Outline of this talk

- Introduction
  - Top quark mass
  - The Tevatron and its collider experiments
  - Previous measurements
  - Experimental challenges
  - The jet energy scale

- Run II Top mass measurements:
  - Template and matrix element techniques
  - The best results from Run II of the Tevatron (CDF)
  - Future

- Conclusions
Introduction
The Top Quark

- Discovered 10 years ago
- Existence is required by the Standard Model (SM), but striking characteristics:
  - Huge mass!
    \[ M_{\text{top}} \sim 175 \text{ GeV} \]
  - Decays before hadronizing
    \[ \tau_{\text{top}} \sim 10^{-24} \text{ sec} \]
  - Special role?
  - Likely related to EWSB
  - Effects from new physics could be more apparent in the top sector
Fermilab’s Tevatron

- World’s highest particle energy collisions
  - ~4 miles circumference protons-antiprotons

- 2 multi-purpose detectors
  - DØ and CDF

  - $\sqrt{s} = 1.8$ TeV
  - Integrated luminosity 120 pb$^{-1}$

- Run II (2001-present)
  - $\sqrt{s} = 1.96$ TeV
  - Integrated luminosity by June, 05:
    - Delivered $> 1$ fb$^{-1}$
    - On tape $> 800$ pb$^{-1}$
    - Analyzed up to $\sim 350$ pb$^{-1}$
Tevatron Performance

Year
Month
2002
1
4
7
10
2003
1
4
7
10
1
4
7
2004
2005
1
4

Total Luminosity (pb⁻¹)

1000
800
600
400
200
0

Store Number

1000
1500
2000
2500
3000
3500
4000

Delivered To tape

1 fb⁻¹
Why Measure the Top Quark Mass?

- \( M_{\text{top}} \) is related to standard model observables and parameters through loop diagrams.

- Many corrections to standard model predictions of electroweak measurements are dominated by \( M_{\text{top}} \).

- **High precision in** \( M_{\text{top}} \) **measurements is needed for:**
  - Consistency check of SM parameters
  - Constrain unknown parameters: \( M_{\text{Higgs}} \)

![Diagram showing the relationship between \( m_W \), \( m_t \), and \( m_H \) with error ellipses for LEP1, SLD data and LEP2 (preliminary), pp data.](image)
Published Top Quark Masses

- Green circle: From EW fits
- Blue triangle: Direct measurement by CDF
- Red triangle: Direct measurement by D0
- Purple square: World average from direct measurements

In 2010 this point should have negligible uncertainties

Lower limits prior to discovery in $e^+e^-$ and $pp$ collisions and from the $W$ boson width

Year:
- 1989
- 1991
- 1993
- 1995
- 1997
- 1999
- 2001
- 2003
- 2005

Top Mass (GeV/c^2): 0 to 240
CDF and D0 applied/created different extraction techniques

At the end of Run I, one of the systematic uncertainties (jet energy scale) was as large as the statistical uncertainty

Run II goal per experiment

\[ \delta M_{top} \approx 2 \text{GeV} / c^2 \quad \text{with} \quad \int L dt \approx 4 - 8 \text{fb}^{-1} \]
Mainly produced in pairs via strong interaction

\[ \sigma(\bar{p} p \to \bar{t} t @ M_{top} = 178 GeV) \approx 6.1 \text{ pb} \]

\( \sim 85\% \)

\( \sim 15\% \)

\[ \approx 1 \text{ top event every 10 BILLION inelastic collisions} \]

Top produced via EWK interaction has \( \sim 1/3 \) of ttbar cross section and has not been observed yet
Final States

- Top decays via electroweak interactions,

\[ Br(t \rightarrow Wb) \sim 100\% \]

- The final state is characterized by the decay of the W bosons

- **Dilepton** (lepton = e or \(\mu\))
  - (5%): 2 leptons, 2 neutrinos

- **Lepton+Jets** (lepton = e or \(\mu\))
  - (30%): 2 b quarks, 2 light quarks, 1 lepton, 1 neutrino

- **All-Jets** (44%):
  - 2 b quarks, 4 light quarks
General characteristics of the events:
- Energetic and central objects
- Lepton = electron or muon
- Two b-quarks
- More jets from ISR/FSR
- Neutrino is undetected

Typical selection:
- $P_T > 15\text{ GeV}$, $|\eta| < 2$
- Well identified $e$ or/and $\mu$
- Some identified $b$-jets
- Several reconstructed jets
- Large missing $E_T$
Run II CDF Detector

Top is a very rich signature: measurements use all pieces of the detector:

- Inner Silicon Tracking
- Tracking Chambers
- Solenoid
- EM and Hadronic Calorimeters
- Muon Detectors

Identify jets from $b$ quarks
Reduce background contributions
Reduce jet combinatorics
Difficult Measurement

- **Background contamination:**
  - Events with real W, Z bosons: W+jets, Z+jets, WW, WZ, ZZ
  - Events with misidentified isolated leptons

- **Not perfect measurements:**
  - Large energy resolution for jets $\Rightarrow 84\%/\sqrt{E_T}$
  - Different ways to assign jets to partons:
    - 2 jets (dilepton) $\Rightarrow$ 2 combinations
    - 4 jets (lepton+jets) $\Rightarrow$ 24 combinations
    - 6 jets (all jets) $\Rightarrow$ 720 combinations
  - Jet energy scale of jets is known to a few percent

- **Uncertainties:**
  - Statistical uncertainty
  - Systematic uncertainty
  - Bias
Jets are complicated objects measured by calorimeter towers and defined by a clustering algorithm.

Instrumental effects:
- Non-linear calorimeter
- Non-instrumented regions
- Non-compensating calorimeter

Algorithm effects:
- Might not capture all particles
- Cone 0.4 use in top physics

Physics effects
- Spectator interactions
- Radiation
- Multiple ppbar interactions
- There are different types of jets

Precision on the determination of jet energies is required by the top mass analyses.
CDF Jet Energy Scale Corrections

- **Calibrate** the calorimeter energy scale

- **Calibrate the calorimeter simulation:** Measure energy \( E \) in the calorimeter for a known particle momentum \( p \) using in situ and/or test beam measurements

- **Jets:** cluster tower energies using jet cone algorithm

- **Corrections:**
  - Obtain **calorimeter-to-particle** corrections using simulated dijet events \( C_{Abs} \)
  - Obtain **particle-to-parton** corrections using Monte Carlo shower in dijet events \( C_{OOC} \)
  - Make jet energy scale **uniform in \( \eta \)** using dijet balance (data and Monte Carlo) \( C_{Rel} \)
  - **Pile-up and underlying event** are measured from data \( C_{MI} \) and \( C_{UE} \)

\[
P_{Parton} = \left[ P_{\text{jet}}^{\text{Calorimeter}} \times (C_{Rel} - C_{MI}) \right] \times C_{Abs} - C_{UE} + C_{OOC}
\]
**Jet Energy Scale Uncertainties**

**JES**: measures how incorrect is our nominal jet energy measurement

**Uncertainties at each correction level:**
- Differences between Monte Carlo and data
- Uncertainties from the method used to obtain the corrections

![Graph showing jet energy scale uncertainties](image)
Jet Energy Scale Uncertainties - Simulation

- In the central region

- Uncertainties from:
  - Hadrons (70% of jet)
    - $p<12$ GeV 2%
    - $12<p<20$ GeV 3%
    - $p>20$ GeV 4%
  - EM particles (30% of jet)
    - $p<60$ GeV 1.5%

- These are later propagated to uncertainties on the jet energy scale
Our understanding of the differences between data and MC in photon+jet is about 3%.

Included in particle-to-parton uncertainties.

\[ p_T^{\text{balance}} = \frac{p_T^{\text{jet}}}{p_T^{\gamma}} - 1 \]
Run II Top Mass Measurements
## Two Different Techniques

<table>
<thead>
<tr>
<th>Template Analyses</th>
<th>Matrix Element Analyses</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Choose a <strong>variable strongly correlated</strong> with the mass of the top quark (<em>Reconstructed</em> $M_{top}$)</td>
<td>• Build a <strong>probability</strong> (ME+PDF+transfer functions) integrating over unmeasured quantities (signal and background)</td>
</tr>
<tr>
<td>• Create <strong>templates</strong> using events simulated with different $M_{top}$ values (+ background)</td>
<td>• Evaluate the <strong>probability of each event</strong> as a function of the top mass</td>
</tr>
<tr>
<td>• Perform maximum <strong>likelihood fit</strong> to extract measured mass</td>
<td>• Calibrate (or not) using the simulation</td>
</tr>
</tbody>
</table>
Template Analysis Overview

- **Signal MC**
- **Background MC**
- **Data**

$\chi^2$ fit: Finds best **Reconstructed $M_{top}$** for the best jet-parton assignment, $p_{Z\nu}$ per event

- **Reconstructed $M_{top}$**

Likelihood fit = $L_{shape} \times L_{background}$

Finds best signal + background templates to fit the data
Increase the Statistical Power

- Improve statistical power of the method dividing the sample in 4 subsamples that have different background contamination and different sensitivity to the top mass

  More b tags are better
  - Increases S:B
  - More “golden” events, where correct jet-parton assignment is found

<table>
<thead>
<tr>
<th>Category</th>
<th>2-tag</th>
<th>1-tag(T)</th>
<th>1-tag(L)</th>
<th>0-tag</th>
</tr>
</thead>
<tbody>
<tr>
<td>j1-j3</td>
<td>$E_T &gt; 15$</td>
<td>$E_T &gt; 15$</td>
<td>$E_T &gt; 15$</td>
<td>$E_T &gt; 21$</td>
</tr>
<tr>
<td>j4</td>
<td>$E_T &gt; 8$</td>
<td>$E_T &gt; 15$</td>
<td>$15 &gt; E_T &gt; 8$</td>
<td>$E_T &gt; 21$</td>
</tr>
<tr>
<td># events</td>
<td>13</td>
<td>56</td>
<td>25</td>
<td>41</td>
</tr>
<tr>
<td>S:B</td>
<td>18:1</td>
<td>4.2:1</td>
<td>1.2:1</td>
<td>0.9:1</td>
</tr>
</tbody>
</table>

More correct combinations with b-tags!

---

6/21/05
Florence Canelli - UCLA
Result on Data

\[
\int \mathcal{L} dt \approx 318 \text{ pb}^{-1}
\]

\[
\begin{align*}
L_{\text{total}} &= L_{\text{2-tag}} \times L_{\text{T-tag}} \times L_{\text{1-tag L}} \times L_{\text{0-tag}} \\
M_{\text{top}} &= 173.2^{+2.9}_{-2.8} \text{ (stat.)} \pm 3.4 \text{ (syst.) GeV} / c^2
\end{align*}
\]
Result on Data

\[ L_{total} = L_{2\text{tag}} \times L_{1\text{tagT}} \times L_{1\text{tagL}} \times L_{0\text{tag}} \]

\[ M_{top} = 173.2^{+2.9}_{-2.8} \text{(stat.)} \pm 3.4 \text{(syst.)} \text{GeV} / c^2 \]

with \[ \int L dt \approx 318 \text{ pb}^{-1} \]
# Systematic Uncertainties Summary

## CDF Run II Preliminary (318 pb⁻¹)

<table>
<thead>
<tr>
<th>Systematic Source</th>
<th>Uncertainty (GeV/c²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jet Energy Scale</td>
<td>3.1</td>
</tr>
<tr>
<td>B-jet energy</td>
<td>0.6</td>
</tr>
<tr>
<td>Initial State Radiation</td>
<td>0.4</td>
</tr>
<tr>
<td>Final State Radiation</td>
<td>0.4</td>
</tr>
<tr>
<td>Parton Distribution Functions</td>
<td>0.4</td>
</tr>
<tr>
<td>Generators</td>
<td>0.3</td>
</tr>
<tr>
<td>Background Shape</td>
<td>1.0</td>
</tr>
<tr>
<td>MC statistics</td>
<td>0.4</td>
</tr>
<tr>
<td>B-tagging</td>
<td>0.2</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>3.4</strong></td>
</tr>
</tbody>
</table>

Systematics dominated by jet energy scale

## Relative to Central
- Corrections to Particles: 2.2
- Corrections to Partons: 2.1
- **Total**: 3.1
Decrease the JES uncertainty

Can we use $W \rightarrow jj$ mass resonance ($M_{jj}$) to constrain the jet energy scale uncertainty (JES)?

$\text{JES} = \text{how wrong our jet calibration is}$

Advantages:

$\text{JES uncertainty begins to scale directly with statistics!}$

$\text{In-situ JES uncertainties: correlations, same environment}$

So use $W \rightarrow jj$ to improve understanding of q jets, therefore b jets, therefore $M_{top}$.
Using $M_{jj}$ to Measure JES

- Similar method to the extraction of $M_{\text{top}}$ using \textit{Reconstructed} $M_{\text{top}}$

- Build templates using the invariant mass $M_{jj}$ of all non-tagged jet pairs for different $\Delta JES$

- Assume $M_W$ (80.4 GeV/$c^2$) and measure the jet energy scale: $\Delta JES$

- Units of $\sigma$: correspond to one standard deviation of jet energy uncertainty

- We could measure $\Delta$ JES assuming a top mass: at 318 pb$^{-1}$ we expect $1.2\sigma$
2-D Template Analysis

- Measurement of $\Delta$JES and $M_{\text{top}}$ simultaneously to take into account correlations: 2-D fit
  - Reconstructed $M_{\text{top}}$ and $M_{jj}$ templates
  - $M_{jj}$ very sensitive to JES, no dependent on $M_{\text{top}}$
  - Each template depend on $M_{\text{top}}$ and $\Delta$JES
  - A priori knowledge is used as a constraint in likelihood (~3%)

$$-\ln L_{\text{JES}} = \frac{(JES - 0)^2}{2\sigma_{\text{JES}}^2}$$

$$L_{\text{total}} = L_{2\text{tag}} \times L_{\text{tagT}} \times L_{\text{tagL}} \times L_{0\text{tag}} \times L_{\text{JES}}$$

(JES nominal=0) Reconstructed $M_{\text{top}}$

(M$_{\text{top}}$=180GeV/c$^2$)
Results in Data

CDF Run II Preliminary (318 pb⁻¹):

\[ M_{top} = 173.5^{+3.7}_{-3.6} \text{(stat. + JES)} \pm 1.7 \text{(syst.)} \text{GeV / } c^2 \]

- Breaks down to \( ^{+2.7}_{-2.6} \text{(stat)} \pm 2.5 \text{(JES)} \)
- 20% improvement in uncertainty due to JES!

Compare to 1-D template expected \( \text{stat+JES} \sim 5.2 \text{ GeV} \)

CDF Run II Preliminary (318 pb⁻¹):

- Median uncert. = 4.2 GeV/c²
- Data: 9.2% of expts w. smaller uncert.
Future Analysis with $M_{jj}$

- With current data the JES prior has a large effect on the final uncertainty.
- Improvement to traditional calibrations of JES expected to be limited in the future.
- Can reach JES uncertainty below 1 GeV/c$^2$ in Run II.
- Total $M_{\text{top}}$ uncertainty: can reach 2 GeV/c$^2$.

Jet Energy Scale from $W\rightarrow jj$

CDF Run II Preliminary

$M_{\text{top}}$ Systematic Uncertainty from $W\rightarrow jj$ Energy Scale Only

Today with JES prior
Matrix Element Overview

In general, the best estimate of a parameter is achieved by comparing the events in data with a probability from theory through maximizing a likelihood:

\[ \hat{L}(M_{\text{top}}) = e^{-N \int \tilde{P}(x; M_{\text{top}}) \, dx} \prod_{i=1}^{N} \tilde{P}(x_i; M_{\text{top}}) \]

where \( x \) is a set of measured variables.

For example:

\[ \tilde{P}(x; c_1, c_2, M_{\text{top}}) = c_1 P_{\text{signal}}(x; M_{\text{top}}) + c_2 P_{\text{background}}(x) \]

Most precise result from Run I (D0) used this method.
Dynamical Likelihood Method

- Maximum likelihood method, where likelihood is built up for each event $i$ as below

  - Integrate over $z1, z2, \text{parton } p_T$
  - Parton Distribution Functions + ISR
  - Matrix element provides complete dynamical event information
  - Replace $p_z^n$ with $W$ propagator factor
  - Transfer functions connect jets to partons

$$L^i(M_{\text{top}}) = \sum I_t \sum I_x \int \frac{2\pi^4}{\text{Flux}} F(z_a, z_b, p_T) \left| M \right|^2 \delta(s_w - (\ell + \nu)^2) w(I_t, x | y; M_{\text{top}}) dx ds_w$$

- Top mass: maximize $\Pi_i L^i(M_{\text{top}})$

- Uses Monte Carlo to correct biases from the background (only 1-tag or more tags sample)
CDF Run II Preliminary (318 pb\(^{-1}\)):

\[ M_{top} = 173.8^{+2.7}_{-2.5} \text{(stat.)} \pm 3.3 \text{(syst.) GeV} / c^2 \]
Comparison of Results

Best Run I result

\[ M_{\text{top}} = 180.1 \pm 5.3 \text{GeV} / c^2 \]

with 125 pb\(^{-1}\)

World average

\[ M_{\text{top}} = 178.1 \pm 4.3 \text{GeV} / c^2 \]

Best Run II results

<table>
<thead>
<tr>
<th>Method</th>
<th>Template</th>
<th>DLM</th>
</tr>
</thead>
<tbody>
<tr>
<td>318 pb(^{-1}) result</td>
<td>173.5 ± 4.1 (“2D”) GeV/c(^2)</td>
<td>173.8 ± 4.2 GeV/c(^2)</td>
</tr>
<tr>
<td>Selection</td>
<td>(\geq 4) jets</td>
<td>= 4 jets, (\geq 1) tag</td>
</tr>
<tr>
<td>Combinatorics</td>
<td>Best (\chi^2)</td>
<td>Use all</td>
</tr>
<tr>
<td>JES</td>
<td>(W \rightarrow jj)</td>
<td>None yet</td>
</tr>
<tr>
<td>Background</td>
<td>Template</td>
<td>Mapping</td>
</tr>
</tbody>
</table>
Implications for Higgs

Constrain on the SM Higgs mass (only based on this measurement):

\[ M_{\text{Higgs}} = 94^{+54}_{-35} \text{GeV} / c^2 \]

(Run I world ave. constraint: \( M_H = 126^{+73}_{-48} \text{GeV} / c^2 \))
Conclusions

- We are moving from discovery to precision measurements of top quark mass

- Challenging final states:
  - Requires to fully use detector capabilities

- Method of extraction of observables are getting far more sophisticated:
  - Making maximal use of the statistics
  - Smarter ways to account for systematic uncertainties

Important tools for LHC

- What is coming? New analysis and in different channels, combination with D0

- We are in good shape and moving towards the Run II goal of 2 GeV/c^2 in the top mass uncertainty

\[ M_{top} = 173.5 \pm 4.1 \text{GeV} / c^2 \]
Backups
Event-by-Event Mass Fitter

- Try all jet-parton assignments
  - Assign b-tag jets to b-quarks
- Keep the mass from assignment yielding lowest $\chi^2$

Reconstructed top mass is a free parameter

$$\chi^2 = \sum_{i=1,4 \text{ jets}} \frac{(\hat{p}_T^i - p_T^i)^2}{\sigma_i} + \sum_{j=x,y} \frac{(\hat{p}_T^{UE} - p_T^{UE})^2}{\sigma_j} + \gamma^2 + \gamma^2 + \gamma^2 + \gamma^2$$

$\gamma = \frac{(m_{jj} - m_W)^2}{\Gamma_W^2} + \frac{(m_{ll} - m_W)^2}{\Gamma_W^2} + \frac{(m_{bjj} - m_t)^2}{\Gamma_t^2} + \frac{(m_{blv} - m_t)^2}{\Gamma_t^2}$
Get analytical functions of reconstructed mass as a function of true top mass

Selected templates (GeV)

Background Templates

- **Background sources:**
  - >=1-tag events:
    - W+jets mistag
    - QCD multijets
    - W+heavy flavor
    - Others (WW/WZ, single top)
  - 0-tag events:
    - W+jets
    - QCD multijets

Use MC events to get background expected distributions:

- **2-tag**
  - Events/5 GeV/c^2
  
- **1-tag (T)**
  - Events/5 GeV/c^2

- **1-tag (L)**
  - Events/5 GeV/c^2

- **0-tag**
  - Events/5 GeV/c^2
Can we use $W \rightarrow jj$ mass resonance ($M_{jj}$) to constrain the jet energy scale uncertainty (JES)?

- JES = how wrong our jet calibration is
- $M_{top}$ measurement sensitive primarily to energy scale of b jets ($W$ mass constraint in $\chi^2$)

**Advantages:**
- JES uncertainty begins to scale directly with statistics!
- In-situ JES uncertainties
- But studies show most uncertainty is shared by light quark and b jets
- Only 0.6 GeV/c$^2$ additional uncertainty on $M_{top}$ due to b-jet-specific systematics

So use $W \rightarrow jj$ to **improve understanding** of q jets, therefore b jets, therefore $M_{top}$
Unbinned likelihood fit:

- Compare data
  reconstructed mass
distributions with templates
  extracted from Monte Carlo
- Background constrained to
  expectations
- Combined fit: multiply
  subsamples likelihood

Likelihood fit sanity check:

Method check: $M_t^{\text{out}}$ vs. $M_t^{\text{in}}$

CDF Run II Preliminary
Red line: $y=x$
Matrix Element Technique

- Determine mass of the top quark evaluating a probability using all the variables in the event, integrate over all unknowns.

\[ P(x; M_{top}) = \frac{1}{\sigma} \int d^n \sigma(y; M_{top}) \, dq_1 \, dq_2 \, f(q_1) \, f(q_2) \, W(x, y) \]

- Sum over all permutations of jets and neutrino solutions.

- Background process probabilities are or not be explicitly included in the likelihood.

- Top mass: maximize \( \prod_i P_i(x; M_{top}) \)
  - Each event has its own probability.
  - Correct permutation is always considered (along with the other eleven).
  - All features of individual events are included, thereby well measured events contribute more information than poorly measured events.

\( d^n \sigma \) is the differential cross section; LO Matrix element

\( W(y, x) \) is the probability that a parton level set of variables \( y \) will be measured as a set of variables \( x \).

\( f(q) \) is the probability distribution than a parton will have a momentum \( q \).
Implications for SUSY

experimental errors 68% CL:
- LEP2/Tevatron (today, m_t; CDF RunII)
- Tevatron/LHC
- LC+GigaZ

M_{W} [GeV]

M_{H} = 113 GeV
M_{H} = 400 GeV

m_{t} [GeV]
Systematics - PDF & Background

- PDF systematics
  - Established procedure in place
  - No improvements expected except as uncertainties on PDFs themselves get smaller

- Background
  - Systematics on background are starting to get important!
  - Lots of aspects of background shape and normalization that can be studied
  - Expect improvements to the extent these systematics are non-negligible
Systematics - Radiation

- Method in hand to use Drell-Yan events to understand and constrain extra jets from ISR
  - Constraint scales with luminosity
  - Easily extendible to FSR
  - BUT that’s not the whole NLO story
  - Need work to understand whether add’l uncertainty is non-negligible

**MC@NLO**
Mapping function (21% background)

Expected statistical uncertainty