1)$
H	**0**
θ	$-$**1**
d_3	**0**
d_3^c	**0**
d_2	**1**
d_2^c	**1**
d_1	**2**
d_1^c	**2**

Respective mass matrix

$$M^d = m_b \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}$$

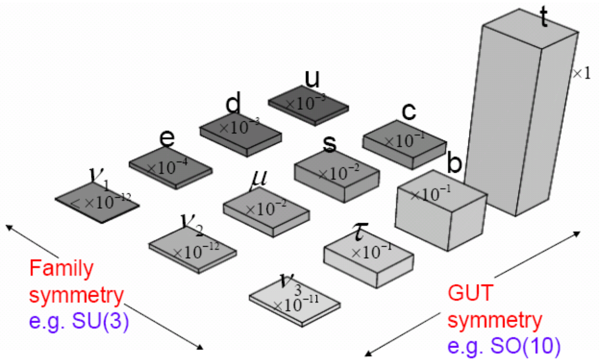
$$\frac{\langle\theta\rangle}{M_X} = \epsilon$$

Non-Abelian?

Reasons

- SM: F.S. $\subset U(3)^6$; $SO(10)$: F.S. $\subset U(3)$
- Specific features (e.g. $M^d_{23} \sim \epsilon^2$) easier to explain
- Lepton (near) HPS mixing strongly suggests non-Abelian

$SO(10) \times SU(3)$?



Objectives

Model features

- Straightforward to embed into GUT / String unification
- Explains observed fermion data (3 generations etc.)
- Explains near HPS lepton mixing

GUT

Pati-Salam

- $SO(10) \rightarrow SU(4) \times SU(2)_L \times SU(2)_R$

GUT leaves some hints

- $SU(4) : q \leftrightarrow l$: $M^d \leftrightarrow M^e$ $(\epsilon_d \leftrightarrow \epsilon_e)$; $M^u \leftrightarrow M^\nu_D$
- $SU(2)_R$: $M^d \leftrightarrow M^u$ $(\epsilon_d \leftrightarrow \epsilon_u)$; $M^e \leftrightarrow M^\nu_D$

$SU(2)_R$ breaking associated with $\epsilon_d > \epsilon_u$

Family assignments

$\bar{\phi}$ and ψ

- The fermions ψ_i, ψ^c_j are triplets of the family symmetry
- The flavons $\bar{\phi}^i_A$ are anti-triplets
- Invariant mass terms: $\bar{\phi}^i_A \psi_i \bar{\phi}^j_B \psi^c_j H$

Desired vevs

$$\langle \bar{\phi}_3 \rangle \propto (0, 0, 1)$$
$$\langle \bar{\phi}_{23} \rangle \propto (0, 1, -1)$$
$$\langle \bar{\phi}_{123} \rangle \propto (1, 1, 1)$$

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Georgi-Jarlskog field

H_{45}

H_{45} (a 45 of $SO(10)$) acquiring a vev:

$$\langle H_{45} \rangle \propto Y = T_{3_R} + (B-L)/2$$

$Y^{d_R} = -1/2 + 1/6 = -1/3$
$Y^{e_R} = -1/2 - 1/2 = -1$
$Y^{\nu_R} = 1/2 - 1/2 = 0$
H_{45} coupling to second generation $\rightarrow m_s/m_\mu = 1/3$

Yukawa superpotential

Leading order terms

$$P_Y \sim (\bar{\phi}_3^i \psi_i)(\bar{\phi}_3^j \psi_j^c) H$$

$$+(\bar{\phi}_{23}^i \psi_i)(\bar{\phi}_{23}^j \psi_j^c) H H_{45}$$

$$+(\bar{\phi}_{23}^i \psi_i)(\bar{\phi}_{123}^j \psi_j^c) H$$

$$+(\bar{\phi}_{123}^i \psi_i)(\bar{\phi}_{23}^j \psi_j^c) H$$

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Mass matrices 1

Term by term

$$P_Y \sim (\bar{\phi}_3^i \psi_i)(\bar{\phi}_3^j \psi_j^c)H$$

Respective Dirac mass matrix

$$M^f = m_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Mass matrices 2

Term by term

$$+(\bar{\phi}^i_{23}\psi_i)(\bar{\phi}^j_{23}\psi^c_j)HH_{45}$$

Respective Dirac mass matrix

$$M^f = m_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & Y^f\epsilon^2 & -Y^f\epsilon^2 \\ 0 & -Y^f\epsilon^2 & 1+Y^f\epsilon^2 \end{pmatrix}$$

Mass matrices 3

Term by term

$$+(\bar{\phi}^i_{23}\psi_i)(\bar{\phi}^j_{123}\psi^c_j)H$$

Respective Dirac mass matrix

$$M^f = m_f \begin{pmatrix} 0 & 0 & 0 \\ \epsilon^3 & Y^f\epsilon^2 + \epsilon^3 & -Y^f\epsilon^2 + \epsilon^3 \\ -\epsilon^3 & -Y^f\epsilon^2 - \epsilon^3 & 1 + Y^f\epsilon^2 - \epsilon^3 \end{pmatrix}$$

And so on...

Unwanted terms?

(Not) spoiling the Yukawa terms

$$P_{spoil} \sim (\bar{\phi}_3^i \psi_i)(\bar{\phi}_{23}^j \psi_j^c) H$$

- Hierarchy spoiled
- Terms like these must be forbidden by added symmetry

Added symmetry (reduced)

Field	$U(1)$
ψ	**0**
ψ^c	**0**
H	**0**
H_{45}	**2**
$\bar{\phi}_3$	**0**
$\bar{\phi}_{23}$	**-1**
$\bar{\phi}_{123}$	**1**

Seesaw and SD revisited

$M_1 < M_2 \ll M_3$

$$m_\nu = \begin{pmatrix} b_1 & c_1 & . \\ b_2 & c_2 & . \\ b_3 & c_3 & . \end{pmatrix} \begin{pmatrix} M_1^{-1} & 0 & 0 \\ 0 & M_2^{-1} & 0 \\ 0 & 0 & M_3^{-1} \end{pmatrix} \begin{pmatrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \\ . & . & . \end{pmatrix}$$

Wanted ν Dirac matrix

$$M_D^\nu = \begin{pmatrix} b_1 & c_1 & . \\ b_2 & c_2 & . \\ b_3 & c_3 & . \end{pmatrix} \propto \begin{pmatrix} 0 & c & . \\ b & c & . \\ -b & c & . \end{pmatrix}$$

Effective neutrino Lagrangian

Effective terms

$$P_\nu \sim \lambda_3(\bar{\phi}^i_{23}\nu_i)(\bar{\phi}^j_{23}\nu_j) \rightarrow @$$

$$+\lambda_2(\bar{\phi}^i_{123}\nu_i)(\bar{\phi}^j_{123}\nu_j) \rightarrow \odot$$

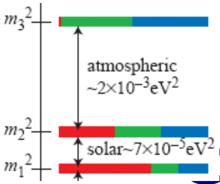
Enforced by effective symmetry:

- $\bar{\phi}_{23} \rightarrow -\bar{\phi}_{23}$
- $\bar{\phi}_{123} \rightarrow \bar{\phi}_{123}$

Vevs reminder

$$\langle\bar{\phi}_{23}\rangle \propto (0, 1, -1)$$

$$\langle\bar{\phi}_{123}\rangle \propto (1, 1, 1)$$



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Getting the effective terms

Leading order terms

$$P_Y \sim (\bar{\phi}^i_{23}\nu_i)(\bar{\phi}^j_{123}\nu^c_j)H \rightarrow ©$$

$$+(\bar{\phi}^i_{123}\nu_i)(\bar{\phi}^j_{23}\nu^c_j)H \rightarrow \odot$$

$$+(\bar{\phi}^i_3\nu_i)(\bar{\phi}^j_3\nu^c_j)H \rightarrow decouple$$

$$P_M \sim [(\bar{\phi}_{123}\nu^c)(\bar{\phi}_{123}\nu^c)](\theta\phi_{123})(\theta\phi_{123}) \rightarrow ©$$

$$+[(\bar{\phi}_{23}\nu^c)(\bar{\phi}_{23}\nu^c)](\theta\phi_{123})(\theta\phi_3) \rightarrow \odot$$

$$+(\theta\nu^c)(\theta\nu^c) \rightarrow decouple$$

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More unwanted terms

(Not) spoiling the effective terms

$$P_{spoil} \sim (\bar{\phi}^i_3 \nu_i)(\bar{\phi}^j_{23} \nu^c_j) H$$

$$\rightarrow P_\nu \sim (\bar{\phi}^i_3 \nu_i)(\bar{\phi}^j_3 \nu_j)$$

Need added effective symmetry e.g.:

- $\bar{\phi}_3 \rightarrow i \bar{\phi}_3$

Again the additional symmetry must be used.

Δ(27) invariants

Transformation properties

Field	Z_3	Z'_3
ϕ_1	ϕ_1	ϕ_3
ϕ_2	$\alpha\phi_2$	ϕ_1
ϕ_3	$(\alpha)^2\phi_3$	ϕ_2

- Allowed: all $SU(3)_f$ invariants (e.g. $\bar{\phi}^i_A\psi_i$)
- Disallowed: some $\Delta(12)$ invariants (e.g. $\psi_i\psi^c_i$)
- Allowed: higher order invariants (e.g. $\bar{\phi}^i\phi_i\bar{\phi}^i\phi_i$)

Δ(27) family symmetry

Why is it interesting?

- small subgroup of $SU(3)_f$
- distinct "triplets" and "anti-triplets"
- forbids the "quadratic" invariant $\psi_i \psi_i^c$
- added invariants useful for vacuum alignment
- discrete family symmetries don't have associated D-terms

Breaking the symmetry

2 generation example

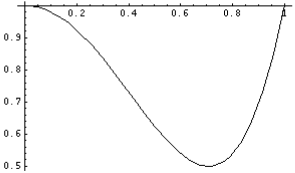
- $V \sim -m^2(\varphi^i \varphi_i^\dagger)$
- Symmetry is continuous, continuum of vacuum states
- No specified direction, just magnitude

Alignment by soft terms example

2 generation example

- Symmetry is discrete, breaks the continuum of vacuum states
- $V \sim -m^2(\varphi^i \varphi_i^\dagger)$
  $\pm m_{3/2}^2(\varphi^i \varphi_i^\dagger \varphi^i \varphi_i^\dagger)$
- Extrema of $|\varphi_1|^4 + |\varphi_2|^4$ with constraint of constant magnitude:
- Positive: $\propto (1,1) \rightarrow V \sim +2v^4/4$
- Negative: $\propto (0,1) \rightarrow V \sim -v^4$

Plot[{x^4 + (1 - x^2)^2}, {x, 0, 1}]



- Graphics -

Minimize[x^4 + (1 - x^2)^2, x]

$\left\{\frac{1}{2}, \left\{x \rightarrow \frac{1}{\sqrt{2}}\right\}\right\}$

$\bar{\phi}_3$ and $\bar{\phi}_{123}$

Quartic term minimisation

- $V \sim -m^2(\varphi^i \varphi_i^\dagger)$
  $\pm m_{3/2}^2(\varphi^i \varphi_i^\dagger \varphi^i \varphi_i^\dagger)$
- For $\varphi = \bar{\phi}_{123}$, positive coefficient yelds $\langle \bar{\phi}_{123} \rangle \propto (1, 1, 1)$
- For $\varphi = \bar{\phi}_3$, negative coefficient yelds $\langle \bar{\phi}_3 \rangle \propto (0, 0, 1)$

Relative alignment

Aligning $\bar{\phi}_{23}$

With $SU(3)$ invariant higher order terms:

- Containing $\bar{\phi}^i_{23} \phi_{123_i}$ with positive coupling
- Term keeping the vanishing component away from the $\bar{\phi}_3$ direction

$\rightarrow \langle \bar{\phi}_{23} \rangle \propto (0, 1, -1)$

The predictions

PMNS angles

- $s^2_{12} \approx \frac{1}{3}{}^{+0.052}_{-0.048}$
- $s^2_{23} \approx \frac{1}{2}{}^{+0.061}_{-0.058}$
- $s^2_{13} \approx 0.0028$

Mixing angles values measured experimentally

- $s^2_{12} = 0.30 \pm 0.08$
- $s^2_{23} = 0.50 \pm 0.18$
- $s^2_{13} < 0.047$

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$\Delta(27)$ family symmetry

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